Thermodynamic limits of the quark-plasma phase

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A model for the quark-plasma phase at finite temperature and density is presented. The assumption that the effective interaction among quarks and gluons increases as density decreases is shown to lead naturally to a critical temperature around 220 – 300 MeV at zero chemical potential. The interpretation of this model in terms of phenomena expected in QCD is discussed.

I. INTRODUCTION

To successfully describe hadronic phenomenology, a model involving quarks must have very different behavior at large and small interquark separation. To explain the scaling behavior of deep-inelastic scattering, the quarks must be only weakly interacting at short distances.[1] To explain the nonobservation of free quarks in accelerator searches, they must be very strongly interacting at long distances. Therefore, if one were able to assemble a large number of quarks to form an ensemble of variable density and temperature, one might expect to find that at high density or temperature (high temperature implying large average momentum and hence a large four-momentum transfer q^2 in an average collision) the quarks exist in a plasma phase which asymptotically approaches an ideal gas. At low density, the quarks would be hidden in a phase consisting of discrete hadrons. One of the questions which many authors have been trying to answer is whether there is actually a phase transition between these regimes and, if so, at what temperatures or densities it will occur.

The purpose of this paper is to discuss a model in which there is a distinct limit to the quark-plasma phase. The basic assumption, which is similar to that made by Olive,[2] is that the interactions among quarks and gluons in the plasma phase can be represented by an effective potential which varies inversely with the density. Requiring a self-consistent solution for the density almost automatically yields a critical temperature and density below which the plasma phase cannot exist.

The model, along with a review of other approaches, is given in Sec. II for the case of equal numbers of quarks and antiquarks. Section III will generalize the model to finite net baryon density, and indicate what conditions one might need in, for example, a heavy-ion collision in order to access this transition. Our conclusions will be given in Sec. IV.

II. MODELS FOR THE QUARK-HADRON TRANSITION

We will specialize immediately to the zero-net-baryon-number-density ($n_B = 0$) case, treating the more general situation in the next section. The $n_B = 0$ situation is of most interest in cosmology, since the early universe presumably passed through the quark-hadron transition about 10^{-5} sec after the big bang.

Soon after the advent of quantum chromodynamics[3] (QCD) it was realized[4-9] that quark matter at high density and high temperature could exist in an unconfined or plasma phase. Kislinger and Morley[4] particularly emphasized that the long-range color forces responsible for confinement would be screened at finite temperature. Since then, properties of quark matter at high temperature and density have been widely discussed.

The best evidence that there exists a deconfining phase transition in finite-temperature QCD comes from an examination of the pure gauge theory on the lattice. Both Monte Carlo calculations[10] and an effective-Lagrangian approach[11] show discontinuous behavior between strong- and weak-coupling regimes as the temperature increases. At high temperature, the energy density has been shown[10] to approach that of an ideal gas. As the temperature is lowered, the specific heat and other quantities such as the effective string tension, show a transition at around $40\Lambda_L$ [in the $SU(2)_c$ example]. This corresponds to a transition temperature of about 215 MeV. The obvious drawbacks of the lattice approach are that one is not assured that the behavior of the theory is the same in the continuum limit and that the effect of quarks is difficult to take into account.

At sufficiently high temperature and density it should be possible to calculate the equation of state of the quark matter using QCD perturbation theory.[4–9] Although this does not contain all the physics of the lattice calculation, the quark degrees of freedom are explicitly included and the calculations provide some useful insight into the behavior of the system.

In the perturbative QCD approach the pressure is expanded as a power series in the QCD running coupling constant g_c:

$$P = P_0 + P_2 + P_3 + \cdots \ , \tag{1}$$

where the subscripts refer to the power of g_c involved. P_0 is clearly the pressure of an ideal gas of quarks. Such calculations[8] have only been carried out for massive quarks to P_3.

The kind of behavior one would hope to see is an ideal gas of quarks at high temperature, and a sharp deviation from ideality at a temperature of a few hundred MeV. Shown in Fig. 1 is the actual behavior for the ratio of P_2 and P_3 to P_0 as a function of temperature in the region of interest. (This calculation is based on Ref. 8, and the reader is referred thereto for details.) It is fairly clear that the series is not particularly convergent since P_3 is actually larger in magnitude than P_2. The sum of the first three terms does approximate an ideal gas largely because of the cancellation between the second and third terms. What effect inclusion of the fourth term would have is difficult to say.

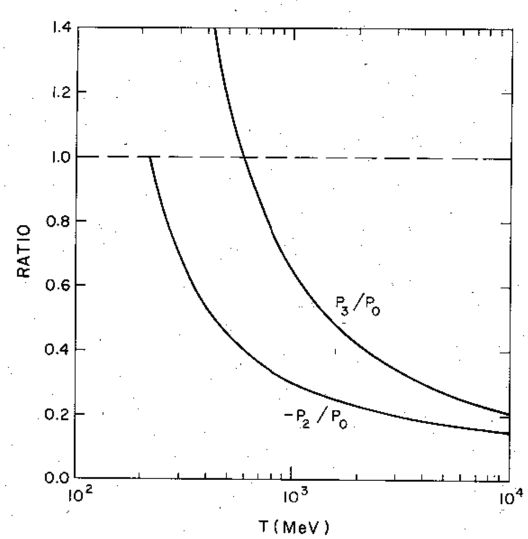

FIG. 1. Ratios of $-P_2/P_0$ and P_3/P_0 in a finite-temperature QCD calculation.

Defining the transition point as that where the perturbative series does not converge is not particularly useful because of the poor behavior of the series even at a temperature as large as 10^4 MeV (recall that the coupling constant varies only logarithmically with q^2). A slightly better definition from the operational point of view would be to define the transition as occurring where $P_i/P_0 > 1$. Such a definition at least restricts it to lie (on the basis of the first few terms) in the 200-to-600-MeV range.

A wholly different approach is represented by the explicit two-phase calculations,[2,12–17] in which one attempts to model the quark-plasma and hadron phases separately, then compare their free-energy densities or entropies as a function of temperature. Even for noninteracting particles, the free energies cross at about 300 MeV as shown in Fig. 2 (depending on how many hadrons one includes; Fig. 2 contains hadrons with a mass less than 1.5 GeV). However, the phases come out the wrong way around: at low temperature the phase is mainly massless gluons, while at high temperature it is the large number of hadronic states available. To reverse the phases, one must include the details of the interactions in each phase. It is clear from Fig. 2 that the free energy curves intersect each other at a fairly shallow angle, so the transition temperature will depend very sensitively on the interactions. Such calculations are difficult to interpret since they rely on totally different models for the description of the two phases. However, they do indicate[2] the possibility of a transition in the 200-to-300-MeV temperature range.

The approach taken here will be to use a mean-field approximation to show that there is a limiting temperature below which the quark phase vanishes. The method is reminiscent of the Bethe solution to the Ising model in condensed-matter physics.[18] The idea is to use a potential for the interaction among quarks and gluons that varies inversely with their number density (as suggested by QCD) and introduce it into the number-density equation either as

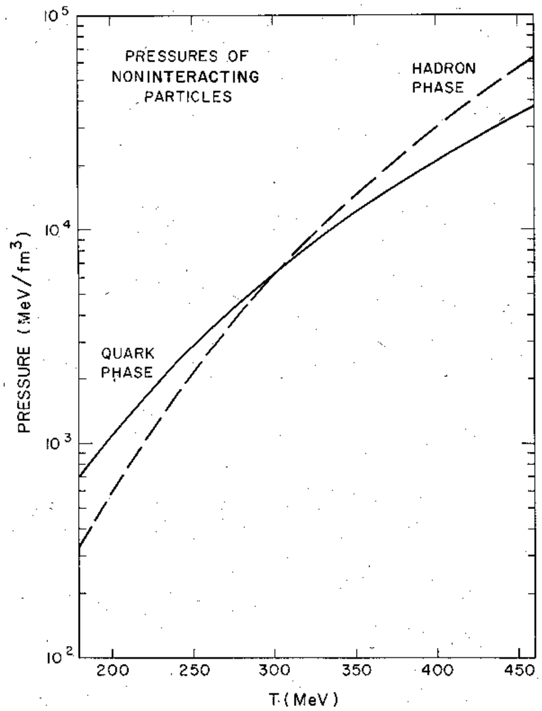

FIG. 2. Ideal-gas pressures for the quark-gluon-lepton phase (full curve) and the hadron-lepton phase (dashed curve). Only hadrons up to a mass of 1.5 GeV have been included.

an effective mass (like a scalar potential)[19] or as a background potential (like the fourth component of a vector potential). We will then show that the equation for the number density has a nonvanishing solution only for a temperature above a certain value.

In a dense system the potential felt by a single quark or gluon will be approximated by the interparticle potential calculated at the average separation distance. Olive[2] suggested using the linear confining potential $V=ar$ for the interparticle interaction in the plasma phase. Following this suggestion we take the interaction energy per particle to be $V=an_{\text{tot}}^{-1/3}$ with $n_{\text{tot}}=\sum_i n_i$ where the n_i are the number densities of the ith quark or gluon spin state. Motivation for this particular choice of V will be given below. It will be useful to define a quantity g as the effective number of quark and gluon spin states at a given temperature. This quantity will be approximately

$$g=n_{\text{tot}}/n_{\text{spin}}\ , \tag{2}$$

where

$$n_{\text{spin}}=\frac{1.202}{\pi^2}\left[\frac{kT}{\hbar c}\right]^3 \tag{3}$$

around the temperature region of interest (200–300 MeV) for the quark masses used below. Then

$$V=an_{\text{tot}}^{-1/3}=a(gn_{\text{spin}})^{-1/3}\ . \tag{4}$$

To obtain a numerical result for the critical temperature, we need to know the coefficient in Eq. (4). This can be estimated from the string model[20] or from an analysis[21] of meson mass spectra. In the massless string model, the energy per unit length is taken to be a constant a. In a frame in which the ends of the string rotate at the speed of light, the angular momentum J and energy E of the string are related via

$$J=(2\pi a)^{-1}E^2\ . \tag{5}$$

This behavior has been exploited for many years in the Regge trajectory analysis of high-energy differential cross sections, with the universal Regge slope α being related to a simply via

$$\alpha=(2\pi a)^{-1}\ . \tag{6}$$

An average value of α would be[22] 0.95 GeV^{-2} and would give a value to a of 0.17 GeV^2. A fit[21] to a part of the meson mass spectrum gave a much larger value to a, namely, 0.30 GeV^2, using a potential that contained an extra constant term to absolutely normalize the masses. We choose the former value of a for the purposes of our calculation here, although clearly there is uncertainty in this choice. We note that Källman[23] has argued that a will also be temperature dependent. Since we are concerned here mainly with showing the criticality of the plasma phase, this and other fine details such as the exact relation between r and n_{tot} (counting nearest-neighbor interactions) or dividing V by 2 (since a is determined for a quark-antiquark pair) will be ignored.

We obtain an equation for the number density by introducing V into the free-particle expression for the number density per spin state at zero chemical potential:

$$n_i=\frac{1}{(2\pi)^3}\int\frac{d^3q}{e^{E_i/kT}\pm 1}\ , \tag{7}$$

where the $+$ $(-)$ refers to fermions (bosons), respectively. The potential is introduced like an effective mass through the replacement

$$E_i\to[(m_i+V)^2+q^2]^{1/2} \tag{8}$$

or like a background potential through

$$E_i \to (m_i{}^2+q^2)^{1/2}+V \; . \tag{9}$$

Results for both approaches will be given.

It is clear that when $n_{\rm tot}$ is very large, as it should be at high temperature, V is very small and the number densities are just those of an ideal, noninteracting gas. As the temperature drops past the mass of the heavy quarks, each will rapidly decrease in number density in turn (since V will still be fairly small) until one is left with only gluons and light quarks. At this point, V will begin to increase and the critical behavior will emerge.

To see this behavior, consider all quarks to be massless. Then if V is large, the number-density equation in the background-potential example becomes, approximately,

$$n_i \sim e^{-V/kT} 4\pi \left[\frac{kT}{2\pi}\right]^3 \int \frac{x^2dx}{e^x} \; , \tag{10}$$

where V contains n dependence as in Eq. (4). This equation has a solution only for a limited range of kT, as can be seen from Figs. 3 and 4. At high T, the right-hand side intersects with the left-hand side at two points in addition to the trivial zero-density point. It is easily checked that the highest density solution is the physical one corresponding to the minimum free energy. At low T, there is only the trivial solution $n=0$. The critical temperature is equal to (for massless quarks)

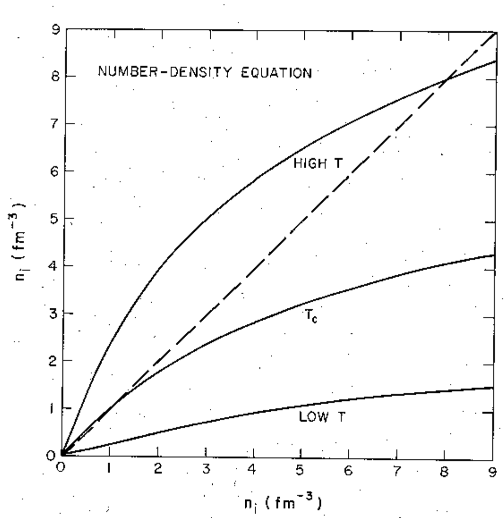

FIG. 3. Behavior of the right-hand side of Eq. (10) as a function of n_i for high (360 MeV), critical, and low (245 MeV) temperature. Three massless quarks are removed.

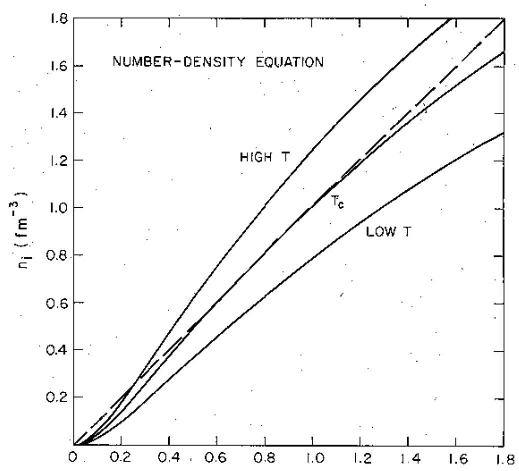

FIG. 4. A somewhat expanded version of Fig. 3 but with different high (319 MeV) and low (295 MeV) temperatures.

$$kT_c = \left[(\pi^2)^{1/3}\frac{e\kappa}{3}\right]^{1/2} , \tag{11}$$

where

$$\kappa = ag^{-1/3} \; . \tag{12}$$

In fact this behavior is quite general for any potential which behaves like

$$V = \kappa n_{\rm spin}{}^{-\delta} \; . \tag{13}$$

One finds

$$kT_c = [(\pi^2)^\delta e\kappa\delta]^{1/(1+3\delta)} \; . \tag{14}$$

As a sample calculation, for three flavors of quarks and eight gluons,

$$g \simeq [(\tfrac{3}{4}\times 3\times 3\times 2\times 2)+(8\times 2)] \; ,$$

where a factor of $\frac{3}{4}$ for the difference between the fermion- and boson-number density has been included. With $a=0.17$ GeV^2 and $\delta=\frac{1}{3}$ this gives a critical temperature of 307 MeV.

This same kind of critical phenomenon is also found using the single-particle energy of Eq. (8), although the detailed behavior is different. Here, one finds that the limiting temperature for massless quarks is

$$kT_c = \left[\kappa^2 2\pi\left[\frac{2e}{9}\right]^3\right]^{1/4} . \tag{15}$$

For the numerical example chosen in the

background-potential case, the critical temperature is reduced to 239 MeV.

When the different quark flavors have different masses, Eq. (7) becomes a set of coupled equations for the quark and gluon densities. It is easily verified that quarks with mass $m \gg T$ effectively decouple from the system. The coupled equations for the quarks and gluons are solved numerically. Choosing quark masses[24] of 8.3, 15, and 300 MeV for the up, down, and strange quarks, respectively, it is found that the critical temperature for the background-potential case is lowered from 307 to 300 MeV, and for the effective-mass case from 239 to 224 MeV. Most of this change is attributable to the proper treatment of the strange-quark number density. The routines used to calculate these temperatures result in an uncertainty of about 2 MeV in the numerical determination of the temperature for a fixed set of input parameters.

It is tempting to associate the critical temperature found here with the temperature for the transition from the deconfined (plasma) phase to the confined (hadron) phase. The critical temperature of the plasma phase would correspond to the limiting temperature of the hadron phase.[25] This behavior closely parallels what is found in lattice QCD. Below T_c, the single-particle energy in our model, and the self-energy of an isolated color source in QCD are infinite, indicating confinement. Above T_c these quantities are finite, indicating that we are in a deconfined phase.

Of course, other interpretations are still possible. For example, explicit two-phase models stress the comparison of free energies. In a two-phase model in which the hadron phase does not possess a limiting temperature, the quark-hadron transition would take place at a temperature higher than the critical temperature calculated in this model for the plasma phase.

A major difference between our calculation and that of Olive is that we couple the quarks and gluons by using the total density in the interaction potential, Eq. (4). This ensures that there is a single critical temperature for the whole system. In contrast, were we to treat the quarks and gluons independently as Olive does,[2] we would be led in this model to the result that the quark and gluon critical temperatures are not the same.

Once the temperature has passed below the transition temperature, the quark-plasma phase no longer exists and the quarks are bound into hadrons. One can check that the temperature which we have obtained is in the appropriate range by using a crude argument based on hadron close packing. If one were to approach the transition by heating up the hadron phase, rather than cooling down the quark phase, then one would expect that the transition would occur about when the hadrons were sufficiently dense that they were "close packed." This density would presumably be close to that at which the phase becomes a color conductor. To estimate this temperature, we will use the condition that the pion density be equal to the reciprocal of its volume. We choose the pion since it will be the most copious hadron in this temperature range. We are assuming, of course, that the quarks are somewhat evenly distributed across the hadronic volume, and not concentrated in its core.

For $kT \gg m_\pi c^2$, the pion-number density as a function of temperature is simply

$$n_\pi = 3n_{\rm spin} \tag{16}$$

for noninteracting pions. Demanding that

$$n_\pi = (\tfrac{4}{3}\pi R_\pi{}^3)^{-1} \tag{17}$$

gives a temperature of 285 MeV for $R_\pi = 0.6$ fm. This value of the temperature justifies our use of the extreme relativistic expression for the number density. For comparison, the nonrelativistic expression for the number density of protons and neutrons, n_N, given by

$$n_N = n_{\bar{N}} = \frac{4}{(\hbar c)^3}\left[\frac{m_N c^2 kT}{2\pi}\right]^{3/2} \exp(-m_N c^2/kT) \tag{18}$$

has a value of 0.17 fm^{-3}. This is the same as that of normal nuclear matter, $n_0 = 0.17$ fm^{-3}. Hence, one can see that the transition should indeed occur in the 200–300-MeV region.

Figure 5 shows the behavior of the change in the effective mass of the quarks as a function of temperature. At high temperature the masses approach the "bare" values. The values of the effective masses at the critical temperature are the bare masses of 8.3, 15, and 300 MeV (for the u, d, and s quarks, respectively) plus about 900 MeV. These are larger than the values usually ascribed to constituent quarks, as one would expect if there is a latent heat associated with the transition. Below the critical temperature, $n_{\rm tot} = 0$ is the solution and Eq. (8) implies the effective quark mass is infinite. This is suggestive of the situation in QCD where a calculation of the confined-quark self-energy, e.g., by solv-

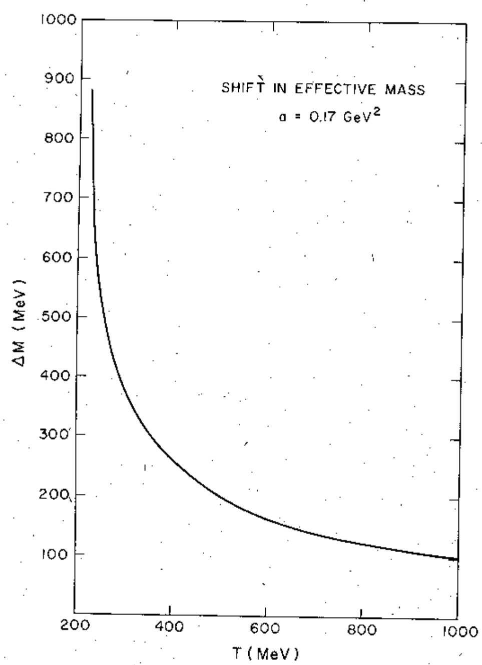

FIG. 5. Change in the effective masses of u, d, and s quarks as a function of temperature at zero chemical potential.

ing the Schwinger-Dyson equation, yields an infinite result.[26,27]

III. FINITE BARYON NUMBER

While the zero-baryon-number-density example is important for the early universe[28] $[(n_B-n_{\bar{B}})/n_B \sim 10^{-10\pm1}$ for this temperature range of the early universe; this ratio does not become significantly larger until the temperature has dropped to the tens of MeV range], for experimental tests or applications to present-day astronomical problems such as neutron-star cores, one must find out the nature of the transition at finite n_B. Bag-model and other estimates[13–17] of this transition density at zero temperature generally give a net baryon-number density of 5 to 20 times n_0.

The formalism used above for $n_B=0$ can easily be generalized to finite n_B by the replacement of the energy in the free-particle Eq. (7) by

$$E_i \to [(m_i+V)^2+q^2]^{1/2} \pm \mu \tag{19}$$

for the effective mass case, or

$$E_i \to (m_i^2+q^2)^{1/2}+V \pm \mu \tag{20}$$

for the background-potential case. The $-$ ($+$) sign of the chemical potential will be used for baryons (antibaryons) so that a positive value of μ will correspond to a baryon excess.

We will first develop an approximation for handling this at $T \to 0$. Again, using the background-potential approach as an example, Eq. (7) for massless quarks with the substitution (20), can be written at low temperature as

$$n_{q_i} \approx \frac{1}{2\pi^2}\int_0^{q_0} \frac{q^2 dq}{e^{(E+V-\mu)/kT}+1} \approx \frac{1}{2\pi^2}\int_0^{q_0} q^2 dq \tag{21}$$

for quarks, and

$$n_{\bar{q}}=0 \tag{22}$$

for antiquarks if $\mu>0$. The integral is truncated at q_0 such that

$$q_0=\mu-V\,. \tag{23}$$

This is obviously equal to

$$n_{q_i}=\frac{1}{6\pi^2}(\mu-V)^3\,. \tag{24}$$

As above, this equation has a critical density below which the real solution vanishes. The chemical potential at this point is

$$\mu_c=2[\kappa(6\pi^2)^{1/3}]^{1/2} \tag{25}$$

corresponding to a critical density of

$$n_{ic}=\left[\frac{1}{6\pi^2}\right]^{1/2}\kappa^{3/2}\,. \tag{26}$$

As an example, consider two flavors of massless quarks. Then $g=(2\times3\times2\times1)$ since the antiquark and gluon densities are very small. The usual factor of $\frac{3}{4}$ can be omitted from g because n_{tot} and n_i involve only fermions. Equations (25) and (26) predict $\mu_c=760$ MeV and

$$n_B=\tfrac{1}{3}\sum_i n_{q_i}=4n_{ic}=1.38 \text{ fm}^{-3} \tag{27}$$

which is about 8 times nuclear density. A similar calculation can be done with the effective-mass approach. The chemical potential at the zero-temperature critical density is now

$$\mu_c=[2\kappa(6\pi^2)^{1/3}]^{1/2} \tag{28}$$

but the critical density (for massless quarks) remains the same as for the background-potential calculation. The results of some zero-temperature

TABLE I. Predictions of quark-hadron transition density at zero temperature. Baryon-number densities are in units of $n_0=0.17\ \text{fm}^{-3}$.

Model	Reference	Baryon density (n_0)
Effective mass Background potential	This work	8
Perturbative QCD	8	26
Modified bag model	13	5
Two-phase QCD	17	10–20

calculations are summarized in Table I.

The region $0 \le T \le T_c$ can be solved numerically. The results are shown in Fig. 6 for the quark masses introduced previously. While the $\mu=0$ case is of interest in the early universe, and the $T=0$ case may be important for neutron stars, is there an intermediate region which might be accessible in the laboratory? One possibility is the collision of relativistic heavy ions. We will follow a method used in Ref. 8 to get an estimate of the kind of energy needed in a near central collision of two heavy ions to access this transition.

Shown in Fig. 7 is the energy-density—vs—baryon-number-density relationship of our two models. The numerical routine used to calculate the energy density did not have excellent convergence,

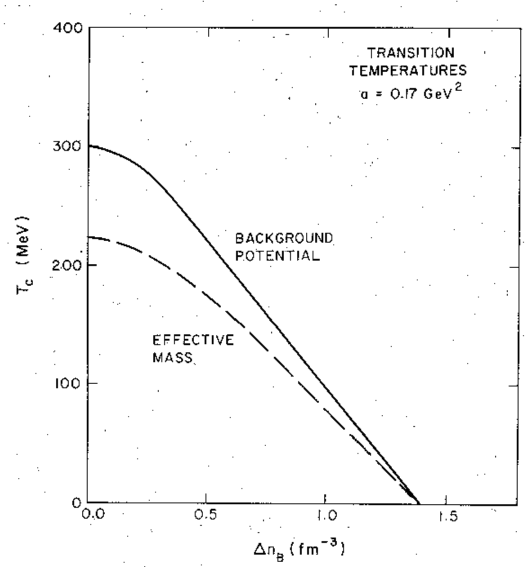

FIG. 6. Quark-hadron transition region for the effective-mass (dashed curve) and background-potential (solid curve) models. The numerical solution worked only in the range $0 \le n_B \le 0.5\ \text{fm}^{-3}$. The curves were then joined to the $T=0$ estimate of Eq. (27).

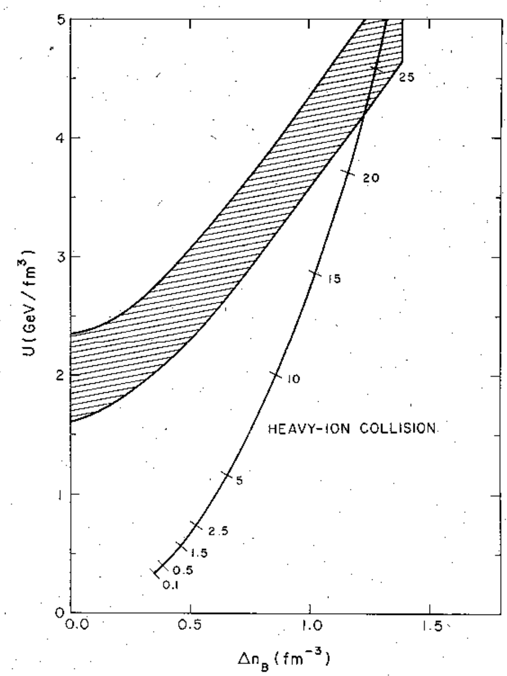

FIG. 7. Energy-density—vs—number-density relation estimated for a heavy-ion collision (full curve) and the two models presented here (cross hatched region). The cross marks on the heavy-ion curve indicate the laboratory kinetic energy per nucleon of the incident heavy ion.

and resulted in a 5% uncertainty in the estimated energy densities.

In calculating these curves, we used a correction procedure suggested by Olive[2] to take care of double-counting the potential. We define the energy density U by

$$U=\sum_i \int E_i dn_i + f(n)\ , \tag{29}$$

where the energy is defined by Eqs. (8) and (9) and

$$dn_i = \frac{d^3 q_i}{(2\pi)^3}\{\exp[(E_i \pm \mu)/kT] \pm 1\}^{-1}\ . \tag{30}$$

The extra term $f(n)$ is determined by the self-consistency condition

$$E_i = \delta U/\delta n_i \tag{31}$$

which yields

$$\frac{\partial f}{\partial n} = -\sum_i \int (\partial E_i/\partial n) dn_i\ . \tag{32}$$

For the background-potential case, one finds

$$f(n)=(\kappa/2)n^{2/3} . \tag{33}$$

For the effective-mass case, we are unable to obtain an analytic expression for $f(n)$. However, it is simple to show that Eq. (33) is an upper bound for $f(n)$ in this case, and so we will use it for calculating the energy density in both models.

The estimated heavy-ion curve is obtained in the following way. In the center-of-mass frame of two identical heavy ions, they will appear to be equally Lorentz contracted into a volume $V(E)$. We will assume that when they collide, all of the nucleons are in this volume, and that all of the center-of-mass kinetic energy is deposited into this volume. The energies quoted in Fig. 7 are the laboratory-frame energies corresponding to these center-of-mass energies. One can see that the curves intersect at about 20–25 GeV/nucleon laboratory energy, which is beyond present-day accelerators but certainly technologically achievable. Again, if one takes the explicit two-phase approach that the critical and transition temperatures are not the same, then the transition densities at nonzero chemical potential will be higher than what we have estimated here. For a discussion of possible experimental signatures of this transition, the interested reader is referred to Ref. 29.

IV. SUMMARY

We have described a model for the quark-plasma phase at finite temperature and density. Although not directly related to QCD, the model closely mimics the behavior anticipated in that theory. Our calculation shows that if the single-particle interaction energy decreases with density there naturally occurs a critical temperature and density below which the plasma phase cannot exist. This results from solving self-consistently for the quark and gluon densities. At high temperature and density the system approaches an ideal gas as suggested by QCD. Below the critical temperature the self-consistent equations yield only the trivial zero-density solution which implies infinite quark and gluon self-energies. The infinite self-energy is reminiscent of the QCD statement that confined quarks and gluons have no mass shell.

Numerically solving the equations for the number density, we find that the transition occurs at 300 and 224 MeV for the background-potential and effective-mass approaches, respectively, at zero-net baryon number. Because of the approximation procedures involved, the numerical determination of the critical temperatures are uncertain to 2 MeV. In the zero-temperature limit, the critical net baryon number density is found to approach ± 1.4 fm^{-3} for both of the models examined. These numbers will depend upon the value chosen for the parameter a and vary approximately like $a^{1/2}$ for the temperature and $a^{3/2}$ for the density. All of these results are similar to those obtained by other authors using different techniques.

A comparison with the energy–baryon-number–density relationship that one might expect to find in near-central collisions of relativistic heavy ions has also been examined. This transition region should be accessible by an accelerator with a beam of at least 20 GeV/nucleon, although there is a 30% uncertainty in this estimate.

Note added. Since this paper was submitted, an extension of the work of Ref. 2 to finite chemical potential has been completed. See K. A. Olive, Nucl. Phys. B198, 461 (1982).

ACKNOWLEDGMENT

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