REGGE TRAJECTORIES IN SU(4)★

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A unitary symmetry analysis of the $J^P = 1/2^+, 1/2^-, 3/2^-, 5/2^+$ and $5/2^-$ baryon octets provides a simple mass formula for the charmed baryons. Members of baryon families (same I, S, C, J, and P) will occur with differences in mass squared of about 2.7, 2.3, 2.1, ... GeV^2 ($C = 1$) or 3.9, 3.2, 2.9, ... GeV^2 ($C = 2$), in agreement with the observed J/ψ family.

The bound state quark model [1] predicts the existence of families of particles with the same B, I, S, C, J and P quantum numbers but different radial wave functions, as one goes to higher mass. The Veneziano model in Regge pole theory [2] has also been used to predict such families. On the basis of assigning several particles in the 1 to 2 GeV resonance region to the same Regge trajectory, it was predicted that the members of these families should have a separation in mass squared of about 1 GeV^2. In the case of the $J^P = 1/2^+$ baryons, this appears to be confirmed.

If one assumes that the slope of the real part of the Regge trajectory (as a function of mass squared) is the same for all particles, then as one goes to higher mass multiplets in SU(3) or SU(4), the splitting within these multiplets should decrease relative to the parent multiplet. For example, if we take the conventional slope of 1 GeV^{-2}, then the ρ-ψ mass difference of 2.33 GeV predicts a ρ'-ψ' difference of 1.99 GeV, the mass of the ψ' being 3.26 GeV. While the mass of the first excitation of the ρ is still open for debate, certainly the mass squared of the first excited state of the ψ is four times the universal Regge slope prediction. It is the purpose of this letter to show that these large differences in mass squared for particles containing charmed quarks are a requirement imposed by SU(4) symmetry and the observed SU(3) trajectories.

It is difficult to analyse this discrepancy on the basis of meson trajectories alone, since the resonance multiplets are incomplete. In addition, there may be problems with the charmed meson mass predictions because of mixing effects [3]. We choose instead to look at the baryons, since the multiplets in the SU(3) domain are more nearly complete. We choose the $J^P = 1/2^+, 1/2^-$, $3/2^-, 5+/2$, and $5/2^-$ multiplets as the basis for our calculations, the multiplet assignments being shown in table 1. There are, of course, ambiguities in assigning the Σ and Ξ resonances to SU(3) multiplets since they appear in both the $\underline{8}$ and $\underline{10}$ representations. We have used the equal spacing rule wherever possible to rule out such ambiguities. The spins of the Ξ particles at 1820 and 1940 MeV have yet to be determined. They can be omitted from the analysis if one assumes that the higher mass Λ in each multiplet is mainly octet. The masses quoted represent the averages of the ranges given by the particle data group [4].

The SU(4) mass formulae for baryons have been investigated by several authors [5]. We reproduce the results here for the $\underline{20}'$ representation in SU(4) which contains the $\underline{8}$ in SU(3):

$$\begin{aligned}
M(\underline{8}) &= m_0 + \alpha Y + \beta[I(I+1) - \tfrac{1}{4}Y^2],\\
M(\underline{6}) &= m_0 - (y-1)\alpha + \tfrac{1}{2}(3y+1)\beta + \alpha Y,\\
M(\underline{3}^*) &= m_0 - (y-1)\alpha - \tfrac{1}{2}(y-1)\beta + (\alpha - \beta)Y,\\
M(\underline{3}) &= m_0 - 2(y-1)\alpha - \beta + (\alpha - \tfrac{3}{2}\beta)Y,
\end{aligned} \tag{1}$$

where we have neglected mixing of the $I = \frac{1}{2}, S = -1$, $C = 1$ states. The charge and hypercharge assignments are taken from ref. [6]. The reduced matrix elements m_0, α and β have been determined for each spin-parity multiplet by the octet members, and are given in table 1. The SU(4) symmetry breaking parameter, y, is fixed by assinging the recently discovered [7] antibaryon at 2.26 GeV as the lightest $J^P = 1/2^+$ charmed

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Table 1
Masses and reduced matrix elements used in the calculation. All quantities in MeV. Data from ref. [4].

J^P	N	Σ	Ξ	Λ	Λ	α	β	m_0
$1/2^+$	939	1193	1318	1116	–	–189	43	1107
$3/2^-$	1520	1670	1820	1690	1519	–150	0	1670
$1/2^-$	1515	1750	1940	1670	1405	–212	15	1720
$5/2^+$	1680	1915	–	1820	–	–164	47	1820
$5/2^-$	1670	1773	–	1830	–	–146	–28	1830

particle $\overline{C}_0$. This yields $y = 8.03$. If we assign the structure at 2.5 GeV as the $J^P = 3/2^+$ charmed particle $\overline{C}_1^*$, then we find $y = 8.3$ for the $\underline{20}$ representation (which contains the SU(3) decuplet), in good agreement. This is higher than the naive quark model prediction of 6.5 [8].

We are now in a position to predict charmed baryon masses. From table 1, we notice that α and β, which we determine from the baryon octets, are roughly constant for the different multiplets. Let us assume that they are indeed constant, for octet average masses in the region 1 to 2 GeV, and use the average values of $\alpha = -0.172$ GeV and $\beta = 0.015$ GeV. The masses of the charmed baryons can then be expressed as a linear function of the N-resonance mass alone, as we show in fig. 1. The detailed mass predictions for the individual spin-parity multiplets and their associated decay rates will be considered elsewhere.

To calculate the masses of the charm families, we take as input the $J^P = 1/2^+$ nucleon excitation at 1430 MeV. From the graph, we see that the masses of the first excitations of the C_0, C_1 and X_N particles are 2.75, 2.99, and 4.19 GeV respectively compared with the parent values of 2.26, 2.50, and 3.70 GeV. Thus, the baryons with one charmed quark show an average increase in mass squared of about 2.73 GeV^2, while those with two charmed quarks have 3.94 GeV^2. This compares very well with the observed ψ, ψ' difference of 3.97 GeV^2 for two charmed quarks in a meson. A second prediction would be that the difference in mass squared between members of a particular charmed particle family should decrease as the average mass of the multiplet increases. As an example, again we look at the $J^P = 1/2^+$ baryons. We assume that the mass squared difference for members of the nucleon family is a constant, given by the N(1430) – N(938) difference. Then, for the baryons with two charmed quarks,

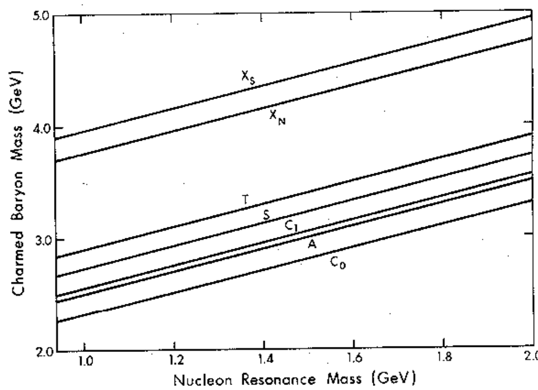

Fig. 1. Average masses of baryons with one charmed quark (C_0, C_1, A, S, T) and two charmed quarks (X_N, X_S) in the $\underline{20}'$ representation, shown as a function of the mass of the N-resonance.

we find an average difference of 3.9, 3.2, and 2.9 GeV^2 between the squared masses of the X, X′, X″, X‴ family. For comparison, the observed differences of the J/ψ family (with mass 3.098, 3.684, 4.1, and 4.414 GeV) are 4.0, 3.2, and 2.7 GeV^2. The agreement is very good. The corresponding differences for the baryons with one charmed quark would be 2.7, 2.3, and 2.1 GeV^2. This would predict, for example, a family of D mesons with mass 1.87 (input), 2.50, 2.92, 3.27, ... GeV.

Insofar as the Chew-Frautschi plot of a charmed particle family with $C = 2$ is concerned, we would expect the initial slope (at the first resonance) to have a value of about 0.25 GeV^{-2}. The slope would increase with mass, asymptotically approaching the nucleon resonance value. It should have about 90% of the N-resonance value when the mass of the X particles is about 30 GeV. Similarly, the singly charmed particles should have an initial slope of about 0.4 GeV^{-2}, reaching 90% of the nucleon slope when mass is about 20 GeV. Extrapolating to lower energy, we would expect trajectories with unit charm to have an intercept at roughly $\alpha(0) = -3/2 \pm 1/2$, higher than expected from a universal slope [9].

We summarize our two main results. We expect the differences in mass squared between successive members of charmed particle families to be larger than that predicted assuming a universal Regge trajectory slope. Equivalently, the mass squared splitting between charmed and uncharmed members of a multiplet should grow with energy. Along a particular trajectory, however, we expect the mass squared differences to decrease with increasing average mass, if the SU(3) members of the multiplet have a constant mass squared difference. Similarly, the rate of growth of mass squared splitting within a multiplet should also decrease as the average mass increases.

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