PHYSICAL REVIEW C VOLUME 29, NUMBER 3 MARCH 1984

Approach to chemical equilibrium in thermal models

David H. Boal*
*Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
and Physics Department, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6*

(Received 14 October 1983)

The experimentally measured (μ^-, charged particle)/(μ^-,n) and (p,n/p,p′) ratios for the emission of energetic nucleons are used to estimate the time evolution of a system of secondary nucleons produced in a direct interaction of a projectile or captured muon. The values of these ratios indicate that chemical equilibrium is not achieved among the secondary nucleons in noncomposite induced reactions, and this restricts the time scale for the emission of energetic nucleons to be about 0.7×10^{-23} sec. It is shown that the reason why thermal equilibrium can be reached so rapidly for a particular nucleon species is that the sum of the particle spectra produced in multiple direct reactions looks surprisingly thermal. The rate equations used to estimate the reaction times for muon and nucleon induced reactions are then applied to heavy ion collisions, and it is shown that chemical equilibrium can be reached more rapidly, as one would expect.

I. INTRODUCTION

Recently, we[1] made an attempt to extend the thermal model traditionally used in heavy ion physics[2] into the domain of proton induced reactions. Of particular interest was the question of whether the high energy tails of the (p,p′) and (p,π) inclusive spectra could be adequately described as arising from a system in thermal equilibrium. The difficulty in applying the thermal model to this region is that the inferred temperatures are sufficiently large that only a subset of the target nucleons can be involved. This is in contrast to the emission of low energy ejectiles (of kinetic energy less than 10 MeV) which show a temperature consistent with the kinetic energy of the incident projectile being spread over most of the nucleus.[3]

In estimating the number of nucleons in the hot source, several approaches were used. Having determined the temperature and source rapidly from an analysis of the data, one can use conservation of energy and momentum to determine over how many nucleons the projectile's energy and momentum are spread. Both estimates give about ten nucleons for the source size in a heavy nucleus such as tantalum or lead. These calculations probably overestimate the number of nucleons in the source, in that the projectile may not have necessarily lost all of its energy and momentum to the target. A different approach would be to integrate the emitted proton spectrum to get the proton multiplicity, and then multiply by two to get the number of nucleons [this assumes the (p,p′) cross section is the same as the (p,n) cross section; see the following]. This calculation gives about three nucleons in the 100—1000 MeV incident kinetic energy range, although the result is energy dependent. This is an underestimate in that many of these hot systems may evolve further and cool down, their evaporative nucleons not contributing to the high energy tails.

Taken at face value, then, these results disagree, although a more detailed calculation will lessen and perhaps eliminate the disagreement. What they do show is that the size of the source is small, an average of the estimates being six nucleons. This is consistent with a geometrical estimate which calculates the impact parameter averaged number of nucleons in a tube, with radius equal to the proton radius, taken along a straight line trajectory through the target nucleus. Although this number of nucleons is small, it is probably enough such that final state interactions among them could take a momentum distribution for the nucleons in the interaction region, which is not too far from thermal, and thermalize it. The purpose of this paper will be to investigate whether the nucleons can reach chemical, as well as thermal, equilibrium.

In heavy ion physics, subthreshold K production has been used to estimate the degree to which chemical equilibrium is achieved. Mekjian[4] has shown that, although pions rapidly thermalize in high energy heavy ion collisions, the estimated lifetime of the state of hot nuclear matter is not sufficiently long to allow kaons to come into chemical equilibrium. Proton induced reactions can use other ejectiles to measure the extent of chemical equilibrium, for example, the ratios of cross sections in the (p,p′)/(p,n) reactions[5] and (p,π^+)/(p,π^-) reactions.[6] Both of these ratios should be about unity for an $N=Z$ target at chemical equilibrium. In fact, after correcting for the N/Z ratio of the target, experimentally both are of the order 2, depending on the target. These values are closer to what one would expect from a model in which the ejectiles are produced in a direct interaction, with only a small amount of multiple scattering present, enough to bring the particles into thermal equilibrium, but not enough for chemical equilibrium.[7]

Another piece of evidence regarding the approach to chemical equilibrium comes from the measurement of neutron and charged particle emission following μ^- capture.[8] In the direct interaction picture, μ^- capture takes place via the reaction

$$\mu^-+\mathrm{p}\rightarrow\mathrm{n}+\nu_\mu\ . \tag{1}$$

Hence, in the absence of multiple scattering, one would expect only neutrons to be emitted, rather than protons. Experimentally,[9–13] the ratio of $(\mu^-,\mathrm{p})/(\mu^-,\mathrm{n})$ for nucleons emitted with energies greater than 15 MeV is observed to be about $\frac{1}{2}$ for light targets such as Al or Cu. [What is usually measured is the charged particle spectrum rather than (μ^-,p). Since the alpha yield is generally smaller than the proton yield,[13] we will equate the charged particle spectrum with the proton spectrum.] Both the (μ^-,p) and (μ^-,n) spectra as a function of ejectile energy look exponential with an apparent temperature of 7–10 MeV (with some deviations from this range). Again, then, a picture emerges in which there is a direct interaction followed by a small amount of multiple scattering, but not sufficient charge exchange to bring the protons into chemical equilibrium.

This paper will treat the $(\mu,\mathrm{p})/(\mu,\mathrm{n})$ and $(\mathrm{p},\mathrm{n})/(\mathrm{p},\mathrm{p}')$ ratios in some detail; the pion production calculations involve at least three body phase space and would make the paper overly long (they will be treated in a later paper on both π and K production). Section II will review muon capture and its description in terms of the thermal model. Calculations of NN collision rates will be presented in Sec. III, and the question of nucleon chemical equilibrium will be addressed. The two ratios of interest mentioned above have only been measured at low temperature and density, different from what one would expect for heavy ion collisions. In Sec. IV, therefore, these calculations are extended to heavy ion densities, and it is shown that chemical equilibrium among the nucleons is reached more rapidly. Because these calculations indicate that the lifetime of the hot interaction region is on the order of 1×10^{-23} sec, in Sec. V we examine the question of whether there is even enough time for thermalization. We will show that the initial momentum distribution of the struck nucleons is not that far from a thermal distribution, so that few subsequent collisions are required to thermalize it. Our conclusions are summarized in the last section.

II. MUON CAPTURE

Neutron emission following μ^- capture has been studied for some time.[8] What we wish to present here is a thermal model approach to the energy spectrum of the emitted nucleon. The interesting physics questions involved in calculating the absolute capture rate will not be addressed here, as they are well covered elsewhere.

In muon capture by a free proton, most of the energy released is carried off by the neutrino, as a consequence of momentum conservation. The recoiling neutron will have a unique energy of 5 MeV. When the capture takes place in a nucleus, an energy spectrum is observed for the emitted neutrons.[1,14–16] At low neutron kinetic energy, the spectrum falls steeply and is describable in terms of evaporation from a source with a small temperature, on the order of an MeV. Above a neutron kinetic energy of about 5 MeV, the falloff is less steep, corresponding to an apparent temperature of 7–14 MeV (data with better statistics tend to favor a lower temperature). In a thermal model, the "hot" interaction region cannot be the entire nucleus, since there is only about 100 MeV of energy available for thermalization to begin with.

The reason that there is a spread in energy, of course, is that the residual nucleus can carry away momentum which the neutrino would otherwise have to carry, without carrying off much energy. Imagine that the muon is captured on a zero temperature Fermi gas of protons. Then, integrating over the momenta $\vec{k}$ of the capturing protons (kinematic labels are shown in Fig. 1) would give an average kinetic energy to the neutron produced in the reaction of about 25 MeV, assuming a separation energy of 10 MeV. Identifying this energy with $(\frac{3}{2})T$ would give a temperature of 17 MeV, not far removed from the range of temperatures indicated by experiments. Hence, the produced neutron would only have to share its energy with one other nucleon to give an average energy consistent with experiment.

The Fermi gas approach will not, however, reproduce the energy spectrum without multiple scattering. It is easily calculated that, if the maximum momentum available in the nucleus is 270 MeV/c (corresponding to a Fermi energy of 38 MeV), then the maximum neutron energy will be 50 MeV. Such a sharp cutoff is not observed experimentally. In a calculation which uses an effective vertex function to account for both a softer momentum distribution than the zero temperature Fermi gas, and multiple scattering, Singer, Mukhopadhyay, and Amado[17] are able to reproduce a temperature of 12 MeV easily. (For other theoretical work on this problem, see Refs. 18–22.) Both of these approaches, then, indicate that only a small amount of multiple scattering is required to produce an apparently thermal spectrum with a temperature of 7–10 MeV.

Before proceeding to a discussion of reaction rates, we should check that the normalization of the thermal model is consistent with the data. The data have been parametrized as

$$\frac{dN}{dT_q}=N_0\exp(-T_q/E_0)\ , \tag{2}$$

where T_q is the kinetic energy of the observed neutron and N is the number of neutrons per capture. The parameter E_0 is closely related to the temperature, which enters into a Maxwell-Boltzmann distribution, (assumed here) as,

$$\frac{dN}{dT_q}=\frac{4\pi}{(2\pi m_{\mathrm{n}}T)^{3/2}}N_{\mathrm{n}}qm_{\mathrm{n}}\exp(-T_q/T)\ . \tag{3}$$

Here, N_{n} is the total number of neutrons emitted. For the charged particle experiments, E_0 is typically in the range

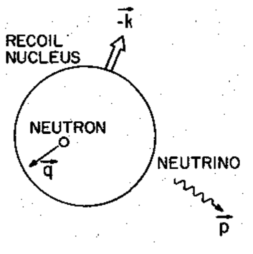

FIG. 1. Kinematic labels for muon capture in a nucleus.

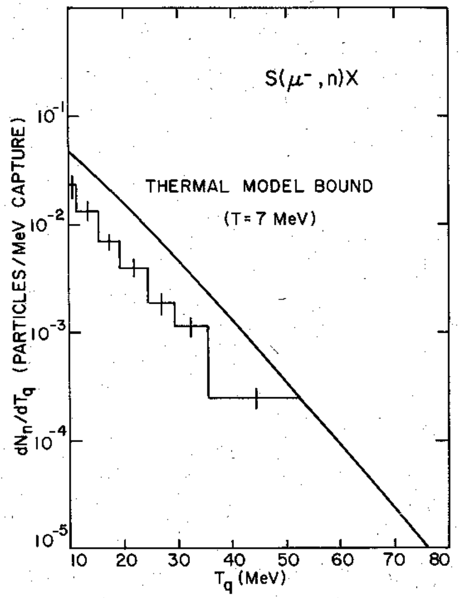

FIG. 2. A comparison of the thermal spectrum expected for a group of two neutrons and a temperature of 7 MeV. The data are from (μ^-,n) on a sulphur target, from Ref. 14.

of 8 MeV, corresponding to a temperature of 7.3 MeV.

As a test, then, we will evaluate Eq. (3) for $T=7$ MeV and $N_n=2$, which is what we expect for the final state after thermalization. Since it is estimated[8] that about half of the captured neutrons go through giant resonance absorption, rather than the direct mechanism assumed here, then the result from Eq. (3) will be divided by 2 in order to compare with data quoted per capture. Data[14] for (μ^-,n) on a sulphur target are shown in Fig. 2.

The theoretical curve represents an upper bound on the data in that not all of the hot zones produced in the direct reaction will emit a neutron in the time frame covered by the 17—7 MeV temperature range. Many of these systems will evolve through to the low temperature regime of classical evaporation phenomena. Indeed, the number of nucleons in the evaporative region is at least double what one would expect by extrapolating the high energy tail. One cannot cleanly interpret this as the number of systems which go on to complete equilibration, but it indicates that a further reduction by two might be expected to make the bound in Fig. 2 into an absolutely normalized prediction. In any event, the data are not inconsistent with a thermal model description. Of course, this may not be the only description.[17–22]

III. REACTION RATES

We have shown in Sec. II that in the direct muon capture reaction, only a very few collisions are required to lower the initial neutron distribution, with an apparent temperature of 17 MeV, to the experimentally observed distribution with an apparent temperature of 7 MeV. We now wish to use the (μ,p)/(μ,n) ratio to estimate the time scale involved in thermalization, and see if this is consistent with only a few collisions occurring. To proceed, we need to know the reaction rates for these processes.

The reaction rate for a particle, which we will label A, to react with a gas of particles B, with number density n_B, is given by

$$\lambda_{AB}=n_B 4\pi\left[\frac{\mu}{2\pi T}\right]^{3/2}\int v^3\sigma(v)e^{-(\mu v^2/2T)}dv\ , \tag{4}$$

where μ is the reduced mass of the AB pair and v is their relative velocity. We have assumed that the temperature is sufficiently low that one can use nonrelativistic kinematics, but sufficiently large that the nucleons have a Maxwell-Boltzmann energy distribution.

To perform the integration, the cross section as a function of energy must be known. Both the pp (which we will assume is the same as the nn) and pn cross sections have the form[23]

$$\sigma=\sigma^0/p_{\rm lab}^{\delta} \tag{5}$$

in the 20—160 MeV incident proton kinetic energy range. Since the exponent δ has a value of 2.2 for both of these reactions, we will take the liberty of fitting these cross sections to the form

$$\sigma(v)=\sigma^0/v^2 \tag{6}$$

for the sake of obtaining an easily manipulated expression. With this fit, $\sigma_{\rm pn}^0=16.7$ mb and $\sigma_{\rm pp}^0=5.1$ mb. Then, λ_{AB} has the particularly simple form

$$\lambda_{AB}=\left[\frac{2\mu}{\pi T}\right]^{1/2} n_B\sigma^0\ . \tag{7}$$

Since the charge exchange cross section is about half the total pn cross section, then

$$\begin{aligned}\lambda(\mathrm{n}\rightarrow\mathrm{n})&=\frac{\rho}{2}\sigma^0(\mathrm{n}\rightarrow\mathrm{n})\left[\frac{2\mu}{\pi T}\right]^{1/2},\\ \lambda(\mathrm{n}\rightarrow\mathrm{p})&=\frac{\rho}{2}\sigma^0(\mathrm{n}\rightarrow\mathrm{p})\left[\frac{2\mu}{\pi T}\right]^{1/2},\end{aligned} \tag{8}$$

where

$$\sigma^0(\mathrm{n}\rightarrow\mathrm{n})=1.35\ \mathrm{fm}^2$$

and

$$\sigma^0(\mathrm{n}\rightarrow\mathrm{p})=0.84\ \mathrm{fm}^2\ ,$$

and ρ is taken to be normal nuclear matter density, 0.17 fm^{-3}.

Defining $\lambda_{\rm el}$ as the n→n rate and $\lambda_{\rm cx}$ as the n→p rate, one can see from Fig. 3 that the time taken to reach thermal equilibrium must be fairly short, 1×10^{-23} sec.

In other words, the initial neutron has spread its energy over one other neutron and one proton in this time period. Indirect experimental evidence supporting this comes from the (μ,pxn) activation experiments.[24] Here, one finds that the number of neutrons, x, accompanying an ejected proton is of the ratio 1:6:4:4 for $x=0,1,2,3$. (The comparison is not exact because the experiment integrates over evaporative protons as well as the high energy tails,

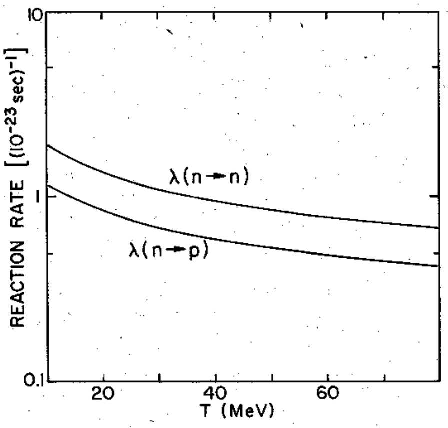

FIG. 3. Estimated reaction rates for n→n (pp and pn elastic scattering) and n→p (pn charge exchange) reactions.

although evaporative proton emission is suppressed with respect to neutron emission because of Coulomb effects.)

To obtain a better estimate of the time scale, we use the time evolution of the proton to neutron ratio. First, we discuss proton induced reactions. In a direct knockout model in which the incident proton scatters off an off-shell proton or neutron to produce the observed nucleon (sometimes by charge exchange), one finds[25] that two high energy protons are ejected for every high energy neutron. This is then the starting abundance before thermalization. Clearly, since the observed[5] (p,n)/(p,p′) ratio is very similar to the starting ratio for a wide variety of targets, there is very little time for multiple scattering. Hence, the lifetime of the high temperature state must be very short, just enough for one or two collisions. Beyond this time, it is likely that either the nucleons escape or they undergo yet another set of collisions which drops the temperature significantly. Similarly, for a variety of targets the energy spectra of the (μ,p) data[9] are about 0.6 times the size of the (μ,n) data[14] for similar energies and targets. We will use the muon data rather than the (p,n) data to estimate the lifetime, since the calculation is cleaner.

We will use λ_{cx} to study the time evolution of a coupled set of charge exchange rate equations:

$$\begin{aligned}\frac{dn_{\mathrm{n}}(t)}{dt}&=[-n_{\mathrm{n}}(t)+n_{\mathrm{p}}(t)]\lambda_{\mathrm{cx}}\,,\\ \frac{dn_{\mathrm{p}}(t)}{dt}&=[+n_{\mathrm{n}}(t)-n_{\mathrm{p}}(t)]\lambda_{\mathrm{cx}}\,.\end{aligned}\tag{9}$$

As before, the n's are number densities. Choosing

$$\lambda_{\mathrm{cx}}=1\times(10^{-23}\ \mathrm{sec})^{-1}$$

in the 10—20 MeV temperature range, the proton to neutron ratio of a system, which is initially energetic neutrons, is shown in Fig. 4. One can see that it takes about 0.7×10^{-23} sec for the p/n ratio to evolve to the 0.6 observed in muon capture.

Furthermore, it is clear that this conclusion will be true at somewhat higher temperatures than those appropriate to the experiments completed thus far. Because the cross sections decrease with lab momentum, the rate also decreases as $T^{1/2}$. Hence, at temperatures of 50—60 MeV, for example, the high temperature states should be similarly short lived. As the temperature is raised beyond this, pion production (which has been omitted here) will become important and the particle number density will increase. This, in turn, will increase the reaction rate and bring the nucleon distributions closer to what one would expect in chemical equilibrium.

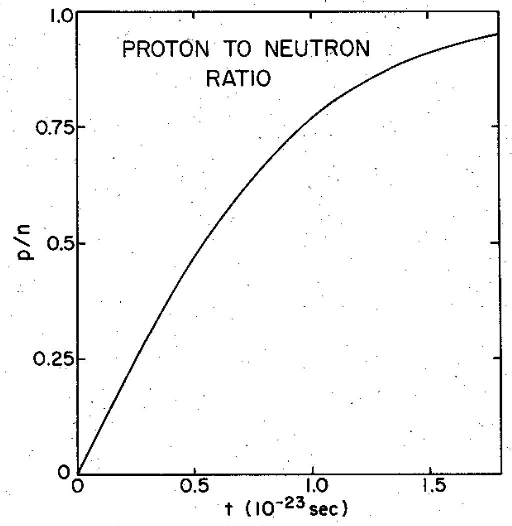

FIG. 4. Estimated time evolution of the proton to neutron ratio for $\lambda_{\mathrm{cx}}=(10^{-23}\ \mathrm{sec})^{-1}$.

IV. HEAVY ION REACTIONS

The tests given above for nucleon chemical equilibrium are more difficult to perform in heavy ion reactions because both the projectile and the target generally have $N\cong Z$, and one would therefore expect the neutron and proton yields to be similar (after correcting for N/Z effects in the target and projectile nuclei, as well as decay products). However, we can extend the calculations presented in Sec. III into the heavy ion domain to see if nucleon chemical equilibrium can be more rapidly achieved than in proton reactions.

At the same incident kinetic energy per nucleon, reactions involving a heavy ion will generally have a higher temperature than ones with a proton. This can easily be visualized in the "rows on rows" approach which would show that there will be more projectile nucleons incident per target nucleon than what one would find in a proton induced reaction. To evaluate the reaction rate at the same kinetic energy per incident nucleon, we need to know the temperature as a function of incident energy. The temperatures found in a thermal model analysis of the (p,p′) reaction on heavy targets are found to depend on incident kinetic energy, K_i, in the following way:

$$T=1.9K_i^{0.46}\ \mathrm{MeV}\,,\tag{10}$$

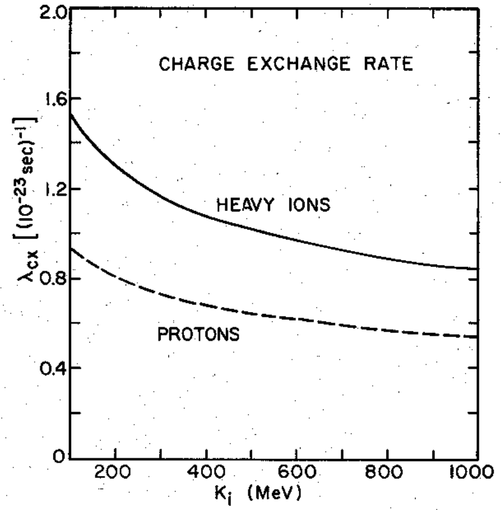

FIG. 5. Comparison of the charge exchange rate for proton and heavy ion induced reactions.

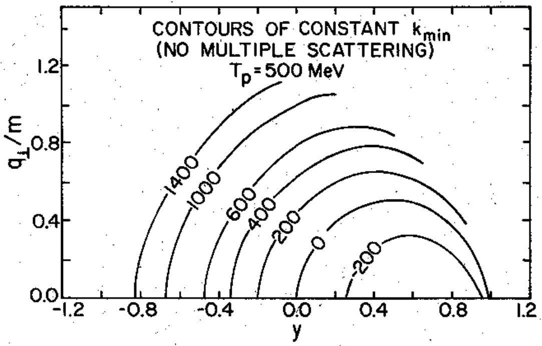

FIG. 6. Contours of constant $k_{\min}$ (label on curves in MeV/c) for an incident proton of kinetic energy equal to 500 MeV on a mass 100 target.

where K_i is in MeV. For heavy ion reactions in the 100–800 AMeV

$$T = 2.1K_i^{0.52}\ \text{MeV} \tag{11}$$

fits the data[26] reasonably well in the energy range of interest. Here, K_i refers to the incident kinetic energy per nucleon.

Using these temperatures, and assuming that the nucleon density in the interaction region of a heavy ion collision is double that of a proton induced reaction, the rates shown in Fig. 5 can be calculated. We have plotted the charge exchange rates for both proton induced and heavy ion induced reactions at the same K_i. As one would expect, the heavy ion reactions will come into both thermal and chemical equilibrium faster than proton induced reactions.

V. RAPID THERMALIZATION

In the above analysis, we have demonstrated that the time scale involved in energetic nucleon emission is very short, and that the nucleons may not reach chemical equilibrium. How is it, then, that the individual proton or neutron spectra look so nearly thermal? The answer is that the knockout process itself gives a spectrum which is not too different from a thermal shape.

For example, in the direct knockout picture,[27–29] the incident proton strikes a target nucleon with momentum $\vec{k}$ to produce the observed nucleon. Since the single particle momentum distribution is usually a rapidly falling function of k, then the magnitude of the cross section will depend significantly on the minimum value of k ($k_{\min}$) which is required to produce the observed particle. In a very crude approximation, then, values of $\vec{q}$ (the observed nucleon momentum) which have the same $k_{\min}$ should have similar cross sections. Shown in Fig. 6 is a plot of selected values of $k_{\min}$ for an incident proton of kinetic energy 500 MeV on a mass 100 target. Only one of the target nucleons is put on mass shell (with momentum $\vec{q}$), the rest of the nucleus recoiling coherently with overall momentum $\vec{k}$. Rather than plot contours of constant $k_{\min}$ against $q_\perp$ and $q_\parallel$ (the perpendicular and parallel components of q with respect to the beam, respectively), the contours are plotted against $q_\perp/m$ and

$$y = \tfrac{1}{2}\ln[(E+q_\parallel)/(E-q_\parallel)]\ .$$

Nonrelativistically, these quantities reduce to the perpendicular and parallel components of the velocity, respectively.

If the source of the ejectiles were truly thermal, then contours of constant Lorentz invariant cross section would be semicircular (in the nonrelativistic limit) with a common origin at the velocity of the source. One can see that the contours in Fig. 6 are surprisingly semicircular. Now, what in fact contributes to a given inclusive spectrum from a heavy target is of the order 3 scatterings of the projectile. That is, the initial state before thermalization begins will be a sum of the secondary nucleons produced in roughly three direct interaction scatterings of the incident nucleon. For the sake of calculation, it will be assumed that an incident proton loses $\frac{1}{3}$ of its energy per collision. We then sum the contributions, each of which is taken to be of the form $\exp(-k_{\min}/k_0)$ (see Refs. 27–29.) The parameter k_0 is assigned a value of 100 MeV/c. The result is shown in Fig. 7. Because the maximum value of y for each collision will decrease with the incident energy, the apparent centers of the circles will also be shifted to smaller y. Inclusion of the NN scattering amplitude, which has its maximum at small momentum transfer, would further accentuate this trend.

Hence, we can see that at the beginning of the thermalization stage the spectra of struck nucleons will already look somewhat thermal, with a source velocity of less than 0.4c (vs a thermal model analysis result of 0.2c). Clearly, it will not take much multiple scattering of the ejected nucleons to produce an approximately thermal spectrum.

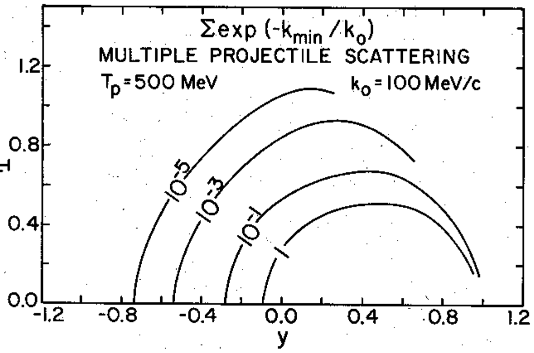

FIG. 7. Contribution of three collisions in a knockout model to $\sum\exp(-k_{\min}/k_0)$, assuming an incident proton with 500 MeV in energy loses one-third of its energy per collision.

VI. SUMMARY

We have used the experimentally measured $(\mu^-,p)/(\mu^-,n)$ and (p,n)/(p,p′) ratios to obtain a limit on the time before freezout of the target nucleons involved in preequilibrium nucleon emission. In the muon case, we estimated that the neutrons produced in the capture process have an initial average kinetic energy of about 25 MeV arising from the Fermi motion of the capturing protons. In cooling to the temperature of 7—10 MeV measured experimentally, the energy of the initial neutron need be spread over only one other nucleon, which occurs in too short a time to bring the target protons into chemical equilibrium. The observed p/n ratio for muon capture allows one to estimate the thermalization time as 0.7×10^{-23} sec. This estimate is subject to uncertainty in the charge exchange rate, whose calculation assumed Maxwell-Boltzmann statistics and neglected the effects of Pauli blocking. The lower the temperature of the system, the more important these effects will become. This situation is contrasted to reactions involving heavy ions, where the rates for achieving chemical equilibrium are about double those for proton reactions at the same incident energy per nucleon. The energy spectrum of nucleons produced in the initial interactions of the projectile was shown to be surprisingly thermal in shape, so it is understandable that these nucleons can come into thermal equilibrium after only a very few subsequent scatterings.

ACKNOWLEDGMENTS

The author wishes to thank the theory group at the Nuclear Science Division of Lawrence Berkeley Laboratory for its hospitality while this work was completed, and for several discussions. He also wishes to thank the Natural Sciences and Engineering Research Council of Canada for financial support. This work was supported by the U. S. Department of Energy under Contract DE-AC03-76SF00098.

---

[*] Permanent address: Physics Department, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6.

[1] D. H. Boal and J. H. Reid, Phys. Rev. C (in press).

[2] For a review, see S. Das Gupta and A. Z. Mekjian, Phys. Rep. 72C, 133 (1981).

[3] For a review, see J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

[4] A. Z. Mekjian, Nucl. Phys. A384, 492 (1982).

[5] B. D. Anderson, A. R. Baldwin, A. M. Kalenda, R. Madey, J. W. Watson, C. C. Chang, H. D. Holmgren, R. W. Koontz, and J. R. Wu, Phys. Rev. Lett. 46, 226 (1981).

[6] D. R. F. Cochran, P. N. Dean, P. A. M. Gram, E. A. Knapp, E. R. Martin, D. E. Nagle, R. B. Perkins, W. J. Shlaer, H. A. Thiessen, and E. D. Theriot, Phys. Rev. D 6, 3085 (1972).

[7] The effects of multiple scattering in pion induced reactions are also significant; see M. M. Sternheim and R. R. Silbar, Phys. Rev. Lett. 34, 824 (1975).

[8] For a review, see P. Singer, *Springer Tracts in Modern Physics* (Springer, Berlin, 1974), Vol. 71, p. 39.

[9] K. S. Krane, T. C. Sharma, L. W. Swenson, D. K. McDaniels, P. Varghese, B. E. Wood, R. R. Silbar, H. D. Wohlfahrt, and C. A. Goulding, Phys. Rev. C 20, 1873 (1979).

[10] H. Morinaga and W. F. Fry, Nuovo Cimento 10, 308 (1953).

[11] Yu. A. Batusov and R. A. Eramzhyan, Fiz. Elem. Chastits At. Yadra 8, 229 (1977) [Sov. J. Part. Nucl. 8, 95 (1977)].

[12] A. O. Vaisenberg, E. D. Kolganove, and N. V. Rabin, Yad. Fiz. 11, 830 (1970) [Sov. J. Nucl. Phys. 11, 464 (1970)].

[13] Yu. G. Budyashov, V. G. Zinov, A. D. Konin, A. I. Mukhin, and A. M. Chatrchyan, Zh. Eksp. Teor. Fiz. 60, 19 (1971) [Sov. Phys.—JETP 33, 11 (1971)].

[14] R. M. Sundelin and R. M. Edelstein, Phys. Rev. C 7, 1037 (1973).

[15] M. H. Krieger, thesis, Columbia University, 1969 (unpublished).

[16] W. O. Schröder, U. Jahnke, K. H. Lindenberger, G. Röschert, R. Engfer, and H. K. Walter, Z. Phys. 268, 57 (1974).

[17] P. Singer, N. C. Mukhopadhyay, and R. D. Amado, Phys. Rev. Lett. 42, 162 (1979).

[18] M. Lifshitz and P. Singer, Phys. Rev. Lett. 41, 18 (1978).

[19] C. Ishii, Prog. Theor. Phys. 21, 663 (1959).

[20] M. Bertero, G. Passatore, and G. A. Viano, Nuovo Cimento 38, 1669 (1965).

[21] H. Überall, Phys. Rev. 139, B1239 (1965), and references therein.

[22] J. Bernabéu, T. E. O. Ericson, and C. Jarlskog, Phys. Lett. 69B, 161 (1977).

[23] Particle Data Group, Lawrence Radiation Laboratory Report UCRL-200000NN, 1970.

[24] A. Wyttenbach, P. Baertschi, S. Bajo, J. Hadermann, K. Junken, S. Katcoff, E. A. Hermes, and H. S. Pruys, Nucl. Phys. A294, 278 (1978).

[25] D. H. Boal, Phys. Rev. C 25, 3068 (1982).

[26] B. V. Jacak (private communication).

[27] R. D. Amado and R. M. Woloshyn, Phys. Rev. Lett. 36, 1435 (1976).

[28] S. Frankel, Phys. Rev. Lett. 38, 1338 (1977).

[29] S. Frankel, Phys. Rev. C 17, 694 (1978).