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Individualism and Differential Calculus

The element of time is a chief cause of those difficulties in economic investigations which make it necessary for man with his limited powers to go step by step; breaking up a complex question, studying one bit at a time, and at last combining his partial solutions into a more or less complete solution of the whole riddle. In breaking it up, he segregates those disturbing causes, whose wanderings happen to be inconvenient, for the time in a pound called *Ceteris Paribus*.... With each step more things can be left out of the pound.... We thus approach by gradual steps towards the difficult problem of the interaction of countless causes. ... It is true that we do treat variables *provisionally* as constants. But it is also true that this is the only method by which science has ever made any great progress in dealing with complex and changeful matter, whether in the physical or moral world.

Alfred Marshall [1926/64, pp.304, 306, 315 (footnote 1)]

Fundamentally, our ultimate method of explanation in economic theory has not changed from that espoused by Marshall. It is merely the partial equilibrium (*ceteris paribus*) explanation of the behavior of any individual decision-maker based on the explicit use of the standard idea of a partial derivative. All non-natural and non-individualist variables (prices, gross national product, etc.) are explained as logical consequences of the behavior of all individuals. The only thing ever disputed is whether one can explain why the economy as a whole would be at an optimum equilibrium state whenever all individuals are in a state of partial equilibrium (i.e. they are maximizing something). Marshall's proposed method for showing that this would be the case breaks the problem of explanation into a sequence of manageable parts such that the

problem can be solved in stages. His method of explanation starts with a very short run where the price is determined solely by demand and ends at a stage where all prices are determined by the natural givens (technology) and the quantities produced and consumed are determined by the given utility functions of the individuals.

It should be noted that the explanation of even one individual in the literal short-run equilibrium is never complete, according to methodological individualist principles, since by definition of the short run the individual in question faces non-natural constraints (income, prices, capital stock, etc.) which are considered changeable only in a longer run. A complete explanation of the individual must ultimately explain these non-natural constraints. Of course, in the long-run equilibrium they can be explained *without giving up the idea of a partial equilibrium for the individual*. It is important to recognize that any individual in long-run equilibrium is thereby also in a short-run (partial) equilibrium!

1. Long-run General Equilibrium and Individualism

Marshallian partial equilibrium analysis is without any serious problems if we restrict our interest to the state of long-run equilibrium. And when we do restrict our analysis to states of Marshallian long-run equilibrium it is indistinguishable from Walrasian general equilibrium analysis. Where the former postulates the isolated individual in a state of personal equilibrium facing long-run equilibrium ‘constraints’ and prices, the latter looks at any pair of individuals and postulates them in an exchange equilibrium facing exogenous ‘endowments’. Both methods of analysis support a methodological individualist view of the world.

1.1. Compatibility of Walrasian and Marshallian Explanations

For Marshallian analysis, a complete explanation of an individual decision-maker would see the economy in a long-run equilibrium. Any individual selected at random will always be in a short-run equilibrium facing long-run equilibrium constraints. The individual consumer, for example, would face an income that is a consequence of that individual’s supply of labor (or other resources) given long-run equilibrium prices. What is explained here is the individual’s demand decision. Since all other variables have long-run equilibrium values, the individual takes market prices as givens and thereby demands the quantity which, when added to the (optimal) demands of all other participants, just brings the total demand into equality with the supply. The important point here is

that we can always explain the behavior of any single individual we select.

The Walrasian explanation of any individual would begin with a vision of an entire economy in a state of general equilibrium that is identical to a long-run equilibrium. There is no suggestion that we must see it in terms of a Marshallian long-run perspective. Here we are able to explain any two individuals (or two goods, two inputs, etc.) selected from a list of all individuals participating in the economy. If it is a general equilibrium then no matter which two individuals (or goods) we select, the two individuals will be in an exchange equilibrium. It is possible to interpret our simple model of Chapter 1 as such an exchange equilibrium. An exchange equilibrium is one where neither individual can gain without the other losing thus there is no mutually acceptable reason for any change. The paradigm of this analysis is the Edgeworth-Bowley box, which represents the allocation of two goods between two individuals (represented by opposing indifference maps) – see Figure 3.1.

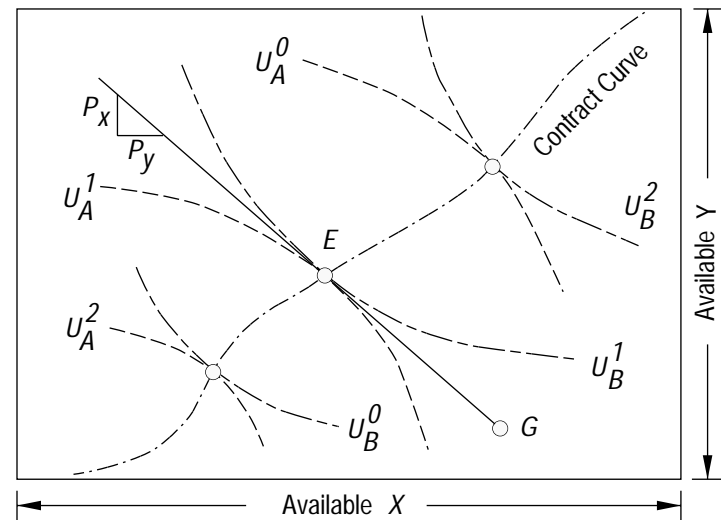


Figure 3.1. Exchange equilibrium

Any exchange equilibrium is represented by a point on the contract curve, that is, on the locus of all points of tangency between the two indifference maps. Whichever tangency point will be the equilibrium allocation depends on the prior initial endowments, also represented by a single point, say G. Whenever there is an exchange equilibrium, the equilibrium prices (and the income distribution) are implicitly

determined. The equilibrium relative price must be equal to the negative value of the slope of the two indifference curves. That is, if our two individuals are Mr A and Mr B, our two goods are X and Y, and MRS_A and MRS_B are the respective negative slopes, then

$$P_X/P_Y = MRS_A = MRS_B.$$

If we have a dollar value for the income of either individual then we can also determine the dollar prices of each good. Already we see a result which is implicit in the Marshallian long-run equilibrium. It would not take much to show that all implications of a Marshallian long-run equilibrium are reproduced in a corresponding Walrasian general equilibrium and vice versa. We note that, like Marshallian analysis, Walrasian analysis can be based on an assumption that each individual calculates a partial derivative. In this case, the slope of the indifference curves is merely the partial derivative that results from holding the level of utility constant during a marginal exchange.

1.2. Individualism and Partial Derivatives

The major importance of explicitly recognizing the use of the partial derivative is that it is the basis for isolating and thereby analyzing the contribution of each individual to the state of equilibrium. For example, in the theory of the firm, the level of output can be *analyzed*, that is, broken down into separate contributions of the individual inputs.

If we are not careful, the partial derivative can also mask individualism from our sight. We will discuss this difficulty first since it will demonstrate how the use of partial derivatives allows us to fulfill the requirements of methodological individualism.

Consider an economy in a state of long-run or general equilibrium and consider any consumer's choice of two goods, X and Y. If the consumer is maximizing utility with respect to these two goods, then he or she will be choosing to consume these goods in such a manner that the marginal rate of substitution (MRS) between them equals their relative price, P_X/P_Y . But, remember that in the state of equilibrium everyone faces the same prices. Thus, in the state of equilibrium all individuals are choosing the same MRS – everyone values the last units bought of every good relative to any other good in exactly the same way. So, is there any real individualism here if everyone is spending their last dollar in exactly the same way?

We must not panic. The appearance of non-individualism can easily be explained away or avoided. It might be said that while all individuals are identical with respect to the marginal demand for X relative to Y, they may differ significantly with respect to the total demand for X relative to

Y. Some individuals may choose a point close to the Y axis and others choose a point close to the X axis. Even though everyone has the same slope for the indifference curve through their chosen point, they may choose different points even if they had the same incomes.

It is not clear whether this way of explaining away any appearance of non-individualism does not also explain away some of the individualistic information content in prices. We need not worry about this in any case since the difficulty, if there is one here, is due solely to the general equilibrium theorist's concept of marginal rates of substitution. The idea of a MRS allows us to see a single individual in a state of equilibrium with respect to any two goods by comparing marginal quantities of those goods rather than calculating the marginal utility of each good. In effect, the individual seems to be in a state of partial equilibrium consistent with the general equilibrium. This now standard conceptual tool was strongly promoted by Hicks in his *Value and Capital* [Hicks, 1939/46] and has dominated microeconomics over the last twenty-five years. The only reason for focusing consumer theory on assumptions about the equilibrium value of the MRS was to avoid assumptions involving the concept of utility as the latter was alleged to be philosophically suspect. Samuelson [1938, 1948, 1950] promoted a method of analysis, Revealed Preference Analysis, which seemed to hold even more promise. With it, supposedly, we could forever avoid the concept of utility, or even the concept of a preference map, by observing and analyzing actual choices and assuming only that the individual, by always knowing what is best, never makes a contradictory choice. For example, if prices do not change, the choices will not change. The promise was forsaken by Houthakker [1950] who showed that any use of Revealed Preference Analysis that would reproduce the usual results of ordinary demand theory based on utility functions must of necessity imply that these two supposedly different approaches are logically equivalent [see further Wong, 1978]. Of course, as a trivial matter, the same holds for the older Hicks-Allen Ordinal Preference Theory based on an assumption of a diminishing MRS rather than a diminishing marginal utility.

If we simply retain the idea of marginal utility – that is, use of the partial derivative of the utility function for each good – then any confusion between individualism and explainable free choice is avoided. To do this we simply recognize the elementary point that the marginal rate of substitution (which is the negative of the slope of the indifference curve) always equals the ratio of the two respective marginal utilities:

$$MRS = MU_X/MU_Y.$$

With this in mind, we could easily say that no two individuals will necessarily have the same marginal utility for the same good and hence individualism would seem to be preserved even on the margin.

This is the most uncomplicated argument for the role of partial derivatives in the service of methodological individualism. It emphasizes that neoclassical explanations are based, not only on maximization, but on the idea of a partial (*ceteris paribus*) equilibrium. Yet, so far it is not a very strong argument so we wish to dig deeper into the fundamentals.

2. Varieties of Individualism in Economic Theory

One of the more fundamental questions that we have just raised concerns how to conceive of individualism whenever it is shown that everyone is identical in some way (e.g. all end up with the same *MRS*). This would seem to put into question just what we mean by individualism. Usually, it is said that everyone has something distinguishing, such as their personal tastes, and that any uniformity between individuals, such as their marginal judgments, is unintentional. This way, it would seem, we can have it both ways. Such may not be the case if we consider the question of why, for example, the marginal productivity of labor diminishes as more labor is hired.

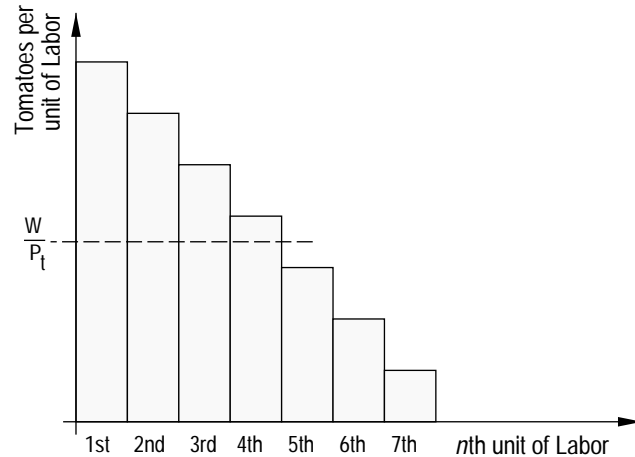


Figure 3.2. Diminishing marginal productivity

On the one hand, we could explain diminishing marginal productivity of labor as the consequence of there being no two individuals alike – pure *liberal individualism*, so to speak. On the other hand, we could

explain it by postulating that all individuals are identical with respect to productivity – pure *egalitarian individualism*, in this case.

2.1. Liberal Individualism

If no two individuals are alike with respect to productivity then the employer can rank them according to their productivity. Figure 3.2 illustrates such a ranking. Let us say that the product produced is tomatoes. If the price of tomatoes (P_t) and the price of each unit of labor (W) is given such that the first person in the ranking harvests more tomatoes than he or she eats (i.e. the productivity of the first person hired is greater than W/P_t) then there is an incentive for the firm to hire the first person. The next ranked person will be less productive and if this person still produces more tomatoes than he or she eats, according to the going wage-rate and price of tomatoes, then this person will be hired too. The firm continues hiring people until the productivity of the next person is less than he or she will eat. Marginal productivity in this case is just the productivity of the marginal individual. And the ranking itself provides the needed display of diminishing marginal productivity of labor.

2.2. Egalitarian Individualism

Now, if we instead claim that all individuals are alike we do not have to give up any hope of explaining diminishing marginal productivity. However, to do so we must yield to effects of some other input which is fixed. For example, let us recognize that we have a fixed amount of land on which to grow our tomato plants – say, ten square meters. If all individuals are alike then no matter which individual is hired first, individual productivity will be the same. The question at issue here is how much will output increase if the input level is doubled – that is, the second person is hired. We know that since there is a fixed amount of land there is a maximum number of people who could stand on the land at the same time and with that maximum number we know the total output is zero since they will be standing on top of all the tomato plants. Since the amount of all other inputs (such as the number of tools) is given and fixed by the *ceteris paribus* requirement of Marshall's method of analysis, when we double the number of people hired they still need to share the tools available and for this reason the output will not double when the input is doubled. So, we can say that whenever the input rises by a certain proportion, if the output rises it does so necessarily by a smaller proportion. This means simply that the ratio of output-to-input, or average productivity, is always falling. Since the average and the margin are the same for the first person hired, a falling average implies a falling margin. And since we know that at some level of input the average productivity is zero (the land is covered with people) we know

the marginal product must be falling from the initial level corresponding to the productivity of the first individual hired. In other words, even though all individuals may be alike we can still explain why the marginal productivity of their collective input is diminishing as more individuals are employed on the same fixed land or with the same fixed inputs.

2.3. *Egalitarian vs. Liberal Individualism*

There is no reason to choose between these two versions of individualism. Either way we can explain why marginal productivity is diminishing with input or output. But we cannot hold both views of the individual since they cannot both be true. The best strategy would be to deny both yet allow ranking where ranking is possible and recognize that the fixity of some inputs always forces a degree of diminishing marginal productivity on the variable inputs. However, we cannot think of all individuals as being identical and at the same time try to explain a world where all inputs are variable, since in this case marginal productivity is fixed and hence not diminishing. If we are going to maintain that prices are given, either we must give up egalitarian individualism or give up universal variability of inputs. It is interesting to note that Paul Samuelson explicitly chose to give up universal variability [Samuelson, 1947/65, p. 85]. By implication he chose to maintain egalitarian individualism.

The only difficulty with maintaining egalitarian individualism in this way is that we beg the question about why some inputs are fixed. Unless it is for some natural reason, our explanation of the individual decision-maker, of the firm in our example, is still incomplete. But even worse, if we give up egalitarian individualism, we have to explain why individuals are different which too easily leads us to psychologistic individualism and its many problems which we noted in the Introduction.

Liberal individualism has the exact opposite problem. If everyone is different, there is the possibility that we could never be able to show that any economy is stable. Since we are, at this point, only interested in seeing how individualism is usually supported in neoclassical models, we will postpone these deeper questions until Part IV where we will consider ways by which they can be overcome.

3. The Long-run Equilibrium as a Special Short-run Equilibrium

What is at stake here is the recognition that all neoclassical explanations of *individuals* are either partial equilibrium or short-run equilibrium

explanations. But they are never complete with respect to methodological individualism unless or until they are *also* long-run or general equilibrium explanations. The best way to think of this is to see that the neoclassical explanation of an individual is that of an individual in a very special short-run equilibrium – namely, the one which would have to exist when the economy as a whole is in a state of long-run (general) equilibrium. The differences between the special short-run equilibrium and just any ordinary short-run equilibrium is most apparent in our explanation of the firm. What we need to see is how much our explanation of the firm depends on techniques of analysis which avoid being nonsensical only by being restricted to the special short-run equilibrium state.

3.1. *Explaining the Firm in a Special Short-run Equilibrium*

Partial equilibrium (or *ceteris paribus*) analysis is so well understood by almost everyone who has taken one or more economics classes that it is taken too much for granted. Some of its more subtle methodological details are often overlooked. Typically, we are taught to consider the individual to be in a state of personal equilibrium in the simple sense that the individual is maximizing his or her utility, wealth, profit, etc., subject to some specified constraints, such that any movement along a continuum away from the equilibrium position will only result in a less than optimum choice. The methodological question that we should always keep in mind is, ‘just what is being explained?’ The variables to be explained are, of course, all the endogenous variables. Which variables are endogenous in any typical economics explanation is not always kept clear. While some of the constraints are clearly exogenous, others are considered to be influenced in some indirect way by the actions of the individual who is often the same one whose behavior is being explained. For example, prices are given to the individual decision-maker but are also influenced indirectly by the demand or supply decisions that the individual makes. Marshall’s short- vs. long-run methodology is designed to keep such things clear. In the short run the individual usually only has one or two variables to choose. By definition of the short run all other variables are effectively put beyond the realm of the individual’s choice. As we noted above, such an explanation of the individual’s choices is incomplete whenever the givens are not natural constraints.

For the moment we would like to avoid discussing the question of completeness of the general equilibrium explanation of the economy. Instead, let us narrow our discussion to the properties of the very special partial equilibrium – the one where there is a state of long-run (general) equilibrium, as well as one where every individual is in a state of short-run equilibrium. In this very special case, all the non-natural constraints

are explained by referring to the presumed state of general equilibrium. Now from the general equilibrium perspective, the essential nature of our typical partial equilibrium explanation is entirely a matter of calculus since the equilibrium choice is also the optimizing choice.

To keep the methodological issues as clear as possible, let us first examine a very simple application of partial equilibrium analysis. Even in the most simple models the methodological fundamentals are fully apparent. Consider, for example, the individual firm (typically treated as if it were a person) which we will claim is maximizing its profit with respect to its level of labor employment – that is, it is in a state of short-run equilibrium. On the basis of this claim alone it immediately follows, as a simple matter of calculus, that the firm's marginal profit with respect to the quantity of labor employed is both zero and diminishing with labor input increments. It turns out that almost everything we have to say about the nature of the firm can be seen as something to support this simple matter of calculus.

Though it is not often stated, there is a presumption here that the possible levels of labor employment can always be represented by points along a continuum. The behavioral explanation is explicitly that the firm increases its level of labor input along a continuum until the marginal profit is brought down to zero, or the firm decreases labor until marginal profit is brought up to zero. What is being explained here is the individual firm's choice of the level of labor input along with the resulting level of output while all other variables are givens. The individual firm need not explicitly calculate its marginal profit but it must have some way of determining what it thinks is a maximum level of profit. For now let us simply assume that it does calculate the marginal profit. This way there is a direct connection between our theory of the firm's behavior and its actual behavior.

If we think of the firm calculating its marginal profit, we can push our simple calculus analysis even further. Specifically, marginal profit is the difference between marginal revenue and marginal cost, and marginal profit is zero when profit is maximum. If we view the firm as deciding its level of output (within a small range of the level appropriate for general equilibrium) then the marginal revenue is just the given price. Narrowing our discussion here to the very special short-run equilibrium that exists at the given long-run equilibrium only means that the given price is just the long-run equilibrium price. Since the marginal revenue is fixed at the level of the given price, for the marginal profit to be diminishing we would have to have the marginal cost rising with output levels and equal to price when marginal profit is zero. Now this only raises the further question about why the marginal cost might ever be

rising – that is, besides being an implication of our presumption of profit maximization with the prices as givens. We must answer this question if we are going to complete even our short-run explanation of the price-taking firm's choice of output level. Again, the explanation must ultimately be based on something exogenous to the firm.

To explain why marginal cost increases with any rise in the output level, we need only continue with our simple calculus analysis. If, in accordance with the Marshallian definition of the short run, the only variable input is labor, then marginal cost of producing more output is merely the cost of the extra labor requirements for the extra output. To calculate the marginal labor requirements we finally reach a fundamental exogenous constraint – namely, the firm's production function which tells the firm its maximum level of output for each potential level of labor input – or equivalently, its minimum level of labor input for each level of output. Since we are examining the very special short-run equilibrium (i.e. the one corresponding to the existence of long-run equilibrium prices), we are assuming that the capital available is the long-run equilibrium amount. So long as the production function represents a natural constraint with the appropriate properties, the short-run explanation of the individual firm's behavior will be complete. That is, the firm will be maximizing (rather than minimizing) profit given the price of labor, if its production function is such that the marginal labor requirements rise with the level of output. Again, we cannot simply assert that the marginal labor requirements must be rising merely because the firm is claimed to be maximizing profit since this assertion would make our explanation of the price-taking firm circular.

The usual way to explain marginal labor requirements is to see them as the inverse of marginal productivity of labor. That is, marginal productivity is the extra output resulting from extra labor input. Being the inverse, a rising marginal labor requirement implies a falling marginal productivity. But this does not yet get us very far towards completing our explanation, since it only begs the question about why the marginal productivity is falling as labor input rises. Fortunately we have already seen above how we can explain diminishing marginal productivity of labor. For now it is enough to recognize that there are many ways we can approach the completion of any short-run explanation so long as they do not violate the requirements of methodological individualism. In the case of our tomato firm, it does not seem to matter since both of the opposing views of individuals' productivities lead to a diminishing marginal productivity curve and thereby a rising marginal cost curve. Once we have explained why marginal cost rises with output levels, our explanation of the price-taking firm is completed. The firm does indeed face a rising marginal cost curve and thus there is a distinct

profit maximizing level of input and level of output. And thus we have a complete, albeit very elementary, short-run equilibrium explanation of the typical individual price-taker firm.

3.2. Analyzing the Firm in an Ordinary Short-run Equilibrium

We may have completed the short-run explanation of the firm – that is, explained the paradigm short-run decision concerning how much labor to hire – but what can we say about the employment of all inputs? It turns out that the only explanation we have for the employment of any other input involves a redefinition of the short run to make the other input the short-run variable input and make labor a fixed factor. This observation, if true, may mean that our ways of accommodating individualism are restricted to long-run equilibrium models. But such an observation may not be obvious in what we have said so far. To determine if it is true, we need to examine the use of calculus concepts that are hidden in some of our typical assumptions about the firm.

To do so, let us consider the analysis of a firm's output into separate individual contributions of each input. And, let us continue restricting our discussion to the special short-run equilibrium which corresponds to a long-run (general) equilibrium. When looking at the firm in any long-run equilibrium we must continually keep in mind that the firm's production function is linear-homogeneous (since all inputs are variable by definition of the 'long run') and thus equation [1.1] is necessarily true. The equation is true even apart from any question of whether profits are maximized or how many inputs there are. Equation [1.1] (which is a consequence of what mathematical economists call Euler's theorem) simply says that whatever is the level of output (X), it can always be calculated by adding together the separate contributions of each input – where each input's contribution is 'measured' by adding together each unit's marginal contribution and where each unit of an input has the same marginal productivity (MPP). This measurement is true only when we assume all labor inputs are identical (i.e. egalitarian liberalism), but certainly in this case the marginal product of any input is just that input's individual contribution. But if we were approaching from a very general viewpoint and paying each unit of any input its marginal product (as if there were as many different types of inputs as there were units) then clearly the marginal product of any particular unit of input is its contribution to output. Since there are no fixed inputs, their sum must be equal to the total output.

Although this analysis seems straightforward, there are some difficulties with the concept of a marginal product which are not often recognized. The marginal product of labor, for example, is always defined as the extra output that results from employing one additional

unit of labor. Why should that unit of labor be credited with all the extra output when it is easily recognized that other inputs helped to produce it? Before answering this let us consider an analogous question for which the answer is widely accepted. If we had a production function with two outputs and one input, there is a well-known accounting problem for such joint products [see Hicks, 1973]. Namely, there is no way unambiguously to allocate the input cost to the two separate outputs except in special cases (i.e. linear production functions). So, we should ask what reason do we have to think that there should not be a similar problem when there are joint inputs? Of course, there is no reason; that is, there is no reason for crediting all resulting extra output to one input when that input is increased *ceteris paribus*.

The concept of marginal productivity is a fiction. Nevertheless, it may be a harmless fiction if we restrict our analysis to long-run equilibria where equation [1.1] necessarily holds, at least, locally. That is, we can still use equation [1.1] to calculate correctly the level of output if we know the inputs' marginal productivities and levels of employment. *But*, this is true only in long-run equilibria!

There is an analogous conceptual problem whenever an individual consumer implicitly thinks that the following is true:

$$(MU_X)(dX) = -(MU_Y)(dY)$$

where dX and dY are the compensating changes to hold the level of utility constant *at the point of equilibrium*. But, calculating marginal utility (MU) using a partial derivative presumes that the change in the level of utility received when one changes the amount consumed of X is not influenced by the *existence* of the good Y and thus may be completely attributed to X . Analogous to the question of marginal productivity calculations, so long as we are examining the individual consumer in the state of equilibrium there will be no chance of any calculation errors.

It may be said that the partial equilibrium method of explaining the economy by isolating each individual and calculating the relevant partial derivatives can avoid obvious errors (allowing for the fictions mentioned above), but this is true *only* when we focus on the very special short-run equilibrium that corresponds to the long-run equilibrium. For some theorists, the partial equilibrium method will still seem to provide a complete methodological individualist explanation as all non-natural givens or constraints facing the individual in question are also explained as having equilibrium values and are thereby results of all other individuals being in a state of short-run equilibrium (they are all maximizing). But for others, the idea of a partial equilibrium may seem