The economic process as a whole is not a mechanical phenomenon … doubts concerning the existence of an invariant parallelism between [business activity and a mechanical clock] are not out of place. However, the alternative idea that the march of the entire economic process can be described by a system of differential equations with clock-time as the independent variable – an idea underlying many macro-dynamic models – is in all probability vitiated ab ovo.

[Georgescu-Roegen 1971, p. 139]

In 1976 John Hicks published an article devoted to Nicholas Georgescu-Roegen with particular reference to a principle attributed to Georgescu: ‘It is a very simple principle: the irreversibility of time. In space we can move either way, or any way; but time just goes on, never goes back. We represent time on our diagrams by a spatial coordinate; but that representation is never a complete representation; it always leaves something out.’ [Hicks 1976, p. 135]. As Hicks recounted, he made many forays into projects that were directed towards involving time but lamented that it had been a struggle to get his economic models ‘in time’. By this he meant not only that time must matter but that time’s essential irreversibility must play a significant role.

The most important distinction Hicks made was between models where economics is in time and models which just include time as a parametric variable. It was not a new distinction as he noted that the Austrian School often focused on how time is dealt with. In particular, he noted that Carl Menger (in conversation with Hicks) objected to how fellow Austrian Eugene Böhm-Bawerk dealt with time claiming that Böhm’s theory was ‘one of the greatest errors ever committed.’ Hicks said [p. 139] that this is because in Menger time is unidirectional. Menger’s theory is an economics in time but Böhm’s is an economics of time, in which time is no more than a mathematical parameter …

Judging by the 75 articles and essays recently collected by Stefano Zamagni and Elettra Agliardi [2004], it seems that the distinction has not been adequately recognized. In their *Time in Economic Theory*, the collected authors – with few exceptions – seem to think the problem is only how to include time in economic models and thus seem unaware of what both Hicks and Georgescu thought was the important problem involving time in economics. Namely, how can we build economic models where time matters because it is irreversible?

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1 Henceforth, I will refer to this as *TiET*. This is a three-volume set with articles organized as ‘methodological issues on time in economics’, ‘time and equilibrium’, ‘time and irreversibility’, ‘time and complex systems’, ‘time and “paradoxes” in game theory’, ‘the allocation of time’ and ‘time, preference reversal and the theory of choice’. It is curious that the famous 1976 Hicks article was not included in this collection since the issue of time in economics is in its title. Nor was his lesser known, but equally relevant, 1973 article included. Similarly, while some sections of Georgescu’s 1971 book are included, the section which I have been quoting from is not.
The physicist Arthur Eddington put the main issue concerning time as one of recognizing 'Time’s Arrow' – and in this light he said:

The great thing about time is that it goes on. But this is an aspect of it which the physicist sometimes seems inclined to neglect…. The classical physicist has been using without misgiving a system of laws which do not recognise a directed time; he is shocked that the new picture should expose this so glaringly.” [1928/58, p. 68].

Both Hicks and Georgescu accepted the necessity to recognize and involve time’s arrow in any realistic economic model. In Georgescu’s view, one must recognize the Second Law of Thermodynamics – which he called the ‘Entropy Law’. Georgescu calls this an evolutionary law. About such a law, he says:

[C]ontrary to the opinion of some biologists, we do not need to discover a single cause for evolution in order to arrive at an evolutionary law. And in fact, almost every proposal of an evolutionary law for the biological or the social world has been concerned with a time’s arrow, not with a single cause.

Of all the time’s arrows suggested thus far for the biological world, ‘complexity of organization’ and ‘degree of control over the environment’ seem to enjoy the greatest popularity. One does not have to be a biologist, however, to see that neither proposal is satisfactory: the suggested attributes are not ordinarily measurable. [1971, p. 128]

Georgescu recognized that the Entropy Law ‘has been, and still is, surrounded by numerous controversies’ [p. 129]. But the important issue is that ‘since the laws of mechanics cannot account for a unidirectional movement, a new branch of physics using nonmechanical explanations had to be created' [ibid.] and the laws of thermodynamics were devised to deal with this. And so, Georgescu set his task to be ‘that of proving that the economic process as a whole is not a mechanical phenomenon’ [ibid.].

While the Entropy Law would seem an obvious way to incorporate time’s arrow and thereby avoid the limitations of mechanical models, it is still an open question as to whether any social theory can be seen in a parallel fashion. As the writer/philosopher Robert Pirsig puts it:

Second Law of Thermodynamics states that all energy systems ‘run down’ like a clock and never rewind themselves. But life not only ‘runs up,’ converting low energy seawater, sunlight and air into high-energy chemicals, it keeps multiplying itself into more and better clocks that keep ‘running up’ faster and faster. [1991, p. 140]

Evolution and time’s arrow

Nevertheless, evolution would seem an obvious way to think of time’s arrow in social science and economics. But even the concept of evolution is open to controversy. Most of the controversy is due to nineteenth-century writers’ attempts to accommodate the notion of man’s evolution. One cause was the Lamarckian notion that all life was evolving toward perfection. As opposed to such a teleological view of evolution that sees progress converging onto the modern state of man, one could agree with Pirsig and see each stage as providing more and more freedom – freedom from the limits imposed by a mechanical world governed by the laws of physics. One need only think of the many ways living beings try to overcome the law of gravity. As Pirsig puts it:
One could show that the degree to which an organism disobeys this law is a measure of its degree of evolution. Thus, while the simple protozoa just barely get around on their cilia, earthworms manage to control their distance and direction, birds fly into the sky, and man goes all the way to the moon. [ibid., p. 143]

So, it would seem that we have to choose between an evolution that is goal directed (perfection) and an evolution that is expanding levels of freedom. Of course, we could go back to Eddington’s third perspective – namely, that what he called time’s arrow ‘makes no appearance in physical science except in the study of organisation of a number of individuals. Here the arrow indicates the direction of progressive increase of the random element’ [1928/58, p. 69].

Building models involving time and change

The question is whether including references to the Entropy Law or to evolution in any form actually accomplishes what Hicks thought was necessary. That is, do such references cause a model to be in time? Can one build models that explain the dynamics of an economy? The question that will remain is whether time is implemented in a manner that it is irreversible and in a manner that its irreversibility can be explained. Let us consider the usual ways time is dealt with in economic models to see whether much has been accomplished toward dealing with the problem of recognizing time’s arrow.2

Time and static models

The number of ways time can be incorporated into any model is limited by the types of statements usually included in the model [see Boland 1989, Chapter 7]. Specifically, time can enter through the statements defining goods or prices and the behavioural functions relating them, through the statements which identify the constraints or givens, through the statements of conditions of ‘equilibrium,’ or, it can be argued [e.g., TiET, Volume I, Introduction, p. xx], through the statements concerning the process of knowing or learning the truth status of any of the above statements.

Note that there is no explicit time in any typical general-equilibrium model. Nonetheless, it is possible to give a temporal interpretation of every competitive equilibrium condition. Each condition can be considered to be a statement which asserts an implicit consistency between the truth of the statements about the givens (viz., the observed values of resources, demand functions, and production functions) and the truth of the statements about endogenous variables (viz., the observed prices and output quantities that are explained by the model) at the same point in time. But this is always a matter of interpretation.

A minimum requirement for any model to be considered an explanation of its endogenous variables is that one can always solve for those variables as (positive) stable functions of the exogenous variables and parametric coefficients of the other givens. Since this is not always possible for some values of the givens, every general-equilibrium model must provide a set of additional conditions for the givens which will assure the solvability of the model for the values of endogenous variables at the same point in time as the givens are observed.

Models which include statements that are only assumed to be true at a specified point in time are static models by definition. Although a model’s logical validity is timeless, its (empirical) truth status is always an open question. Therefore, with respect to any given model, today’s values of the endogenous variables may be shown to be consistent with today’s values of the

2 The remainder of this section is based on my 1978 article which explicitly addressed this problem.
exogenous variables, but tomorrow their respective values may not be consistent. Since dynamic processes obviously refer to more than one point in time, the explanatory usefulness of a static model would seem rather limited.

*Time-based variables*

General-equilibrium models can easily be modified to accommodate time and still allow for equilibria. The classic model for which there is a proof of the existence of a general equilibrium while recognizing time is the one due to Kenneth Arrow and Gerard Debreu [1954] which offers a means of overcoming the temporal limitation of static models by dating all variables with subscripts and building models which cover many points in time. In these models any good, say a hamburger $H_t$ at time $t = t_o$, is not the same hamburger $H_t$ at time $t = t_1$. Of course, in such a model we have many more goods than one could observe at any one point in time. But formally, such a model is still just a version of a static general-equilibrium model except that the number of goods and equilibrium equations has been multiplied by the number of points in time being considered. This form of equilibrium model implies that the explanation of the endogenous variables is essentially static for the entire period of time over which the goods are defined. There are no dynamics to be explained here because nothing is changing. The values of the endogenous variables at any point can be shown to follow from the values of the exogenous variables statically given at the unique initial point in time. The individual makes his or her only decision at that one point in time [see Smale 1976 (*TiET*, Volume I, Chapter 25)].

*Time-preference or the economics of time*

Another method of including time is to make time a ‘commodity’ – such as ‘leisure’ time or ‘waiting’ time. Examples are Gary Becker’s theory of allocating time [1965 (*TiET*, Volume III, Chapter 2)] and Böhm-Bawerk’s period of production [1889]. In both models, time is spent on production, and increasing the time spent implies increasing the costs. In the Becker model the costs are the opportunities lost, and the amount of time is allocated to produce household benefits (e.g. meals, shopping, etc.) such that utility is maximized over all possible uses of the time endowment. Similarly, in the Böhm-Bawerk model the costs are the needed working capital, which increases with waiting time. Time is allocated to waiting until the product is considered finished. The optimum waiting time will maximize the profit rate.

The difficulty with this approach is that time is another exogenously scarce resource which can be uniquely and optimally allocated; thus the time allocation is viewed as another static variable that has been uniquely determined when it is logically consistent with other static and exogenous givens. Again, nothing is changing during the period of time considered. Neither Becker’s nor Böhm-Bawerk’s approach can avoid the static nature of the givens (the constraints, the tastes, the production functions, time available, etc). As with the typical general-equilibrium model, the endogenous variables are statically fixed by the exogenous givens. There is no reason for historical change; hence it cannot be explained.

*Variable ‘givens’ or lagged variables*

As an alternative to the above approaches one might attempt to determine the time-path trajectory of the endogenous variables. Given that the solution of a model represents its explanation, the only way the endogenous variables can change over time is either by one or more of the exogenous variables changing or by some of the parameters of the logical relationships autonomously changing (or both). The population’s growth rate in Nicholas Kaldor’s growth model [1957] is an example of the former, and what Hicks [1976] called an
‘autonomous invention’ or a non-neutral change in technology might be an example of the latter [see Boland 1992, Chapter 7]. However, usually in economics the logical relationships are assumed not to change over the relevant time period. The entire explanation of historical changes is usually invested in the exogenous changes of the givens. The changes in the givens may be represented by movements along their fixed trajectories. Thus if some of the static givens of the typical general-equilibrium model are replaced by time-path trajectories for a specified time period, the result will be derivable trajectories for the endogenous variables over the same time period. With this method of including time we have only replaced a point in time with a static sequence of corresponding points in a fixed period of time. The solution will be a fixed trajectory, that is, a fixed sequence of changing values.

Of course one does not necessarily have to assume that the time period of the exogenous variables is the same as that of the endogenous variables. One could assert that some of today’s exogenous variables may be yesterday’s endogenous variables [Nerlov 1972]. An example of this approach is the John von Neumann [1937/45] balanced growth model. With a lagged-variable approach one can derive a time-path trajectory for the endogenous variables. However, the position of the trajectory over a given period of time will depend only on the initial set of values for the exogenous givens. The initial values of the givens are essentially the only exogenous variables of the model over the whole time period.

On the surface the direct approach of including an exogenous time-path for the givens, or the indirect approach using lagged variables, looks like a solution to the problem of explaining historical change. But a closer examination will show this to be an illusion. In the exogenous trajectory approach the endogenous variables are changing only because the exogenous variables are changing. In the case of lagged variables the position of an endogenous variable on its trajectory is uniquely determined merely by the length of time transpired since the initial givens were established, and the position of the trajectory itself is uniquely determined only by the initial values of the exogenous givens. In both cases the trajectories of the endogenous variables are exogenously fixed. The only ‘dynamics’ of the model are exogenous. Since exogeneity results from one’s explicit choice not to explain the givens or their behaviour, the dynamic changes in the model are not explained. In other words one would be still relying on a statically given time-path trajectory which is fixed over the relevant time period. There is no explanation as to why it is that trajectory rather than some other.\(^3\)

As noted by both Hicks and Georgescu, in economics a point in time is treated logically the same way a point in space is treated. There is nothing (such as real time’s irreversibility) which distinguishes time from space [Hicks 1976, p. 135; Georgescu-Roegen 1971, p. 130]. It is argued here that the dynamics in all the above approaches appear to be an illusion created by an arbitrary labeling of one or more exogenous variables. If one is going to explain any historical process with a fixed trajectory, then one must be able to explain that fixity as well.

**Flow variables**

The criticism raised against the approaches that add time by appropriately defining certain variables can be extended to those approaches that add a time-differential equation to an

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\(^3\) We might, for example, assume the given path was such that the exogenous variable grew at a constant rate. But if we are asked why we did not assume an increasing rate, we cannot justify our assumption solely on the grounds that it yields the observed time-path of the endogenous variables. The truth of our assumptions regarding exogenous givens must be independent of our conclusions regarding endogenous variables [Boland 1989, Chapter 6].
otherwise static model. One of the problems in using equilibrium models to explain prices is that observed prices may not yet have reached their equilibrium values. Thus it is often argued that we need an explanation of the disequilibrium behaviour of the endogenous variables [Barro and Grossman 1971]. Typically, a theory of price adjustment is attached to our static equilibrium models. The basic approach is to add a differential (or difference) equation which gives the rate of change of the price as a function of the amount by which the two sides of one of the equilibrium equations deviate from equality prior to reaching equilibrium [Frisch 1936; Arrow 1959]). In market demand and supply analysis this usually is an equation of the form: \( \frac{\partial p}{\partial t} = f(S_t - D_t) \), where \( \frac{\partial f}{\partial (S_t - D_t)} \) is negative and \( f(0) = 0 \). But, unless this additional equation is explained, the dynamics are purely improvised and arbitrary. A make-shift differential equation for the ‘dynamics’ of the market does not say who changes the price or why it is being changed. Until one can say why the price has changed (rather than describing how much it should change) one has explained neither the process of disequilibrium change nor the dynamics of the market.

As significant as some may consider such criticism to be [Gordon and Hynes 1970; Boland 1977], matters are even worse for the determination of the equilibrium level of prices. Most models which include time-differential equations only guarantee a solution in the long run. Such models (including both ‘adaptive expectations’ and ‘rational expectations’ models) are incapable of yielding a determinant and non-arbitrary solution for the prices at points of real (calendar or clock) time where equilibrium has been reached. If one means by ‘in the long run’ that it takes anything approaching an infinite amount of time to yield a determinant solution, one is in effect conceding that one does not have a real time explanation of the observed behaviour of the endogenous variables. To assert the existence of a long-run equilibrium when its attainment requires an infinite length of time is to imply simply either that time does not matter or that one has no explanation.

Timing choices

There is one obvious case where time is directly involved: namely, where the choice to be made is to choose a time to act. Models that address this issue would seem to have successfully addressed the issue of irreversibility of time with reference to investment decisions [see TiET, Volume II, Part I]. But, this is an illusion and does not address the problem outlined by Hicks. Recall that the Böhm-Bawerk model was an explanation of a timing decision. In his case, it was decisions that might involve choosing when to cut a tree down for lumber or when to open a keg and bottle the wine. In both cases, the dynamics are exogenous. In the case of investment decisions the dynamics are also exogenous but, unlike Böhm’s model, the source of the dynamics is unknown. The choice in Böhm’s model is mechanical. Investment timing decisions can vary depending on how one views the future. Of course, one could take a mechanical approach to investment decisions based on the cost of investment funds but as John Maynard Keynes argued this is not necessary or likely. That is, when one is optimistic an early investment decision will be made but when one is pessimistic the investment might be postponed. In this case...

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4 Another version of this approach is to make the rate of change of price a function of the difference between intended and unintended inventory levels. But this approach leads to instability in some markets (cf. Hicks 1956, p. 148). Moreover, it does not explain the price behaviour. Although one can interpret price-makers’ behaviour as some sort of learning process where the rational maximizing decisions are statically behind the facts [cf. Gordon and Hynes 1970, p. 377], one still needs a long-run argument to guarantee stability.

5 That is, for the trees it is a matter of the biological determination of how fast trees grow; and for the wine it is a matter of the biological and chemical determination of both how and how fast particular wines age.
sense, the timing of an investment is endogenous so long as one builds into the model the explanation as to how one might choose to view the future.

Nevertheless, the problem remains as to whether the model embodies irreversibility. In the real world one cannot expect exogenous variables to all return to their starting values before a decision is made. This is true of Böhm’s model as it is in any investment model when compared to the real world. But there is nothing inherent in such a model that prevents one from resetting the exogenous variables to their original values and starting over. Georgescu thought that every economics model must involve production and as such must involve the Entropy Law with the recognition that resetting the exogenous variables does not recover the energy used up – the same issue raised by the recognition of hysteresis [see TiET, Volume II, Part I]. But Entropy per se is not necessary to include so long as history is part of the model and its irreversibility is explicitly recognized.

Towards an explanation of dynamics and irreversible change

[E]conomic problems arise always and only in consequence of change... [T]he economic problem of society is mainly one of rapid adaptation to changes in the particular circumstances of time and place.

Friedrich Hayek [1945, pp. 82–3]

Being reactive to dynamics is not necessarily what Hicks was questioning. It is the endogeneity of the change that is at issue. Within neoclassical economics the only reason for change is the recognition that one is not maximizing. And it is exactly this issue that makes explaining dynamics difficult for mechanical models based on the conditions for equilibrium.

Frank Hahn has for many years discussed the difficulties surrounding the reliance on equilibrium methodology. In a 1989/90 essay [TiET, Volume I, Chapter 23] he laments:

Current economic theory by and large avoids dynamics, at least non-equilibrium dynamics. This has the virtue of allowing orderly argument and conclusion. But such order is bought at too high a price. ... Once it was recognized that we must study sequence economies, it also became urgent to include expectations in the description of the agent. ... here we need to know something of the process of learning in order to get determinate answers. ... only economies that are very large in the technical sense allow one to make traditional price-taking assumptions. It seems clear that in the world firms must predict not only prices but also demand. [p. 125]

Hahn recognized that Kenneth Arrow’s 1959 article long ago raised the issue of the dynamics of price adjustment. Arrow in particular argued that while one can show that prices in a state of equilibrium can easily be explained with the usual neoclassical assumptions, such an explanation is not enough – how prices are adjusted to reach an equilibrium must also be explained. Stated in a more fundamental way, if one is to explain why prices are what they are (in equilibrium), one must also explain why they are not what they are not (out of equilibrium). Hahn goes on to suggest:

... dynamics should be viewed as a learning process, both about demand conditions and about the strategies of near competitors. Once again, when an equilibrium is defined relatively to such processes, it seems that they are indeterminate unless history – that is, information – is explicitly modelled and known. The path of history is the outcome of individual decisions and in turn helps to fix the latter. ... the information available to agents at any time is determined by the particular path followed. The economy could
have followed a different path and generated quite different information. There is something essentially historical in a proper definition of equilibrium and of course in the dynamics itself. [pp. 125–6]

Hahn here is repeating much of the challenge that Hayek, Georgescu and Hicks were issuing. But he is also making the point that once the need to address learning is recognized, we must recognize the path-dependency of the final prices that are determined in any market process. Unfortunately, while one can find many examples of articles where learning is recognized, few address the idea of learning in a manner that both entails inherent path-dependency and allows the dynamics to be truly endogenous.

When it comes to learning, the main problem is that too many model builders think facts speak for themselves in an inductive manner. Whenever facts do not speak for themselves, one must make assumptions. Assumptions can easily lead one to outcomes that differ from the ones that a mechanical neoclassical model will reach. Robert Clower [1955, 1959] provided an excellent example when he discussed the decision process of an ‘ignorant’ monopolist. There is no reason to think that a textbook monopolist knows the market’s demand curve. In Clower’s model the monopolist proceeds in a trial-and-error manner by forming expectations as to what the market can absorb at a trial price and if the monopolist’s expectation is false, the actual price obtained will be different. The key question raised is: How does the monopolist interpret the information coming back from the market? In Clower’s model, the monopolist interprets every differing price as evidence that the demand curve has shifted from the one that was expected. And the model’s monopolist assumes that the demand curve is linear when in fact it is not. The result is that it is possible for the an equilibrium to be reached (i.e., a fixed point) where expectations are fulfilled but the firm (unknowingly and contrary to intentions) is not actually maximizing profit. Where this equilibrium is located depends on the assumptions made by the monopolist and the false assumptions can mean a false equilibrium.

As Mordecai Kurz [TiET, Volume II, Chapter 20] points out, induction is exogenous learning – that is, facts speaking for themselves without any decisions to be explained. As such, the issue raised by Hahn means that when building a model to explain the dynamics of an economy and its markets, one must make assumptions about how decision makers learn in a non-inductive manner and thereby make the dynamics and path-dependency endogenous. Unless the model builder is willing to recognize simultaneously many different ways to process information – that is, many different learning strategies (e.g., more than just assuming everyone is a Bayesian learner [see Boland 2003, Chapter 8]), little progress can be expected for the future of dealing with time in economics in a realistic way. Recognizing learning can help to solve the problem of time’s arrow since one cannot unlearn. But, as Hahn concludes:

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6 And there is no reason to think any two individuals will choose the same assumptions – particularly, when it comes to interpreting observational evidence. I have explicitly dealt with this by hypothesizing different assumptions about learning [see Boland 1992, Chapter 11]. For example, one could be a ‘sophisticated inductivist’, a ‘conventionalist’, an ‘instrumentalist’, a ‘scepticist’ when it comes to learning. Each involves a different dynamic behaviour.

7 That is, the monopolist thinks he is maximizing profit but his incorrectly inferred marginal revenue (calculated from the assumed straight-line demand curve that will not necessarily be tangent to the real demand curve) does not equal his easily calculated marginal cost. In other words, the monopolist reaches an equilibrium where profit is not really being maximized. For more on Clower’s ignorant monopolist and the use of false assumptions, see Boland [2002].

8 See footnote 6, above.

9 This is the solution proposed in my 1978 article.
Obviously a long road stretches ahead. It is a risky and unruly road, but I hope that many more theorists with a bent for rigorous theorizing will embark on it – if for no other reason than that turning the old wheel is now too easy.10

References


10 So, the question needs to be asked about the purpose of collecting 75 articles on time in economics. Does this collection in any way help one to deal with the problem of time’s arrow or even to negotiate Hahn’s ‘risky and unruly road’? Other than Hahn’s article and an abbreviated version of Joan Robinson’s Thames Paper [TiET, Volume II, Chapter 8] and possibly Paul David’s article [TiET, Volume II, Chapter 12], few if any of articles in this collection will help economic model builders put their models in time.
Hicks, J.R. [1956] ‘Methods of dynamic analysis’, in (anon.) 25 Economic Essays in Honour of Erik Lindahl (Stockholm: Ekonomisk Tidskrift), 139–51