The $\rho^- \to \pi^- \gamma$ Decay Rate and Vector-Meson Dominance (*).

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The recent measurement of the $\rho^- \to \pi^- \gamma$ decay width (1) has caused much concern in that it is approximately three times smaller than anticipated using quark or higher-symmetry models (2). We note that this rate is in direct disagreement with simple vector-meson dominance (VMD) and the well-known $\pi^0 \to \gamma\gamma$ rate, without the use of higher symmetries or singlet-octet mixing (3). Using two methods, we estimate the corrections to simple VMD due to higher-mass vector mesons and find that they are insufficient to remove the disagreement.

The coupling constants $g_{\pi^0\gamma\gamma}$ and $g_{\rho\pi\gamma}$ for the processes $\pi^0 \to \gamma\gamma$ and $\rho \to \pi\gamma$ are related through VDM by

$$g_{\pi^0\gamma\gamma} = \frac{2e g_{\rho\pi\gamma}}{g_\rho}, \tag{1}$$

where e is the electric charge and g_ρ is related to the direct ρ-photon coupling vertex $G_{\rho\gamma}$ via

$$G_{\rho\gamma} = e\frac{m_\rho^2}{g_\rho}. \tag{2}$$

Although g_ρ can be measured directly through the decays $\rho \to l\bar{l}$, the value so obtained has a rather large error associated with it. Fortunately, simple VMD provides the relation

$$g_\rho = g_{\rho\pi\pi}, \tag{3}$$

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(1) B. GOBBI, J. L. ROSEN, H. A. SCOTT, S. L. SHAPIRO, L. STRAWCZYNSKI and C. M. MELTZER: *Phys. Rev. Lett.*, **33**, 1450 (1974).

(2) B. T. FELD: *Models of Elementary Particles* (New York, N. Y., 1969) reviews the quark model. See for example ref. (13,16) for higher symmetry models.

(3) A. KOTLEWSKI, W. LEE, M. SUZUKI and J. THALER: *Phys. Rev. D*, **8**, 348 (1973).

where $g_{\rho\pi\pi}$ is the $\rho\to\pi\pi$ coupling constant. Since this decay rate is now known [4] with greater accuracy, it is the one we shall choose for determining g_ρ. We wish to point out that eq. (1) follows from VDM and the isospin invariance of the strong interactions alone and does not depend on the imposition of a higher symmetry on meson-photon coupling constants. Neither does it depend on the spectrum of the isoscalar vector mesons that dominate the isoscalar component of the electromagnetic current.

From eq. (1) one computes

$$\frac{\Gamma(\rho^-\to\pi^-\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{g^2_{\rho\pi\pi}/4\pi}{6\alpha}\left(\frac{m^2_\rho-m^2_{\pi^-}}{m_{\pi^0}m_\rho}\right)^3 . \tag{4}$$

The coupling constant $g_{\rho\pi\pi}$ is determined by using the ρ-meson mass and width found by Roos [4], who was able to achieve consistent results from $\pi\pi$ phase shift and pion form factor data. We find

$$\frac{g^2_{\rho\pi\pi}}{4\pi}=2.93\pm0.02 . \tag{5}$$

For the $\pi^0\to\gamma\gamma$ decay width we adopt [5]

$$\Gamma(\pi^0\to\gamma\gamma)=(7.89\pm0.38)\text{ eV} . \tag{6}$$

The resulting value for the $\rho\to\pi\gamma$ decay width

$$\Gamma(\rho^-\to\pi^-\gamma)=(91\pm4)\text{ keV} \tag{7}$$

is more than five standard deviations away from the experimental result [1]

$$\Gamma_{\text{exp}}(\rho^-\to\pi^-\gamma)=(35\pm10)\text{ keV} . \tag{8}$$

One would expect that the generalized vector-meson dominance model [6] (GVMD) would yield corrections to eq. (1). The higher-mass resonances (predicted, for example, by the Veneziano model [7]) change eq. (1) into a series

$$g_{\pi^0\gamma\gamma}=2e\sum_i\frac{g_{\rho_i\pi\gamma}}{g_{\rho_i}} , \tag{9}$$

where the sum is over all the $I=1$, $J^{PC}=1^{--}$ mesons ρ_i. The quantity g_{ρ_i} is the generalization of g_ρ (cf. eq. (2)) for these mesons. Here, one no longer has the equality

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(4) M. Roos: Helsinki University Report No. ISBN 951-45-06-3-0 (1975), finds $m_\rho=(776.3\pm0.4)$ MeV and $\Gamma(\rho\to\pi\pi)=(154.5\pm1.0)$ MeV.

(5) This is the weighted average of the recent determination of A. Browman, J. DeWire, B. Gittelman, K. M. Hanson, D. Larson, E. Loh and R. Lewis: *Phys. Rev. Lett.*, **33**, 1400 (1974) and the tabulation given by the Particle Data Group: *Phys. Lett.*, **50** B, 1 (1974).

(6) J. J. Sakurai: *Proceedings of the Canadian Institute of Particle Physics Summer School* (Montreal, 1972), p. 437.

(7) See J. L. Peterson: *Phys. Lett.*, **2** C, 155 (1971), for a review.

$g_{\rho_i} = g_{\rho_i \pi\pi}$ but rather the sum rule

$$1 = \sum_i \frac{g_{\rho_i \pi\pi}}{g_{\rho_i}} . \tag{10}$$

Again, eq. (9) is independent of the spectrum of $I = 0$, $J^{PC} = 1^{--}$ mesons.

Let us first examine eq. (9) from a theoretical standpoint. Triangle anomalies in the partially conserved axial vector current can be used to evaluate possible corrections to eq. (9). In such an approach [8] the coupling constant $g_{\rho_i \pi\gamma}$ factorizes:

$$g_{\rho_i \pi\gamma} = g_{\rho_i \pi\pi} C_{\pi\gamma} , \tag{11}$$

where $C_{\pi\gamma}$ is a constant independent of ρ_i. This result is obtained in fermion loop models [9] as well, provided that F universality [10] is maintained. This factorization is also independent of higher-symmetry arguments.

Combining eqs. (9)-(11), we obtain

$$g_{\pi^0 \gamma\gamma} = 2eC_{\pi\gamma} \sum_i \frac{g_{\rho_i \pi\pi}}{g_{\rho_i}} = 2e \frac{g_{\rho\pi\gamma}}{g_{\rho\pi\pi}} , \tag{12}$$

a result identical to eq. (1). This rather model-dependent conclusion shows that GVMD does not change the VMD result.

We may estimate the correction in other ways. For example, the results of a duality analysis of the pion form factor data [11] indicate that the sum of the higher-order terms of eq. (10) is -11% of the first term $g_{\rho\pi\pi}/g_\rho$. Thus GVMD rate predictions could change VMD predictions by 22%, lowering our $\rho^- \to \pi^- \gamma$ result to 70 keV or raising it to 110 keV. This is still at least three standard deviations from the experimental value.

The analogous decay $\bar{K}^{*0} \to \bar{K}^0 \gamma$ has also been measured recently [12]. Again, the width found is approximately three times smaller than expected from the quark model. If we assume that the $\bar{K}^{*0} \to \bar{K}^0 \gamma$ decay width is related to that of $\rho \to \pi\gamma$ through SU_3, we find that

$$\Gamma(\bar{K}^{*0} \to \bar{K}^0 \gamma) = (206 \pm 9)\ \text{keV} , \tag{13}$$

again using $\pi^0 \to \gamma\gamma$ as input. This is more than three standard deviations away from the experimental value

$$\Gamma_{\text{exp}}(\bar{K}^{0*} \to \bar{K}^0 \gamma) = (75 \pm 35)\ \text{keV} . \tag{14}$$

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(8) P. G. O. FREUND and S. NANDI: *Phys. Rev. Lett.*, **32**, 181 (1974); R. TORGERSON: *Phys. Rev. D*, **10**, 2951 (1974).

(9) References to very early work may be found in R. TORGERSON: *Phys. Rev. D*, **10**, 2951 (1974). For the baryon loop model, see, for example, R. ROCKMORE: *Phys. Rev. D*, **11**, 620 (1974) and references contained therein; see also B. EDWARDS and A. N. KAMAL: *Phys. Rev. Lett.* (to be published), for its application to PVγ interactions without the imposition of F universality.

(10) J. J. SAKURAI: *Ann. of Phys.*, **11**, 1 (1960); M. GELL-MANN: Caltech Report No. CTSL-20 (1961) (published in M. GELL-MANN and Y. NE'EMANN: *The Eightfold Way* (New York, N. Y., 1964)).

(11) P. L. BRUNINI, F. RIMONDI and G. VENTURI: *Lett. Nuovo Cimento*, **10**, 693 (1974). We quote results derived from their model labelled $2a$ since it is in closest agreement with known meson decay data.

(12) W. C. CARITHERS, P. MÜHLEMAN, D. UNDERWOOD and D. G. RYAN: *Phys. Rev. Lett.*, **35**, 349 (1975).

It is of interest to compare the value for the $\rho \to \pi\gamma$ decay width found here to that found in other calculations. In four recent approaches, nonet symmetry ([13]), SU_3 with broken nonet symmetry ([14]), nonet symmetry with broken SU_3 ([15]) and strong PCAC anomalies ([16]), a $\rho \to \pi\gamma$ decay width close to 80 keV is found, in good agreement with what has been found here. It should also be noted that the broadest interpretation of the $\rho^- \to \pi^-\gamma$ experiment implies a range for $\Gamma(\rho^- \to \pi^-\gamma)$ from (30 ± 10) keV to (80 ± 10) keV. This upper limit is not in disagreement with VMD. The desirability of a new measurement of $\rho \to \pi\gamma$ is clearly indicated since it tests VMD alone. In addition, if VMD passes this test, more precise information on $K^* \to K\gamma$ rates will enable us to properly assess the assumption of SU_3 symmetry for pseudoscalar-vector-vector interactions.

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([13]) A. Bramon and M. Greco: *Phys. Lett.*, **48** B, 137 (1974).
([14]) D. H. Boal, R. H. Graham and J. W. Moffat: University of Toronto Report (July 1975
([15]) B. Edwards and A. N. Kamal: *Phys. Rev. Lett.* (to be published).
([16]) R. Torgerson: *Phys. Rev. D*, **10**, 2951 (1974).

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