Evidence for SU(3) octet mixing

Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2E9
(Received 1 June 1976; revised manuscript received 5 August 1976)

The strong decays of the two strange axial-vector mesons Q_{c}(1289) and Q_{c}(1404) are examined within the context of SU(3). It is found that the decays can be successfully explained by treating the Q's as mixed states of two pure C = +1 and C = −1 SU(3) octets. The vector–axial-vector–pseudoscalar S-wave coupling constants are calculated to be approximately 2.8 GeV for the A_{1} multiplet and 4.2 GeV for the B multiplet, and the mixing angle approximately 48°.

Evidence has recently been presented supporting the existence of strangeness–one axial-vector mesons in the region 1300–1400 MeV. These particles, called Q_{c} and Q_{b}, are observed from partial-wave analysis to have masses 1289 and 1404 MeV, respectively. It is tempting to assign these particles to two different SU(3) octets whose I = 1 members are the A_{1}(1100) and B(1235). The charge-conjugation parity of the neutral and non-strange members of the Q_{c}, A_{1} multiplet would be even, while that of the Q_{b}, B multiplet would be odd. This assignment would be favored by the SU(6) ⊗ O(3) quark model, which predicts two axial-vector multiplets with even and odd parity.

However, the decays of the Q's are not easily incorporated into a scheme with approximate SU(3) symmetry of the coupling constants. For example, the ratio $\Gamma(Q_{c} \to pK)/\Gamma(Q_{c} \to K^+\pi)$ should be about 1/3 according to SU(3) and phase-space considerations; however, it is observed to be at least 10, even with the most liberal interpretation of errors. A possible way around this difficulty would be to construct a model with SU(3)-symmetry breaking. In a model of SU(3) breaking via a $\lambda_{a}$ spurion, one finds that in order to suppress the $Q_{c} \to K^+\pi$ decay one needs large SU(3) breaking, that is, the SU(3)-breaking parameters are as large as the SU(3)-preserving ones. This is certainly not in accord with our previous experience with SU(3).

The near degeneracy of the mean mass of the two axial-vector multiplets suggests the possibility of mixing between them. This idea was proposed some time ago by several authors when the characteristics of the Q's were much less well defined. If, as has been suggested, the two multiplets considered here have different C parities, then invariance of the strong interactions under G parity would dictate that only the Q's, which are eigenstates of strangeness and therefore not of G parity, would mix. Thus, the axial-vector–vector–pseudoscalar (AVP henceforth) vertex involving the Q's will contain both f- and d-type coupling.

The Lorentz-covariant decay amplitude for $A_{1} \to VP$ is given by

$$T = g_5 \varepsilon_{A}^{*}\varepsilon_{V}^{*} + g_6 \varepsilon_{A}^{*}\varepsilon_{P}^{*}\varepsilon_{V}^{*},$$

where the $\varepsilon$'s and the $\gamma$'s are the polarizations and momenta of the vector and axial-vector mesons. We express the mixing of the strange members of the A_{1} and B octets via the angle γ:

$$b_{5}(Q_{c}) = i f_{K}^{*} K_{A}^{*} \cos \gamma + d_{K} \varepsilon_{A}^{*} \sin \gamma,$$

$$b_{5}(Q_{b}) = -i f_{K}^{*} K_{A}^{*} \sin \gamma + d_{K} \varepsilon_{B}^{*} \cos \gamma,$$

with similar expressions for $g_{5}$.

Helicity amplitudes proportional to those introduced by Colglazier and Rosner are easily constructed from $b_{5}$ and $g_{5}$:

$$H_{0} = \left[ g_{5} m_{A} q^{2} + (m_{\pi}^{2} + q^{2})^{1/2} g_{5} \right] / m_{\pi},$$

$$H_{1} = g_{5},$$

where $m_{\pi}$ and $m_{A}$ are the masses of the vector and axial-vector mesons, and q is the center-of-mass momentum of the decay process. In terms of $H_{0}$ and $H_{1}$, the decay rates are given simply by

$$\Gamma(A \to VP) = \frac{q}{24\pi m_{A}} \left( H_{0}^{2} + 2H_{1}^{2} \right).$$

A compilation of $B \to \omega \pi$ data yields $g_{5}^{2}/g_{5}^{2} = -2.90$ GeV$^{-2}$. While definitive data on the A_{1} multiplet are lacking, one may estimate $g_{5}/g_{5}$ via a quark-model sum rule involving the $H$'s:

$$\frac{H_{1}}{H_{0}^{2}} = \left( \frac{H_{0}}{H_{1}} \right)_{\omega \pi \to \omega \pi} - 1.$$  

We obtain $g_{5}/g_{5} = 2.58$ GeV$^{-2}$.

The decay processes to which we apply these formulas are listed in Table I. The small values for $\Gamma(Q_{c} \to K^{+}\pi)$ and $\Gamma(Q_{c} \to pK)$ imply that $g_{5} = g_{5}$ and $\gamma = 45^{\circ}$. (These would be equalities if the two rates vanished.) These conditions also imply that $\Gamma(Q_{c} \to pK)/\Gamma(Q_{c} \to K^{+}\pi) = 0.4$, which is plausible if one includes the large systematic error in the $Q_{c} \to pK$ rate. The first five decay rates in the table are used for a minimum-$\chi^{2}$ fit, while the re-
maining three are predictions. The errors chosen for the minimization in the decays of the Q's are the systematic ones. The Q decay rates depend critically on the Q mass, because of the small phase space available. Because there is a 25-MeV systematic uncertainty associated with the Q mass, we have chosen a value of 1300 MeV for our calculations. Raising or lowering the mass by 10 MeV changes the ρK and ωK rates accordingly by about 20%. We show the solutions found for the fit with and without the D-wave contribution included. In the latter case, there are two solutions with roughly the same χ², so both are given. We note that it is a good first approximation to ignore the D-wave contribution. We have listed here only those solutions with positive coupling constants and mixing angle in the first quadrant. Other simple ambiguities exist due to choice of quadrant for γ and sign of $g_A^q/g_E^q$, but they yield the same results for the processes listed in Table I. As more data become available, one will hopefully be able to distinguish between these solutions.

Some A → SP decays of the Q's have also been observed. Simple calculation shows that these rates would be given by

$$\Gamma(Q_{11} \rightarrow P_f S_k) = \frac{2}{3} \frac{q^3}{m_{Q_1}^3} \left( h_A d_{1f} c_{1k} + h_B f_{1f} s_{1k} \right)^2 \quad (6)$$

$$\Gamma(Q_{21} \rightarrow P_f S_k) = \frac{2}{3} \frac{q^3}{m_{Q_2}^3} \left( -h_A d_{1f} s_{1k} + h_B f_{1f} c_{1k} \right)^2 \quad (6)$$

where $h_A$ and $h_B$ are dimensionless coupling constants defined analogously to $g_A$ and $g_E$. The fact that the $\pi K$ channel is more strongly coupled to $Q_1$ than $Q_2$ also supports a nonzero value for the mixing angle, although a numerical analysis is not yet possible.

We are indebted to Dr. T. A. Lasinski and Experimental Group B at SLAC for making their analysis available to us prior to publication.

---

**TABLE I.** Predicted and observed decay widths of the Q's. Solution I, which includes the D-wave contribution, has $g_A^q = 2.78$ GeV, $g_E^q = 4.20$ GeV, and $\gamma = 47.8\%$. Solutions II and III, with no D wave, have $g_A^q = 3.26$ GeV, $g_E^q = 3.57$ GeV, $\gamma = 54.7\%$ and $g_A^q = 2.85$ GeV, $g_E^q = 3.64$ GeV, $\gamma = 45.1\%$. The first error in the observed-width column is statistical, while the second (in parentheses) is systematic. The vector mixing angle is taken to be 37.3°. [D. H. Boal and R. Torgerson, Phys. Rev. D 10, 2991 (1974).]

<table>
<thead>
<tr>
<th>Decay</th>
<th>Predicted width (MeV)</th>
<th>Observed width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1 \rightarrow \rho K$</td>
<td>62.7</td>
<td>59.3</td>
</tr>
<tr>
<td>$Q_1 \rightarrow K^*\pi$</td>
<td>6.9</td>
<td>6.3</td>
</tr>
<tr>
<td>$Q_1 \rightarrow \omega K$</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$Q_1 \rightarrow K^*\pi$</td>
<td>139</td>
<td>144</td>
</tr>
<tr>
<td>$B \rightarrow \omega \pi$</td>
<td>123</td>
<td>123</td>
</tr>
<tr>
<td>$Q_1 \rightarrow \omega K$</td>
<td>18.1</td>
<td>15.1</td>
</tr>
<tr>
<td>$Q_1 \rightarrow \omega K$</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>$A_1 \rightarrow \rho \pi$</td>
<td>155</td>
<td>154</td>
</tr>
</tbody>
</table>

² See Ref. 4.
³ See Particle Data Group, Ref. 3.

---

*Work supported in part by the National Research Council of Canada.
³For a review of previous work, see Particle Data Group, Rev. Mod. Phys. 48, 51 (1976); Yu. Antipov et al., Nucl. Phys. B86, 365 (1975); S. Tovey et al., ibid. B95, 109 (1975); G. Otter et al., ibid. B95, 365 (1975).
⁷See Ref. 5 for a review. For a recent discussion of the role of SU(3) breaking in radiative decays, see Ref. 6 and D. H. Boal, R. H. Graham, and J. W. Moffat, Phys. Rev. Lett. 36, 714 (1976).
¹⁰See Collaglizer and Rosner, Ref. 8.

---