Parameter-free hard-meson prediction of the K_{l3} form factors*

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Using the results of a previous on-mass-shell analysis of current-algebra sum rules for $K\pi$ scattering we are able to make parameter-free predictions for the K_{l3} form factors. The results are compared with recent experiments and found to be in reasonably good agreement.

I. INTRODUCTION

It has long been recognized that the K_{l3} decays are a potential source of valuable information about hadron dynamics. As a result they have been extensively studied[1] by both theorists and experimentalists. In this note we will present parameter-free predictions for the K_{l3} form factors. They are obtained from three-point functions, all parameters of which were previously determined[2] in an analysis of $K\pi$ scattering based on current-algebra techniques.[3]

In BGW we investigated on-mass-shell current-algebra sum rules for the $K\pi$ scattering amplitude. When evaluated at threshold, these become corrected soft-pion and soft-kaon relations for the S-wave $K\pi$ scattering lengths. The object of the study was to estimate the sizes of the correction terms in these relations and to get an idea of the mass-extrapolation errors associated with soft-meson calculations; in particular, this allowed a direct comparison of the relative sizes of kaon- and pion-mass-extrapolation effects in the same process. The results turned out to be both consistent and reasonable, encouraging us to have some confidence in the present predictions, which are based on the parameters determined there.

The predictions for the K_{l3} form factors come about in the following way: The corrections to the soft-meson results were calculated in BGW using a dispersive approach. It was assumed that the dispersion relations were dominated by the $K^*(890)$ and a $J^P=0^+$ κ meson. In this approximation the relevant absorptive parts are given, essentially, by the products $\langle K|A^\mu(0)|K^*\rangle \times \langle K^*|A^\nu(0)|K\rangle$ and $\langle K|A^\mu(0)|\kappa\rangle\langle\kappa|A^\nu(0)|K\rangle$ [or $\langle\pi|A^\mu(0)|K^*\rangle\langle K^*|A^\nu(0)|\pi\rangle$ and $\langle\pi|A^\mu(0)|\kappa\rangle \times\langle\kappa|A^\nu(0)|\pi\rangle$] in the case of corrections to the soft-pion (soft-kaon) sum rules, where $A^\mu(x)$ is the $\Delta S=0$ ($|\Delta S|=1$) axial-vector current. These matrix elements were obtained from a hard-meson current-algebra analysis[4] of the family of three-point functions involving the $\Delta S=0$ and $|\Delta S|=1$ axial-vector currents, the $|\Delta S|=1$ vector current, and their divergences.[5] By means of certain consistency conditions[2] all unknown parameters describing this family of three-point functions could be fixed within fairly narrow ranges, thus allowing the absorptive parts to be determined and estimates of the correction terms to be made. Now, since the matrix element $\langle\pi|V^\mu(x)|K\rangle$, where $V^\mu(x)$ is the $|\Delta S|=1$ vector current, can be expressed in terms of members of the same family of three-point functions, the K_{l3} form factors can be predicted.

To summarize, our model for the K_{l3} form factors has the basic features that (a) the structure of the form factors is derived from a hard-meson analysis of the appropriate family of three-point functions, and (b) all unknown parameters are fixed from an independent analysis of $K\pi$ scattering.

A number of previous investigations[6-17] have made use of (a) to calculate the K_{l3} form factors and there have also been models for them based on other types[18-23] of on-mass-shell current-algebra analysis. However, we feel that the present work is sufficiently different and interesting to justify yet another contribution along these lines. In the first place the form factors obtained in the present model differ in detail from those found previously. Furthermore, since the model incorporates generally believed properties such as current algebra and pole dominance of amplitudes, and is so tightly constrained, both by the physically reasonable values of most of the parameters and by necessary consistency conditions[2] for the rest, its predictions should be confronted with the data. As will be seen the model is qualitatively successful in general and, in several instances, quantitatively fairly accurate.

In Sec. II we will give the explicit expressions for the K_{l3} form factors. The determination of the values of the parameters appearing in these expressions will be described and our predictions given in Sec. III. A comparison of these predictions with the experimental data will be made in Sec. IV.

II. EVALUATION OF THE K_{l3} FORM FACTORS

The description of the K_{l3} decays $K\to\pi+l+\bar\nu_l$ depends on the matrix elements $\langle\pi_a(q)\,|\,V_c^\lambda(0)\,|K_b(p)\rangle$, where a, b, and c are SU(3) indices, p and q are the kaon and pion four-momenta, respectively, and $V_c^\lambda(x)$ is the $|\Delta S|=1$ hadronic weak vector current. This matrix element can be expressed in terms of the form factors $f_\pm(t)$ as[24]

$$(2\pi)^3(2q^0 2p^0)^{1/2}\langle\pi_a(q)\,|\,V_c^\lambda(0)\,|K_b(p)\rangle = if_{abc}[(p+q)^\lambda f_+(t)+(p-q)^\lambda f_-(t)] \quad (a=1,2,3;\ b,c=4,\ldots,7)\,, \tag{1}$$

where $t=-(p-q)^2$ and the f_{abc} are the SU(3) structure constants.

The K_{l3} form factors can be related to the vertex functions $\Gamma_\mu(q,p)$ and $\overline{\Gamma}(q,p)$ defined by[2]

$$\int d^4x\,d^4y\,e^{-iq\cdot x}e^{ip\cdot y}\langle T\{\partial_\mu A_a^\mu(x)\partial_\nu A_b^\nu(y)V_c^\lambda(0)\}\rangle_0$$
$$= if_{abc}\frac{F_\pi F_K m_\pi^2 m_K^2}{g_{K^*}(q^2+m_\pi^2)(p^2+m_K^2)}\,\Delta_{K^*}^{\lambda\eta}(k)\Gamma_\eta(q,p)+id_{abc}\frac{F_\pi F_K m_\pi^2 m_K^2 F_K k^\lambda}{(q^2+m_\pi^2)(p^2+m_K^2)(k^2+m_K^2)}\,\overline{\Gamma}(q,p)\,, \tag{2}$$

where $k=p-q$ and d_{abc} is the usual symmetric SU(3) tensor. The matrix element (1) is obtained from Eq. (2) by multiplying both sides by $(q^2+m_\pi^2)(p^2+m_K^2)$ and then taking the limits $q^2\to-m_\pi^2$, $p^2\to-m_K^2$, with the result

$$(2\pi)^3(2q^0 2p^0)^{1/2}\langle\pi_a(q)\,|\,V_c^\lambda(0)\,|K_b(p)\rangle = if_{abc}\frac{1}{g_{K^*}}\,\Delta_{K^*}^{\lambda\eta}(k)\Gamma_\eta(q,p)+id_{abc}\frac{F_K k^\lambda}{k^2+m_K^2}\,\overline{\Gamma}(q,p)\,. \tag{3}$$

The properly symmetrized vertex functions can be obtained[25] from BGW. They have the form

$$\Gamma_\mu(q,p)=Ap^\sigma q^\tau[g_{\tau\sigma}(p+q)_\mu+(2+\delta_{K^*})(g_{\tau\mu}k_\sigma-g_{\sigma\mu}k_\tau)-g_{\tau\mu}p_\sigma-g_{\sigma\mu}q_\tau]+B(p+q)_\mu+Cq^\sigma\Delta_{K^*\sigma\mu}^{-1}(k)+Dp^\sigma\Delta_{K^*\sigma\mu}^{-1}(k)\,, \tag{4}$$

where the constants A, B, C, and D are defined by

$$A=\frac{C_{A_1}C_{K_A}}{2g_{A_1}g_{K_A}F_\pi F_K}\frac{m_{K^*}^2}{g_{K^*}}\left(\frac{g_{K_A}}{g_{A_1}}+\frac{g_{A_1}}{g_{K_A}}\right),\quad B=\frac{g_{K^*}}{2F_\pi F_K}\,,\quad C=(F_\pi^2-C_{A_1})B\,,\quad D=(F_K^2-C_{K_A})B\,,$$

with

$$C_x=g_x^2/m_x^2\,,$$

and

$$d_{abc}\overline{\Gamma}(q,p)=\frac{f_{abc}}{2F_\pi F_K F_K}\left[2p\cdot q(C_{A_1}-C_{K_A})-F_K^2(m_K^2-m_\pi^2-m_K^2X_K-m_\pi^2X_\pi)-\frac{k^2+m_K^2}{m_K^2}(F_\pi^2 m_\pi^2 Y_\pi+F_K^2 m_K^2 Y_K)\right], \tag{5}$$

where

$$X_\pi=-\frac{2}{c\sqrt3}\,\omega_\pi\frac{m_K^2}{m_\pi^2},\quad X_K=-\frac{2}{c\sqrt3}\,\omega_K\frac{m_K^2}{m_K^2},\quad Y_\pi=\frac{cf_{c8e}d_{eab}}{f_{abc}\omega_\pi},\quad\text{and } Y_K=\frac{cf_{c8e}d_{eab}}{f_{abc}\omega_K}.$$

In the above the indices a, b, and c take on the values indicated in Eq. (1) and

$$\omega_\pi=\frac{\sqrt2+c}{\sqrt3}\quad\text{and}\quad\omega_K=\frac{\sqrt2-\frac12 c}{\sqrt3}\,,$$

where c is the symmetry-breaking parameter introduced by Gell-Mann, Oakes, and Renner[26]; we use their value of $c=-1.25$.

After a little algebra Eqs. (1), (3), (4), and (5) lead to

$$2F_\pi F_K f_+(t) = F_\pi^2 + F_K^2 - F_\kappa^2 - C_{K^*} + \tfrac{1}{2}(1+\delta_{K^*})\frac{m_{K^*}^2}{m_{A_1}^2 m_{K_A}^2}(g_{A_1}^2 + g_{K_A}^2) + \frac{g_{K^*}^2}{m_{K^*}^2 - t}\left[1 - \tfrac{1}{2}(1+\delta_{K^*})\frac{m_{K^*}^2}{C_{K^*} m_{A_1}^2 m_{K_A}^2}(g_{A_1}^2 + g_{K_A}^2)\right] \quad (6)$$

and

$$2F_\pi F_K f_-(t) = C_{K^*}\frac{m_K^2 - m_\pi^2}{m_{K^*}^2 - t}\left[(1+\delta_{K^*})\frac{m_{K^*}^2}{2C_{K^*}}\left(\frac{C_{A_1}}{m_{K_A}^2} + \frac{C_{K_A}}{m_{A_1}^2}\right) - 1\right] + \frac{1}{m_\kappa^2 - t}\left[(F_K^2 - F_\pi^2)(m_\kappa^2 - m_K^2 - m_\pi^2) - F_\kappa^2(m_K^2 - m_\pi^2 - m_K^2 X_K - m_\pi^2 X_\pi)\right] - \left(\frac{F_\pi^2 m_\pi^2 Y_\pi + F_K^2 m_K^2 Y_K}{m_\kappa^2}\right). \quad (7)$$

Note that at $t=0$ Eq. (6) reduces to the Glashow-Weinberg relation[27]

$$f_+(0) = \frac{1}{2F_\pi F_K}(F_\pi^2 + F_K^2 - F_\kappa^2)$$

whereas, in the SU(3) limit, it takes the form obtained by Gerstein and Schnitzer.[10]

III. PARAMETERS AND PREDICTIONS

Values have been obtained in BGW for all the parameters appearing in the expressions (6) and (7) for the K_{l3} form factors. Where possible they were taken from the latest data compilation.[28] In particular we used $m_\pi = 138$ MeV, $m_K = 496$ MeV, $m_{K^*} = 891$ MeV, and $F_\pi = 92$ MeV. The value $\Gamma(K^* \to K\pi) = 50$ MeV leads[2] to $\delta_{K^*} \approx -1.0$. In addition we assumed $m_{A_1} = 1100$ MeV and[29] $m_{K_A} = 1320$ MeV.

The parameter g_ρ can be determined from the leptonic decay rate of the ρ^0, $\Gamma(\rho \to l^+ l^-)$. If one writes[30]

$$g_\rho^2/m_\rho^2 = 2\gamma F_\pi^2$$

[where $\gamma = 1.0$ corresponds to the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[31]] the $\rho^0 \to e^+e^-$ rate[28] leads to $\gamma = 1.4 \pm 0.2$, while the $\rho^0 \to \mu^+\mu^-$ rate[28] gives the higher value $\gamma \cong 2.0$. In BGW a slightly modified set of first Weinberg sum rules[32] was used to relate g_{A_1}, g_{K_A}, and g_{K^*} and F_κ to g_ρ.

Several consistency conditions were employed to determine F_K/F_π, F_κ/F_π, and m_κ, as well as to pick out a range of values of γ for $\delta_{K^*} \approx -1.0$. For a discussion of these consistency conditions the reader should consult BGW. The conditions led to $F_K/F_\pi = 1.22$, $F_\kappa/F_\pi = 0.45$, $\gamma = 1.4$, $m_\kappa = 1200$ MeV, and $\Gamma(\kappa \to K\pi) = 500$ MeV, the latter predictions being especially gratifying in view of the recent observation[33] of a κ candidate with similar mass and width. The values of all parameters used for our K_{l3} predictions are summarized in Table I. Because we have taken $\delta_{K^*} = -1$ the values of m_{A_1}, m_{K_A}, g_{A_1}, and g_{K_A} are not relevant for our predictions [see Eqs. (6) and (7)] and hence are not included in Table I.

In experimental analyses the K_{l3} form factors are usually parametrized as linear functions of t

$$f_\pm(t) = f_\pm(0)\left(1 + \lambda_\pm \frac{t}{m_\pi^2}\right). \quad (8)$$

It is common to assume that the product $f_-(0)\lambda_-$ is small (and this is borne out in the present study)

TABLE I. Values of the parameters used in Eqs. (6) and (7) to obtain the predictions in Eq. (13). The underlined values can be extracted directly from data compilation (Ref. 28). The remaining values were obtained in Ref. 2 from an independent analysis of $K\pi$ scattering.

Parameter	Value
m_π	138 MeV
m_K	496 MeV
m_{K^*}	891 MeV
m_κ	1200 MeV
F_π	92 MeV
F_K	112 MeV
F_κ	41 MeV
g_{K^*}	0.132 GeV^2
δ_{K^*}	−1

so that the scalar form factor $f_0(t)$ defined by

$$f_0(t)=f_+(t)+\frac{t}{m_K^2-m_\pi^2}f_-(t) \tag{9}$$

can be parametrized

$$f_0(t)=f_0(0)\left(1+\lambda_0\frac{t}{m_\pi^2}\right). \tag{10}$$

Finally, a quantity of considerable interest is

$$\xi(t)=\frac{f_-(t)}{f_+(t)}. \tag{11}$$

Note that from Eqs. (8)–(11) it follows that

$$\xi(0)=\frac{m_K^2-m_\pi^2}{m_\pi^2}(\lambda_0-\lambda_+). \tag{12}$$

Our predictions, based on the parameters listed in Table I, are

$$\begin{aligned} f_+(0)&=0.937,\\ f_-(0)&=0.102,\\ \xi(0)&=0.109,\\ \lambda_+&=0.036,\\ \lambda_-&=-0.059,\\ \lambda_0&=0.040. \end{aligned} \tag{13}$$

Because of (11) and (12) only four of these are independent. In the following section we will compare these predictions with the experimental data.

Moderate changes in the parameters of expressions (6) and (7) about the values of Table I do not greatly affect most of our predictions. For the sake of illustration let us first consider the consequences of allowing F_K/F_π to have the values[34] 1.15 and 1.27. In both cases $f_+(0)$ decreases to ~0.88. For $F_K/F_\pi=1.15$ $\xi(0)$ decreases insignificantly to 0.107 while λ_+ and λ_0 increase by small amounts to 0.040 and 0.042, respectively. However, when $F_K/F_\pi=1.27$ $\xi(0)$ becomes 0.255, λ_0 increases to 0.053, with λ_+ decreasing to 0.035. If, together with a change in F_K/F_π, we vary δ_{K^*} and γ in reasonable ranges ($1.3\lesssim\gamma\lesssim1.6$, $-1.2\lesssim\delta_{K^*}\lesssim-0.9$) along their consistency curve,[2,30] λ_+ changes by only a few thousandths and $\xi(0)$ changes by a few hundredths in a direction opposite to λ_+ [see Eq. (12)], with $f_+(0)$ and λ_0 remaining unchanged. Amusingly enough it appears that the parameter values listed in Table I, which, as we have said, are optimal values from the standpoint of the $K\pi$ scattering analysis of BGW, also lead to experimentally agreeable K_{l3} predictions.

IV. COMPARISON WITH EXPERIMENT

Perhaps the least controversial of the K_{l3} parameters is λ_+. Most experimental studies have found λ_+ to be small and positive. In recent experiments, for example, Brandenberg *et al.*[35] get $\lambda_+=0.019\pm0.013$, and Wang *et al.*[36] obtain $\lambda_+=0.040\pm0.012$. In a high-statistics experiment involving a Dalitz-plot analysis of 1.6×10^6 $K_L^0\to\pi\mu\nu$ decays, Donaldson *et al.*[37] find $\lambda_+=0.030\pm0.003$. The latter authors also performed world averages of previous results concluding that $\lambda_+=0.029\pm0.005$ from $K^+\to\pi^0e^+\nu_e$ experiments and $\lambda_+=0.032\pm0.004$ from $K_L^0\to\pi e\nu$ analyses. In a more recent, but less accurate, experiment Buchanan *et al.*[38] find $\lambda_+=0.044\pm0.006$ from a Dalitz-plot analysis of $K_L^0\to\pi e\nu$ decays. Our predicted value of $\lambda_+=0.036$ is in rather good agreement with the bulk of the results discussed above. It should be noted that our prediction for λ_+ is somewhat closer to the world-average value and that of Donaldson *et al.* than is the result $\lambda_+=m_\pi^2/m_{K^*}^2\cong0.22$ based on naive K^* dominance of $f_+(t)$.

Although the slope of $f_+(t)$ is fairly well determined, its intercept at $t=0$ is not known. Few experiments give a measurement of $f_+(0)$. Buchanan *et al.*[38] find $f_+(0)=0.96\pm0.07$ in an unconstrained fit to this quantity, indicating that it could be less than its SU(3) value of 1.0. Our prediction of[2] $f_+(0)=0.94$ is consistent with this result and not too far from what one would expect from more general arguments.[39]

The form factor $f_+(0)$ is fairly strongly correlated experimentally with the ratio F_K/F_π, since[40]

$$\frac{F_K}{F_\pi f_+(0)}=1.25\pm0.03. \tag{14}$$

We predict that[2]

$$\frac{F_K}{F_\pi f_+(0)}=1.30, \tag{15}$$

which is close to (14).

For a number of years many experiments obtained negative values for the parameter λ_0. Indeed, from an analysis of $K_{\mu3}$ Dalitz-plot and polarization experiments completed before their review, Chounet *et al.*[1] arrived at a value of $\lambda_0=-0.11\pm0.03$. This caused considerable consternation because it disagreed so greatly with the current-algebra result[41]

$$f_+(m_K^2)+f_-(m_K^2)=F_K/F_\pi, \tag{16}$$

which, using Eqs. (8)–(10) and (14) implies that

$$\lambda_0\cong0.02, \tag{17}$$

assuming a linear extrapolation to $t=m_K^2$ is valid.

However, $K_{\mu3}$ together with K_{e3} branching-ratio measurements have produced results which are consistent with (17). Donaldson *et al.*[37] have averaged existing $K_{\mu3}$ and K_{e3} branching-ratio measurements and find $\lambda_0 = 0.011 \pm 0.010$ from the $K^+_{\mu 3}/K^+_{e3}$ ratio and $\lambda_0 = 0.035 \pm 0.010$ from the $K^0_{\mu 3}/K^0_{e3}$ ratio.

From their own Dalitz-plot analysis Donaldson *et al.* obtained $\lambda_0 = 0.013 \pm 0.005$ from an unparametrized fit using $f_+(t)$ and either $f_0(t)$ or $\xi(t)$, while they found $\lambda_0 = 0.019 \pm 0.004$ from a two-parameter $[\lambda_+, \lambda_0; f_+(0) \equiv 1.0]$ fit. The latter type of fit is preferred by Donaldson *et al.*, because the correlation between λ_+ and λ_0 is much less than between $f_+(t)$ and $f_0(t)$ or $\xi(t)$ and also the error analysis is more straightforward. [It also turns out that $f_0(0)/f_+(0) \approx 1.07$ in the former fit.]

Buchanan *et al.*,[38] in a band-coupled approach, used several different input forms for $f_+(t)$ to extract $f_0(t)$ and $\xi(t)$. Employing a linear parametrization with $\lambda_+ = 0.044$ (from their K_{e3} analysis) they obtained $\lambda_0 = 0.024 \pm 0.013$ for a constrained $[f_+(0) \equiv 1.0]$ fit and $\lambda_0 = 0.032 \pm 0.010$ for the unconstrained fit mentioned above.

Our predicted value of $\lambda_0 = 0.040$ is consistent with the result of the unconstrained fit of Buchanan *et al.* and is not incompatible with their constrained fit; however, it is quite a bit larger than either of the results of Donaldson *et al.* In Fig. 1 we have compared the predicted $f_0(t)$ with the unconstrained data of Buchanan *et al.* The agreement is seen to be quite good; however, in view of the rather accurate determination of λ_0 by Donaldson *et al.*, the t dependence predicted for $f_0(t)$ in our model is probably too strong.

Returning briefly to Eq. (16) we predict the left-hand side to be ~1.47. We have mentioned above that our prediction for F_K/F_π is 1.22. Thus, a rather large correction to the soft-pion theorem is generated by going on the mass shell.

The quantity $\xi(0)$ has long been of great interest. In the limit of exact SU(3) symmetry $f_-(t) \equiv 0$. For approximate SU(3) symmetry one might reasonably expect $f_-(0) \approx 0$, which implies [for $f_+(0) \approx 1$] that $|\xi(0)|$ is small. It is interesting to note that this is guaranteed by Eq. (12) if λ_+ assumes its world-average value and λ_0 its current-algebra value (17).

Until fairly recently the experimental picture indicated that the contrary might be true. Chounet *et al.*[1] concluded from the bulk of experimental results available to them that $|\xi(0)|$ was large, with $\xi(0) \approx -1$. Donaldson *et al.*,[37] averaging the earlier, together with more recent, data, found that $\xi(0) = -0.94 \pm 0.21$ from $K^+_{\mu 3}$ polarization experiments and $\xi(0) = -0.69 \pm 0.19$ ($K^0_{\mu 3}$ polarization[42]). However, branching-ratio and Dalitz-plot measurements yield smaller values of $|\xi(0)|$. Donaldson *et al.* averaged the values obtained from the former type of experiments obtaining $\xi(0) = -0.37 \pm 0.13$ ($K^+_{\mu 3}/K^+_{e3}$ ratio) and $\xi(0) = 0.09 \pm 0.13$

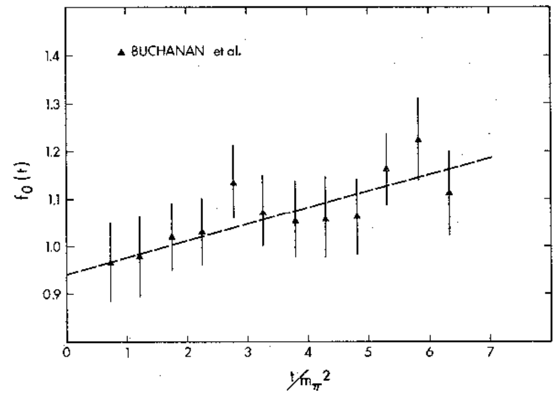

FIG. 1. Prediction for the scalar form factor $f_0(t)$ (dashed line) compared to the data of Buchanan *et al.* (Ref. 38).

($K^0_{\mu 3}/K^0_{e3}$ ratio), both determinations depending on their value of $\lambda_+ = 0.03$.

In their own experiment Donaldson *et al.* obtained $\xi(0) = 0.00 \pm 0.04$ from the unparametrized fit and $\xi(0) = -0.11 \pm 0.03$ in the two-parameter fit. Buchanan *et al.*[38] found $\xi(0) = -0.20 \pm 0.15$ when $\xi(t)$ was assumed to be constant and $\xi(t) = -(0.34 \pm 0.31) + (0.03 \pm 0.07)t$ when a linear t dependence was assumed.

Our predicted value of $\xi(0) = 0.109$ is again more consistent with the less accurate results of Buchanan *et al.* than with that of Donaldson *et al.* (if we discount the latter's unparametrized fit). In Fig. 2 the predicted $\xi(t)$ is compared with the results of Buchanan *et al.* and with the unparametrized data of Donaldson *et al.* Although the two

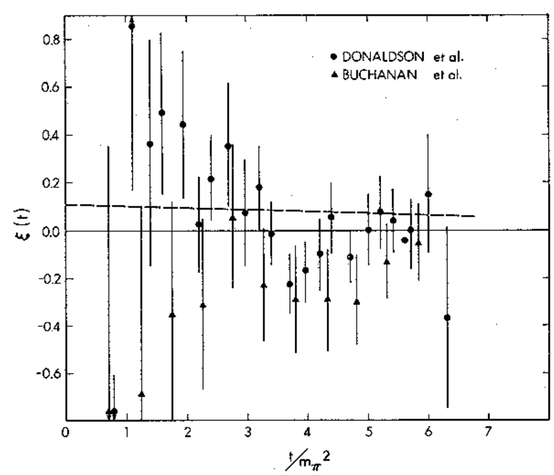

FIG. 2. Prediction for $\xi(t)$ (dashed line) compared to the data of Donaldson *et al.* (Ref. 37) and Buchanan *et al.* (Ref. 38).

sets of data are fairly consistent and the agreement of our $\xi(t)$ with the data is not too bad, nevertheless the predicted $\xi(0)$ is probably too far from the -0.11 ± 0.03 preferred by Donaldson *et al.*

As there is insufficient data on λ_- to draw any conclusions regarding either its sign or magnitude, our value in Eq. (13) must remain an untested prediction.

In conclusion, it should be clear from the above discussion that the present hard-meson model for the K_{l3} form factors is in at least qualitative agreement with most of the data. Reasonably good quantitative results are obtained for λ_+, $F_K/[F_\pi f_+(0)]$, and $f_+(0)$. While our predictions for $f_0(t)$ and $\xi(t)$ are not in close accord with the accurate data of Donaldson *et al.*,[37] they are at least consistent with experimental trends over the past several years.

Note added in proof. Since this manuscript was submitted, an analysis of the muon polarization in the decay $K_L^0\to\pi^-\mu^+\nu_\mu$, based on more than 200 000 events, has been published by A. R. Clark *et al.*, [Phys. Rev. D 15, 553 (1977)]. Their value of $\xi(0)=0.178\pm0.105$ is in good agreement with ours, and is significantly higher than those results obtained previously. We thank Dr. G. Shen for bringing these results to our attention.

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