EXPERIMENT 1.
STATISTICAL ASPECTS OF NUCLEAR DECAY

(Report due 1 week after completion of experiment)

There is, associated with all measurements, a degree of uncertainty as to the true value of the quantity being measured. Although more, and better, experiments can be devised to measure the quantity, they can only succeed in decreasing the possible error in its measurement. It is of prime importance, therefore, to indicate the probability that the result is within certain limits (sometimes refer to as the confidence level). In some instances, due to the complexity of the situation, the experimenter can only estimate the accuracy of his results. However, it is possible in principle, if not always practical, to compute probabilities on a strictly mathematical basis.

The problem of measuring nuclear phenomena presents itself differently in that the processes are random in nature and quite often involve so few occurrences that their own statistical nature becomes apparent. Many faulty conclusions have been reached by experimenters who have tried to use their results without accounting for the normal statistical fluctuations inherent in their measurements. The understanding of statistical theory therefore becomes imperative when dealing with nuclear processes.

The study of statistics is based upon the probabilities for certain events to occur. These probabilities can be grouped into frequency distributions which then describe the occurrence of the events. For example, the probability of drawing an ace from a full deck of cards is just 4/52. Assuming a successful draw, the probability of drawing another ace from the deck is now 3/51. However, the probability of initially setting out to draw two aces in succession is (4/52)(3/51). Probabilities can be calculated for many more such events and the results can be represented as a frequency distribution which, in turn, can be used to give the probability for the occurrence of any particular event, such as drawing three aces in a row.

The binomial distribution can be considered the basis of all frequency distributions in that all other distributions can be derived from it. It rigorously applies to phenomena in which the numbers of occurrences are integers. An analytic approximation to the binomial distribution is
given by the normal distribution. Since values for the occurrences need not be integral, the normal distribution is generally applied to continuously variable quantities and as an approximation of the discontinuous functions. A limiting case of the binomial distribution is given by the Poisson distribution. It describes all random processes whose probability of occurrence is small and constant and thereby applies to essentially all observations of nuclear phenomena. Mathematical expressions are associated with these distributions to characterize them. Among these expressions, the mean value and standard deviations are of prime importance since all others are functions or continuations of these two.

It is also possible to set up criteria to determine how well the distribution of observed occurrences follows a theoretical distribution. Pearson's $\chi^2$ (chi-square, pronounced "kai"-square) test is one of the better ones and indicates the probability that a repetition of the observations would show greater deviations from the theoretical distribution believed to govern the process. By selecting certain limits to this probability, a set of data can be said to follow, or not to follow, the proposed distributions. Conversely, the validity of a set of data can be determined if the probability distribution is known for the phenomena.

In this experiment, the ability of the experimenter to pipette reproducibly a series of replicate samples will be tested, and then the statistical nature of nuclear decay will be illustrated.

Triplicate samples will be prepared from the same stock radioactive solution, and one of the samples will be subjected to a detailed statistical analysis for the two different count rates. The other samples will be used to check the reproducibility in preparation. A comparison of the results will aid the student to judge more accurately when and where allowances for the statistical nature of nuclear phenomena need be taken into account. However, before performing this experiment, students are urged to familiarize themselves with (or review) the statistical concepts that have been just mentioned.

1.1 Experimental

Before starting any experimental work, sign in a pencil dosimeter; record the required information in the special forms provided in the lab.
1.1.1 PREPARATION OF THE WORK SPACE

Before starting any work with radioactive material, the work space where the handling will take place has to be prepared in order to minimize the exposure to the handler, minimize the risk of contamination and in case of a spill, to minimize the spread of contamination.

Exposure is minimized by obeying the following principles summarized as “time, distance, shielding and prevention of contamination”; the instructor will elaborate on these points.

One aspect of contamination control is to prepare your working space properly. This involves:

- Taping a layer of disposable absorbent paper (absorbing side always up) on the working surface,
- Setting a working tray itself lined with absorbent paper (absorbing side up),
- Installing at convenient locations in or next to the work space containers to received contaminated items generated during the experiment (liquid waste containers, solid waste container and sharp container)
- Checking that the foot operated waste container is functioning properly,
- Locating the bucket containing the spill kit,
- Verifying that portable monitoring equipment is functioning properly.

The actions just listed will have to be repeated for every experiment.

1.1.2 SOURCE PREPARATION

The preparation of radioactive samples will be the object of future experiments. For the present purpose, three samples will be prepared by evaporation of a solution labeled with a radioactive species ($^{137}$Cs or $^{32}$P) deposited on filter paper.

Ordinary volumetric apparatus can be used for handling and sampling moderate volumes of most radioactive solutions. However, special precautions should be taken to guard against spillage and contamination. The experimenter should never use his mouth in pipetting; various forms of propipette or autopipettes are available and must be used. The experimenter must
also wear some form of water-repellant gloves; surgical or plastic gloves are recommended.

If volumes smaller than 1 ml are to be measured and transferred, adjustable pipettes are generally used. These pipettes cover the range between 1-1000 µl.

A set of adjustable Gilson pipettors (0-20 µl, 10-200 µl, 50-1000 µl) and a set of Eppendorf pipettes, with disposable plastic tips, will be used to transfer solutions from 5 µl to 1000 µl volumes. These pipettes are precision instruments and are expensive; however, they will deliver reproducibly a given volume, *only* if they are operated properly as described next.

*To withdraw liquid:*

- Fit tightly a new tip of the proper size (use a new tip for each new withdrawal),
- Dial the required volume,
- Dip the tip into the solution to sample,
- Depress the plunger (no further) until the first resistance is met,
- While keeping the tip inside the solution to be sampled, wet the inside of the tip with the liquid to withdraw by releasing and depressing gently the plunger a couple of times (but do not depress further than this point of first resistance),
- Gently release the plunger completely to withdraw the preset volume of liquid.

*To expel the liquid:*

- Position the tip above the receiving container,
- Expel the liquid by depressing gently the plunger until the first resistance is felt,
- Depress the plunger further all the way to expel any droplet which may be still in the tip.

Properly used, the reproducibility of these instruments is in the range 0.1 to 0.6%, depending on the volume setting.

(In the following, the sources to be used and the exact amount will be specified by the instructor)
After demonstrating a proficiency in the use of micropipettes, the student will be given an active solution from which to make three identical samples (labeled A, B and C) of the assigned radioactive species deposited on 2.5 cm filter paper (the instructor will specify the exact volume to use).

Label three aluminum counting planchettes (counting cards) with your name, date and isotope name, and on each one, center a little piece of double-sided adhesive tape where the filter paper disk can be secured. Inject the required amount of radioactive solution on each piece of filter paper and let dry under an infrared lamp, but care should be taken not to over-heat the sample. The dried samples on the filter paper are finally covered with a thin plastic film (Saran Wrap™ or equivalent) taped onto the planchette. The samples are now protected from the atmosphere and the rest of the laboratory is protected from the samples.

1.1.3 DETECTION SYSTEM

A beta silicon solid-state detector system will be used in these experiments. A block diagram of the set-up is shown in Fig. 1.

The operation of the detector system will be demonstrated. The detector system is EXTREMELY SENSITIVE to changes in high voltage. Caution must always be exercised when changing the high voltage to the detector.

- Never change the controls on the master high voltage power supply.
- Be careful of the high voltage. Do not stick your fingers into the equipment.
- Be careful not to touch the surface of the detector in any way, but if you do, do not try to clean it yourself. Notify the instructor immediately.
- Do not knock the detector.
- Under no circumstances attempt to disassemble the equipment.
- If you encounter any problems, notify the instructor.
In counting radioactive samples, it is often necessary to correct data because of unwanted “background counts”. These may be the result of cosmic rays, a second nearby source, or contamination of the equipment. It is therefore necessary to always measure and know the background so that a true count rate can be determined.

**Background count rate.** Determine the background for the beta counter each day you use it and record it in your notebook. Also, as a check, you may want measure the count rate given by a standard source positioned on the same shelf and write it in your notebook. This count rate should be constant from day to day and any significant variation may indicate some malfunction in the counting set-up.

**Sample position.** Select one of your samples (sample A) for the statistical analysis. The count rates should be selected such that $\approx 1 - 5$ cps and $\approx 10 - 50$ cps are obtained. First, find which two shelf positions in the beta counter sample holder will produce such count rates by performing a series of 10 s counts with the sample positioned on each shelf in turn; in the following, these two shelves will be referred to as the “10cps position” and the “1cps position” (even if the actual count rates are not quite these numbers).

**Measurements.** The very nature of the statistical process requires a number of determinations. It is therefore necessary to obtain replicate measurements (ten replicates give a good sample) of the count rates of the source in each of the two positions.

- For sample A in the “10cps position”, obtain ten replicate measurements of the number of counts accumulated each for 10 s; repeat with that **same** sample A in the “1cps position”.

![Block diagram of the beta detection system.](image)
Count, once each, sample B then sample C in the “10cps position” for long enough as to obtain $\approx 1000$ counts. Record the actual number of counts and the corresponding duration of counting.

Finally, count the first sample (sample A) positioned on at least five other different shelves, again for the length of time necessary to obtain $\approx 1000$ counts. Record each time the shelf number, the actual counts and the corresponding duration of counting.

With a ruler, measure the distance from the detector to each of the shelf positions.

Record in your note book the active area of your detector.

Once all your measurements have been performed, take apart carefully your sample, discard the filter paper into the active solid waste container and put the planchettes into the cleaning solution provided.

Before leaving, perform wipe tests of your working area.

### 1.2 Results and Discussion

First tabulate all your experimental results collected during the lab in the format provided in the spreadsheet `expt1_results.xls` ([www.sfu.ca/~brodovit/files/nusc346/templates/](http://www.sfu.ca/~brodovit/files/nusc346/templates/)); when you hand in your report, submit a copy of this file to the instructor as an email attachment.

For sample A, assume a Poisson distribution and calculate the following for the two sets of observed counts (use the raw counts, not the count rates):

1. The experimental mean value.
2. The predicted standard deviation.
3. The experimentally observed standard deviation.
4. The standard error (error on the mean).
5. The predicted probable error.
6. The observed probable error.
7. The $\chi^2$ (chi-squared) value.
8. The probability, $P$, that $\chi^2$ should exceed its observed value.

Present the above results as a table.
Discuss whether the observed distributions are consistent (or not) with a Poisson distribution. Next, calculate and compare the net count rates (count rates corrected for background) and expected standard deviations for the three samples in comparable positions. Determine whether the three samples are identical, within the limits of counting statistics and comment on the agreement (or disagreement) between the results; take into account error propagation.

For a given sample, the observed count rate, \( R \), (in count per second – cps – for example) is related to the true decay rate or activity \( A \) (in disintegration per second – dps or Bq) by:

\[
R \text{ (in cps)} = \varepsilon A \text{ (in dps)}
\]  

where \( \varepsilon \) is the net detection efficiency. The term \( \varepsilon \) is itself the product of several terms each associated with the various factors which affect the actual number of events recorded by the detector – energy of the radiation, efficiency of the detector, thickness and shape of the source, scattering effects to mention only some of these factors. In addition, the geometry, \( \text{ie} \), the position of the source relative to the detector, will obviously affect the number of events detected; the corresponding contribution to the factor \( \varepsilon \) is the geometric efficiency \( g(d) \). For a source at distance \( d \) from the detector, equation (1) may be rewritten as:

\[
R(d) = \varepsilon' g(d) A
\]  

in which the geometric efficiency \( g(d) \) appears explicitly, while other factors are regrouped as \( \varepsilon' \).

For a point source located at a distance \( d \) from a circular detector of radius \( r \), elementary geometry consideration\(^1\) predicts that the geometric efficiency is given by:

\[
g(d) = \frac{1}{2} \left( 1 - \frac{d}{\sqrt{d^2 + r^2}} \right)
\]  

Note that if \( d \gg r \) (ie, if the source is at a distance \( d \) much larger than the size of the detector), eqn. 3 can be approximated by

\[\text{ ---}
\]

\(^1\) See derivation in one on the appendices.
\[ g(d) \approx \frac{r^2}{4d^2} \]

which emphasizes the well known \(1/d^2\) variation of count rate with distance for a given radioactive source.

Using all the data collected with the sample A in different shelf positions, plot the experimental count rates \(R(d)\) against the corresponding \(g(d)\) calculated using eqn. 3. Determine to what extent equations (2) and (3) are satisfied, and to what extent values of \(g(d)\) calculated with eqn. 4 may be more (or less) appropriate; explain.

Be sure to display the data and results (and the corresponding uncertainties) in appropriate graphs and tables. Are your results consistent with a Poisson’s distribution? A brief discussion of the pertinent aspects of statistical analysis and how it should be applied to measurements of nuclear decay process should also be included.

From what you have learned, and given a radioactive sample having a count rate of the order of 1000 cpm (count per minute), to get the true count rate more accurately would it be better (or worst, or same) to measure the activity of this samples ten times, each for 1 min duration compared to a single measurement lasting 10 min? Justify your answer by estimating in each case the expected standard deviation of the determination.

In your discussion define and/or explain briefly

- Frequency distributions
- Binomial distributions
- Poisson distributions
- \(\chi^2\) test of distribution
- Mean
- Median
- Standard deviation
- Standard error
- Probable error
- Propagation of errors
- Degrees of freedom
− Detection efficiency

The results of the wipe tests should appear in your report.

1.3 References


