Matching for Social Mobility with Unobserved Heritable Characteristics*

Chris Bidner[†] and John Knowles[‡]

(Preliminary Version: Please do not circulate)

June 26, 2018

Abstract

When there is a genetic basis for economic inequality, social mobility across generations will be a function of the intensity of marital sorting on genotypes. We construct a model of equilibrium sorting in which incomplete information about genotypes is the only force precluding perfect segregation on genotypes. Social mobility therefore depends on the ability of singles to infer the genotype of potential spouses from observables such as education and family background. Our model explains why public policies of redistribution or education may have little effect on long-run social mobility, consistent with the findings of Clark [2014]. The analysis reveals a novel channel through which grandparents and more distant ancestors affect current outcomes.

Keywords: Family Economics, Inequality, Household Formation, Marriage.
JEL Classification: D10, D19, D31, D80, D83, H31, H52, I24, J11, J12, J18

*We are grateful for constructive comments from the SFU Brownbag seminar. This research was supported by the Social Sciences and Humanities Research Council of Canada.

†Department of Economics, Simon Fraser University, 8888 University Drive, Burnaby, B.C. Canada V5A 1S6. cbidner@sfu.ca

‡Department of Economics, Simon Fraser University, 8888 University Drive, Burnaby, B.C. Canada V5A 1S6. jaknowle@sfu.ca

1 Introduction

The role of genetics in explaining economic inequality has become increasingly accepted in the social sciences. The accumulation of evidence, from sibling studies and genome analysis (GWS) suggests that inherited ability is determined by the action of many genes and influences in turn a wide diversity of outcomes, including education and financial sophistication. The recent literature on economics of inequality suggests widespread concern about the implications of the genetic view for explaining empirical patterns of inequality and for the design of redistributional policy.

Genetic transmission implies an important role for an element of choice that is often overlooked by economic models of social mobility: the selection of a mate. In this paper we propose a simple model of marital matching in which mate selection is motivated by concerns for the genotype that will be passed onto the children of the union. The key element of our approach is incomplete information: genotypes are unobserved and must be inferred from public signals, such as human capital or labor-market productivity.

Direct evidence, in the form of GWS results, suggests that spouse's genotypes do tend to be correlated, but are silent on the mechanism or motivation that gives rise to these results. There is a theoretical literature on the equilibrium determination of who marries whom (marital sorting), but this literature generally ignores the possibility that marriage is motivated by the production of children. Conversely, economic models of intergenerational transmission, following Becker and Tomes [1979], generally assume away variation in marital sorting on genotype. Two exceptions should be noted: Feldman et al. [2000], who allow for the genetic impact of marital sorting, in their model of intergenerational mobility, and Zak and Park [2002], who model sorting on an exogenous vector of characteristics, including genes.

In our matching model, singles observe the human-capital attainment of potential spouses, which permits imperfect inference about their genotypes. The precision of these inferences can be increased by Bayesian updating on the family history of a potential spouse. In its full generality, this framework is far more complex than standard matching models: family history involves many lineages, matching is multi-dimensional, and the distribution of genotypes evolves endogenously with each generation. Incorporating a general treatment of any of these features would a technically formidable challenge.

To avoid this complexity, we instead simplify the matching process so that, under the standard additive model of polygenic inheritance, the equilibrium sorting implies PAM on all observable variables; under complete information, this would yield the segregation case alluded to above. We then show that the equilibrium takes a particularly simple form; the family history of any potential spouse is summarized by a scalar variable, which we call "status", that follows a simple and tractable law of motion. This variable represents the equilibrium expectation of the mean genetic score for ability, prior to observing the signal. Effectively, spouses sort on posterior beliefs about genotype.

We find that the model economy has a unique stationary equilibrium, and characterize the resulting parent-child and husband–wife correlations of genotypes. Thus we can characterize the resulting intergenerational mobility and inequality, in terms of genetic ability, human capital and income.

Unsurprisingly, any changes to the environment that make family status more informative will increase sorting on genotype, reducing social mobility — even though sorting on family status, by construction, remains constant. Thus if technological change makes ability more important as a determinant of output, the equilibrium matching will be more assortative, and consequently social mobility, in terms of ability will fall.

This 'information principle' carries over to the analysis of social policies. Clark [2014] has argued that the long-run social mobility of families is both surprisingly low and surprisingly uniform across countries and across changes of social regimes. We show that, when social policies are common knowledge, redistribution does not affect sorting on genotype, and hence the long-run social mobility is unaffected. This invariance is nevertheless consistent with the 'Gatsby curve' of Corak [2013], the strong cross-country relationship between father-son income correlations and measures of after-tax income inequality, such as the Gini coefficient. When redistribution policies are common knowledge, rational agents are able to unwind the effects of the policy and infer the original income from the after-tax income, so redistribution does not affect inferences about genotype, preserving the informativeness of the family-status variable. Thus a country like Sweden with a high degree of redistribution, can have low inequality and high parent-child mobility at the same time as having, due to strong sorting on genotype, low long-term mobility.

A similar argument applies to education policies. Suppose that education is a function of both ability and parental income. If education substitutes for genetic ability in the production of income, then education subsidies might reduce sorting on genotype and thus increase social mobility. Alternatively if education and genetic ability are complements, then meritocratic education policies might by, by magnifying the effects of genetic inequality, reduce long-run social mobility. However if both parental income and the education policy are common knowledge, rational agents

[0] It may also be worth pointing out that under the AGM, if fertility variation is unrelated to ability, as it is in our analysis below, then marital sorting has no effect on average ability of a population; thus our analysis has no implications for racial differences in means.

are still able to unwind the policy effects, so inference of the agent's ability from the family income history is unaffected. PAM on family status thus implies that long-run social mobility is unaffected by such education policies.

Parental concern over family status provides new incentives for human-capital investment. Suppose that the investment is observed only by the parent; then the child's signal will be distorted, resulting in a better matching for the child, and hence a higher genotype for the grandchild. We show that the resulting equilibrium in our model is unique and is characterized by over-investment in education. We also show that the model can be extended to allow for imperfect sorting on observables; our equilibrium is therefore consistent with empirical spouse correlations of education and ability.

1.1 Related Literature

Our paper is most strongly related to the literature on social mobility (both theoretical and empirical) and to the literature on equilibrium matching.

Our paper complements a large empirical literature on intergenerational mobility dating back to Behrman and Taubman [1985] by providing a theory that incorporates marital sorting on genetic endowments. This possibility was acknowledged as important by Becker and Tomes [1979] but their model, and later versions assumed each child had only one parent and human capital investment was the only endogenous variable underlying mobility. Aiyagari et al. [2000] incorporated marital sorting into this approach. Kremer [1998] proposed a Markovian model of parental sorting and argued that the effects of sorting on inequality were negligible, given plausible levels of intergenerational persistence of education. Sorting matters in our model precisely because genetic ability is more persistent than noisy measures of ability such as education. Feldman et al. [2000] does propose a model of child outcomes with two-parent genetic effects but does not specify a marital sorting mechanism to generate the distribution of parents.

The main evidence for genetic ability consists of sibling studies. The results of dozens of twin studies since the 1980s are summarized by Polderman et al. [2015], who concludes that 70 per cent of cognitive- ability variation is explained by genetic heritability. An important study comparing biological and adopted children, Sacerdote [2007] suggests that genetic effects on education attainment are three times stronger than environmental effects.

Recent advances in genome analysis also support two key pieces of our argument. Using enormous genetic datasets, Okbay et al. [2016] have identified dozens of specific DNA snippets that predict education attainment, while Barth et al. [2017] show that the education scores derived from these snippets also predict sophisticated financial behavior, independently of the effects on income and education. Evidence for spouse correlations at the genetic level include Robinson et al. [2017] and Conley et al. [2016]. Psychological evidence that spouses are correlated in cognitive ability dates back at least to Phillip et al. [1987].

The weakness of the link between long-run social mobility and measured father-son correlations is demonstrated by Clark [2014], who documents the astonishing persistence of family status across centuries and concludes that the invisibility of genes in the matching process is the main guarantee of social mobility.

Contrary to most economic models of marital sorting, the main variable of interest in our model, the genotype, is not observed by any of the participants in the matching process. This makes our theory similar in a spirit to the static models of Chade and Eeckhout [2017] who have matching with symmetric type uncertainty, or Anderson [2015] and Anderson and Smith [2010] who extend that approach to a state variable that evolves over time. These are one-dimensional models however that abstract both from the phenotype-genotype distinction that plays a key role here and hence from the problem that inferences for any one agent are based on many family histories, rather than a single lineage. Limited progress has been made on characterizing multi-dimensional matching with complete information; this is summarized in Lindenlaub [2014] .

2 Model

We now develop a model of a matching market in which singles would like to maximize the expected income of their descendants. This makes people with superior genotypes more desirable than others, holding other variables constant. To ensure tractability, we make a number of functional form assumptions in the structural model such that the reduced form of the model can be expressed as three log-linear equations. This structure ensures that the equilibrium sorting implies PAM on all observable variables, which leads in turn to a simple rule that governs the Bayesian evolution of beliefs. Later we extend the model to allow for imperfect sorting on observables.

The equations describing the genetic basis of ability and human capital follow directly from the standard model of quantitative genetics, the Additive Genetic Model (AGM) , as described by Feldman et al. [2000]. Quantitative traits, such as ability, are assumed in the AGM to be the result of the additive effects of many genes. This implies the effects of both parents on the child are equal and independent of the effects of other genes or the environment. Thus each genome can be assigned a genetic score for ability, and by a LLN result, the model implies normality of the (log) ability distribution as the number of genes goes to infinity.

2.1 Fundamentals

2.1.1 Population

Time in the model consists of an infinite succession of discrete periods : $t = 0, 1, 2, ...$ People in the model live for one period as adults. At $t = 0$ there exists a unit mass of each sex, who are paired off into a unit mass of households, which is indexed by $i \in [0, 1]$. Each household i has two identical offspring, one of each sex; each offspring adopts the family index and joins the adult population in period $t + 1$. Males and females are paired in each period, once again forming a unit mass of households, which we can index by the husband's name $i \in [0, 1]$.[1] We let i' denote the maiden name of the mother in family i. Note for every type of adult at each time t there exists an equal mass of each sex.

Singles in each generation are differentiated by an unobservable variable, which we call 'ability' and is determined by a person's genome. There is also an observable variable, or phenotype, that is a function of genotype and other inputs.

2.1.2 Genotype

At date 0, nature endows both agents in family i (i.e. the son and daughter) with a 'genotype', summarized by ability $\theta_{i0} \in \mathbb{R}_+$. This common ability is drawn from a normal distribution; $\theta_{i,0} \sim N(0, \bar{\gamma}_0)$ where $\bar{\gamma}_0 > 0$. Our assumption that siblings get the same ability realization is for simplicity–it is not essential but allows us to abstract from the possibility that sibling outcomes are informative about one's genotype. The genotypes of subsequent generations are given by:

$$\theta_{it} = b \cdot [\theta_{i,t-1} + \theta_{i',t-1}]/2 + \upsilon_{it} \tag{1}$$

where $b \in (0, 1)$ and $\upsilon_{it} \sim N(0, \sigma_\upsilon^2)$ is a shock. Recall that i' is the 'maiden index' of the mother in family i, so that $\theta_{i',t-1}$ is her ability.

2.1.3 Phenotype

The phenotype consists of an observable variable, human capital, which, in combination with the spouse's phenotype, and public policy, determines household income. This part of the structural model is fully described in the Appendix, but here we present the reduced-form equations.

The log of the human capital of agent i in generation t, denoted $x_{it} \equiv \ln X_{it}$, is produced by ability θ_{it} (nature), the log of parental investment $h_{i,t-1}$ (nurture), and

[1] Since there is no asymmetry by sex, the female's index would work just as well. The emphasis on the male lineage is motivated by the empirical literature which focuses on traits that are easier to measure for the male lineage, such as last names and earnings.

"economic luck" ε_{it}. We allow for redistribution of parental investment as a policy tool (via an expansion of public schooling, say). The reduced-form relationship is given by:

$$x_{i,t} = \alpha_0' + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot h_{i,t-1} + \varepsilon_{i,t}, \tag{2}$$

where $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$. The parameter α_1 captures the extent to which ability (nature) matters for human capital production. The parameter α_2 captures the extent to which parental investment (nurture) matters for human capital production, but also inversely captures the extent of parental investment redistribution. For instance, "equality of opportunity" arises when $\alpha_2 = 0$. The constant, α_0', simply accounts for redistribution (i.e. ensures that the resource constraint binds).

It is useful to consider a special case whereby all families invest the same proportion of their income, z. We will go on to show that this is optimal and derive the optimal z. But holding it fixed for now helps clarify the main arguments to follow. Fixing z allows us to write

$$x_{i,t} = \alpha_0 + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot y_{i,t-1} + \varepsilon_{i,t}, \tag{3}$$

where $\alpha_0 = \alpha_0' + \alpha_2 \cdot \ln z$.

The log income of a household consisting of a member of family i and a member of family i' in period t depends on household output as well as on redistributive factors. Household output depends on the human capital of household members. Redistribution of output arises from two sources–a progressive taxation system, as well as deviations from meritocracy whereby rents are acquired by virtue of parental wealth. The reduced-form relationship has log household income being a function of average household human capital and average household parental income:

$$y_{i,t} = \beta_0 + \beta_1 \cdot [x_{i,t} + x_{i',t}]/2 + \beta_2 \cdot [y_{i,t-1} + y_{i',t-1}]/2. \tag{4}$$

The inverse of the parameter β_1 reflects the extent of redistributive taxation in the model, while the inverse of the parameter β_2 reflects the extent of meritocracy. The constant β_0 simply ensures that the resource constraint holds (total output equals total income).

2.1.4 Preferences

Agents have preferences over their own consumption and the infinite sequence of consumption of their descendants. Specifically, agents care about the present discounted value of utilities:

$$U_{i,t} = u(C_{i,t}) + \mathbb{E}\left[\sum_{\tau=1}^{\infty} \delta^{\tau} \cdot u(C_{i,\tau})\right] \tag{5}$$

where $\delta \in (0,1)$ is the discount factor and $C_{i,t}$ is the consumption of household i in period t. We impose log utility, which, given that all households invest a fraction z of their income in the human capital of offspring, implies that:

$$U_{i,t} = y_{it} + \mathbb{E}\left[\sum_{\tau=1}^{\infty} \delta^{\tau} \cdot y_{i,\tau}\right] + U_0, \tag{6}$$

where $U_0 \equiv \ln(1-z) + [\delta/(1-\delta)] \cdot \ln(1-z)$ is a constant. Thus, when families invest a fixed share of their income, log utility implies agents act as if they care about the present discounted value of (log) family income.

2.1.5 Information Structure

In the date t singles pool, each member of family i (there are two, one male and one female) is characterized by $\omega_{it} \equiv \{\Psi_{it}, x_{it}, y_{i,t-1}\}$, where Ψ_{it} is a distribution describing beliefs about θ_{it}. Beliefs Ψ_{it} are formed by first forming a prior based on the genetic transmission equation and the beliefs associated with the agent's parents $(\Psi_{it}, \Psi_{i't})$. This prior is then updated via Bayes' rule on the basis of observed human capital $x_{i,t}$ and parental income $y_{i,t-1}$.

2.1.6 Equilibrium

Let Ω be the set of possible realizations of ω_{it}, and let $\omega_t \equiv \{\omega_{it}\}_{i \in [0,1]}$ describe the realized set of characteristics on offer by one side of the marriage market (it is the same for each side by construction) at date t. A *matching* $m_{\omega_t,t} : \Omega \to \Omega$ describes the characteristics of the spouse that is assigned to a male as a function of their own characteristics. That is, a male with characteristics ω_{it} is to marry a female with characteristics $m_{\omega_t,t}(\omega_{it})$.

A *matching equilibrium* is a collection of matchings, $\{m_{\omega_t,t}\}_{\omega_t,t}$, such that at each (ω_t, t) the matching $m_{\omega_t,t}$ is (i) stable: there is no unmatched pair such that both prefer to marry each other over their assigned partner (at least one strictly), and (ii) feasible: for all measurable $\tilde{\Omega} \subseteq \Omega$ the measure of males with $\omega_{it} \in \tilde{\Omega}$ equals the measure of females with types in the image of $\tilde{\Omega}$ under m.

An *equilibrium* is an investment rule (mapping disposable income into parental human capital investment) and a matching, such that investments are optimal given the matching and the matching is stable and feasible given the investment rule.

3 Analysis and Results

3.1 Segregation Matching Equilibrium

Our assumptions of a large population and gender symmetry are immensely valuable here since they allows us to consider a simple candidate for matching equilibrium: segregation. That is, $m_{\omega_t,t}(\omega_{it}) = \omega_{it}$: agents marry someone with identical characteristics.[2] This allows us to focus attention on equilibrium ability sorting holding fixed the sorting patterns along other dimensions (such as human capital and parental income). In the appendix (section B) we verify that segregation indeed constitutes a matching equilibrium.

Under segregation, the equations of motion for human capital and income simplify to:

$$
\begin{aligned}
x_{i,t} &= \alpha_0 + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot y_{i,t-1} + \varepsilon_{i,t} \\
y_{i,t} &= \beta_0 + \beta_1 \cdot x_{i,t} + \beta_2 \cdot y_{i,t-1}.
\end{aligned}
$$

Using the former in the latter gives a relationship only involving ability and income:

$$y_{i,t} = \pi_0 + \pi_1 \cdot \theta_{i,t} + \pi_2 \cdot y_{i,t-1} + \varepsilon^{\varepsilon}_{i,t}$$

where

$$\pi_0 \equiv \beta_0 + \beta_1\alpha_0 \tag{7}$$

$$\pi_1 \equiv \beta_1\alpha_1 \tag{8}$$

$$\pi_2 \equiv \beta_1\alpha_2 + \beta_2 \tag{9}$$

$$\varepsilon^{\pi}_{it} \equiv \beta_1 \cdot \varepsilon_{it}. \tag{10}$$

3.2 Steady State

A steady state arises when the variance of ability σ^2_θ (dispersion), the husband-wife ability correlation ρ^{HW}_θ (sorting), and the parent-child ability correlation ρ^{PC}_θ (persistence) are all independent of time. In a steady state, the type transmission equation alone allows to derive the variance of types and the parent-child type

[2] This should not be interpreted as siblings marrying. Rather, it is a large-population idealization of marrying someone (from a different family) with 'similar' characteristics.

correlation as functions of the husband-wife correlation:[3]

$$\sigma_\theta^2 = \frac{\sigma_\upsilon^2}{1 - \frac{b^2}{2} \cdot (1 + \rho_\theta^{HW})} \tag{11}$$

$$\rho_\theta^{PC} = \frac{b}{2} \cdot (1 + \rho_\theta^{HW}) \tag{12}$$

Our interest lies in deriving the husband-wife ability correlation. To this end, let $\phi_{it} \equiv \int \theta d\Psi_{it}(\theta)$ denote an agent's expected abilty, and express an agent's ability as the sum of this expectation and a belief error:

$$\theta_{it} = \phi_{it} + \varepsilon_{it}^\gamma.$$

Notice that the variance of belief errors ε_{it}^γ at date t, denoted γ_t, must fall between zero (when beliefs are perfectly accurate) and σ_θ^2 (when beliefs are completely uninformative). In a steady state where the variance of belief errors is independent of time, denoted γ, we have:[4]

$$\rho_\theta^{HW} = 1 - \frac{\gamma}{\sigma_\theta^2}. \tag{13}$$

The steady-state values of $(\rho_\theta^{HW}, \sigma_\theta^2, \rho_\theta^{PC})$ are therefore given by the solution to the three numbered equations above, as we now report.[5]

Proposition 1 *If the variance of belief errors, ε_{it}^γ, is constant over time and denoted γ, then the dispersion, persistence and sorting properties of ability are given by:*

$$\sigma_\theta^2 = \frac{\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2}}{1 - b^2} \tag{14}$$

$$\rho_\theta^{PC} = b \cdot \frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}} \tag{15}$$

$$\rho_\theta^{HW} = \frac{\sigma_\upsilon^2 - \gamma \cdot (1 - \frac{b^2}{2})}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}}. \tag{16}$$

[3] These are the equations underlying Kremer (1997), where the husband-wife correlation is taken as exogenous. For instance, if we had instead assumed that type were observed then segregation would imply $\rho_\theta^{HW} = 1$ and therefore $\sigma_\theta^2 = \sigma_\upsilon^2/[1 - (b^2/2)]$ and $\rho_\theta^{PC} = b$.

[4] The intuition for the dependence on σ_θ^2 is as follows. Consider two agents that end up with the same posterior belief. A given dispersion of belief errors tells us the expected difference in the pair's genotypes. A given expected difference will have a large impact on the correlation if the distribution of ability is tight (e.g. there is a higher chance that the 'top' genotypes will end up with the 'bottom' genotypes). The impact on the correlation is small if the distribution of ability is disperse (e.g. matches will tend to be more 'local' relative to the range of abilities in the population).

[5] The fact that $\gamma \in [0, \sigma_\theta^2]$ ensures that each of the quantities in the proposition fall within the appropriate ranges (i.e. variance is non-negative and correlations are between -1 and 1. Specifically, $\gamma \in [0, \sigma_\theta^2]$ ensures that $\rho_\theta^{HW} \in [0, 1]$, that $\rho_\theta^{PC} \in (b/2, b)$, and that $\sigma_\theta^2 \in [\sigma_\upsilon^2/(1-b^2/2), \sigma_\upsilon^2/(1-b^2)]$.

Each of these outcomes is increasing in the precision of beliefs (i.e. is decreasing in γ).

The proof is contained in the derivations in appendix section D. Figure 1 illustrates how ρ_θ^{PC} and σ_θ^2 are jointly determined by γ (using (13) and (11)).

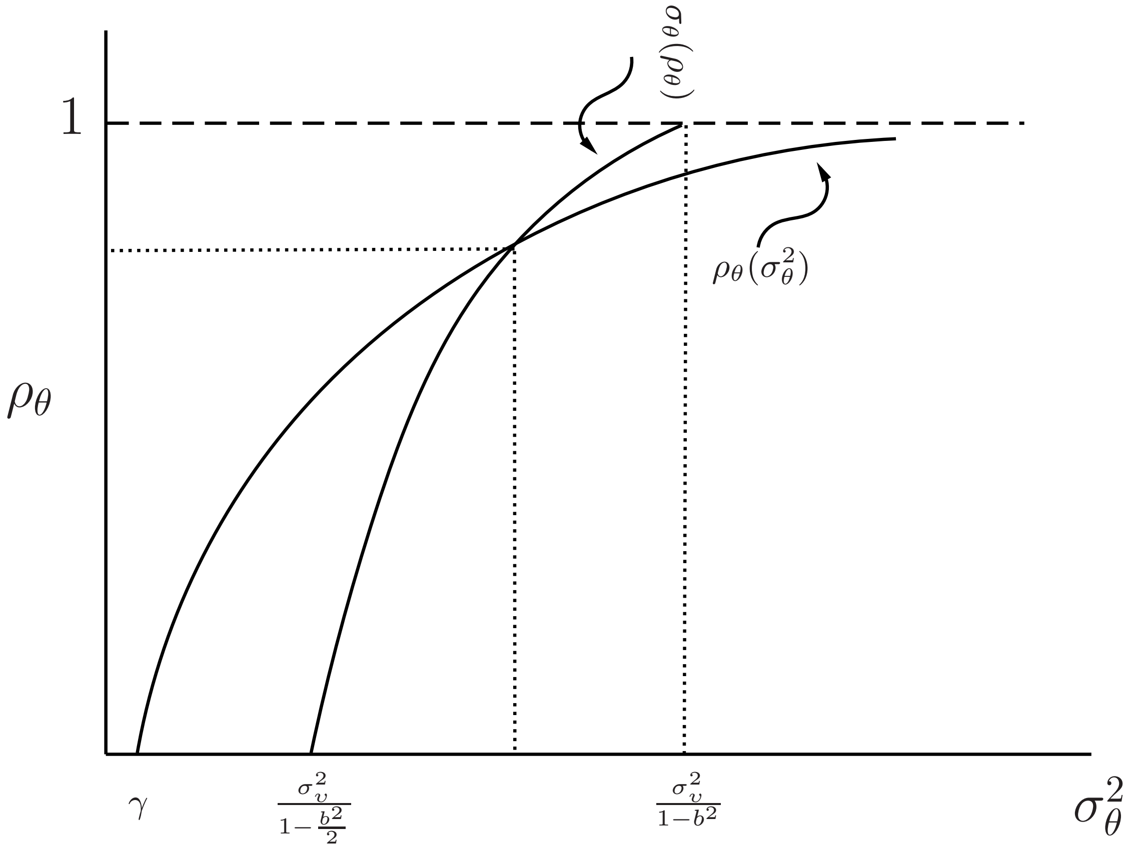

Figure 1: Long-Run Genotype Sorting and Dispersion

While understanding the factors driving the sorting, persistence, and dispersion of ability is important in its own right, we are also interested in the consequences for social mobility. To this end, we have the following.

Proposition 2 *Given a steady-state variance of ability, σ_θ^2, the dispersion and persistence properties of income are given by:*

$$\sigma_y^2 = \frac{\left(\frac{1+b\pi_2}{1-b\pi_2}\right)\alpha_1^2\beta_1^2 \cdot \sigma_\theta^2 + \beta_1^2\sigma_\varepsilon^2}{1-\pi_2^2} \tag{17}$$

$$\rho_y^{PC} = \frac{\left(\frac{b+\pi_2}{1-b\pi_2}\right)\alpha_1^2 \cdot \sigma_\theta^2 + \pi_2 \cdot \sigma_\varepsilon^2}{\left(\frac{1+b\pi_2}{1-b\pi_2}\right)\alpha_1^2 \cdot \sigma_\theta^2 + \sigma_\varepsilon^2}. \tag{18}$$

Both of these outcomes are increasing in σ_θ^2 and thus are increasing in the precision of beliefs (i.e. is decreasing in γ).

The proof is contained in the derivations in appendix section D.

The expression for the persistence of income, ρ_y^{PC}, makes clear the role of heritable ability in social mobility: a positive intergeneration income correlation would be observed *even if* parental income had no causal impact on income (i.e. even if $\pi_2 = 0$).

We stress that the steady state precision of beliefs, γ, is endogenously determined, and therefore fully analysing the properties of a steady state requires that the information structure is sufficiently tractable that the steady state precision of beliefs can be characterized. We now turn to this.

3.3 Steady State Precision of Beliefs

Suppose that beliefs about the types of those of the previous generation are described by a normal distribution with an agent-specific mean and common variance: $\theta_{i,t-1} \sim N(\phi_{i,t-1}, \gamma_{t-1})$. Then the type transmission equation implies that prior beliefs about the current generation are also normal, given by

$$\theta_{it} \sim N(\bar{\phi}_{it}, \bar{\gamma}_t) \tag{19}$$

where $\bar{\phi}_{it} = b \cdot [\phi_{i,t-1} + \phi_{i',t-1}]/2$ and $\bar{\gamma}_t = (b^2/2) \cdot \gamma_{t-1} + \sigma_v^2$. These prior beliefs are then updated on the basis of an agent's human capital, x_{it}, and parental income, $y_{i,t-1}$. Whilst this sort of Bayesian updating can quickly become complicated, our structure allows us to conduct this updating in a very tractable manner. By re-arranging the human capital equation we get

$$s_{i,t} \equiv \frac{x_{it} - \alpha_0 - \alpha_2 \cdot y_{i,t-1}}{\alpha_1} = \theta_{it} + \xi_{i,t}, \tag{20}$$

where $\xi_{i,t} \equiv \varepsilon_{i,t}/\alpha_1$. That is,

$$s_{it} \sim N(\theta_{it}, \sigma_\xi^2) \tag{21}$$

where $\sigma_\xi^2 = \sigma_\varepsilon^2/\alpha_1^2$. From (19) and (21) standard results tell us that the posterior is also normal:

$$\theta_{it} \mid s_{it} \sim N(\phi_{it}, \gamma_t), \tag{22}$$

where

$$\gamma_t \equiv \frac{\sigma_\xi^2 \cdot \bar{\gamma}_t}{\sigma_\xi^2 + \bar{\gamma}_t}, \tag{23}$$

and

$$\phi_{it} \equiv \lambda_t \cdot \bar{\phi}_{it} + (1 - \lambda_t) \cdot s_{it}, \tag{24}$$

where

$$\lambda_t \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2 + \bar{\gamma}_t} \tag{25}$$

is the weight that agents place on the prior.

Since prior beliefs at date $t = 0$ are of the form indicated in (19), with $\bar{\phi}_{i0} = 0$ and $\bar{\gamma}_0$ given, we have that posterior beliefs about generation $t = 0$ are normal with a common variance as indicated by (22). But then our supposition regarding beliefs about the prior generation holds for generation $t = 1$, leading to the conclusion that their posterior beliefs are also given by (22), validating the supposition for generation $t = 2$ and so on. It therefore follows that the variance of beliefs evolves according to the following difference equation:

$$\gamma_t = \frac{\sigma_\xi^2 \cdot [(b^2/2) \cdot \gamma_{t-1} + \sigma_\upsilon^2]}{\sigma_\xi^2 + (b^2/2) \cdot \gamma_{t-1} + \sigma_\upsilon^2}. \tag{26}$$

It is straightforward to see that this implies global convergence. In particular, the steady state variance of belief variance, denoted γ, is the unique positive solution to

$$\gamma = \frac{\sigma_\xi^2 \cdot [(b^2/2) \cdot \gamma + \sigma_\upsilon^2]}{\sigma_\xi^2 + (b^2/2) \cdot \gamma + \sigma_\upsilon^2}. \tag{27}$$

Proposition 3 *The variance of beliefs converges to the steady state value, γ, which satisfies (27). The steady state variance of beliefs, γ, is (i) increasing in $\sigma_\xi^2 \equiv \sigma_\varepsilon^2/\alpha_1^2$, (ii) increasing in b and σ_υ^2, and is (iii) independent of α_2, β_1, and β_2.*

The proof follows from the above discussion with the comparative statics being easily derived from (27).[6]

Given γ we can derive the steady state weight that agents place on their prior, λ. This value indicates the importance that agents place on family background relative to individual performance. We have that $\lambda_t \to \lambda$ where

$$\lambda \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2 + (b^2/2) \cdot \gamma + \sigma_\upsilon^2}. \tag{28}$$

[6]The right side of (27) is increasing in σ_ξ^2 (which is $\sigma_\varepsilon^2/\alpha_1^2$), implying that so too is γ. The right side of (27) is increasing in σ_υ^2 and in b, implying that so too is γ.

Proposition 4 *The long-run relative importance of family background, λ, is (i) increasing in $\sigma_\xi^2 \equiv \sigma_\varepsilon^2/\alpha_1^2$, (ii) decreasing in b and σ_υ^2, and is (iii) independent of α_2, β_1, and β_2.*

Notice that an increase in signal noise, σ_ξ^2, in steady state raises both the variance of beliefs and the relative importance of family background. On the other hand, an increase in b or σ_υ^2 in the steady state also raises the variance of beliefs yet reduces the relative importance of family background.

We are now in a position to discuss the factors that shape the sorting, persistence and dispersion of ability by combining propositions 1 and 3. We are also in a position to discuss the factors that shape the social mobility and inequality by combining propositions 2 and 3. We now turn to such issues.

4 Discussion of Results

4.1 Economic Environment

By construction, an agent's inherited ability is not affected by their environment. Nevertheless, the environment shapes the dispersion and persistence of ability in a society. It does this by changing the availablity of information about types, thereby influencing the sorting of types. The model is useful for identifying which aspects of the environment can be expected to have an impact and which aspects will not.

Corollary 1 *An increase in $\alpha_1/\sigma_\varepsilon^2$ strengthens the sorting, persistence and dispersion of ability.*

Thus an increase in the return to ability α_1 (holding luck constant) reduces within-household ability heterogeneity (both in terms of husbands and wives and in terms of parents and children) but increases between-household ability heterogeneity. To be sure, this is because an increase in the return to ability makes an agent's human capital a more reliable signal of ability and this fact facilitates stronger sorting in the marriage market. Another plausible mechanism has to do with frictional matching: a greater return to ability provides incentives to search more intensely for a high ability partner. To distinguish the two mechanisms, we note that the competing explanation would also predict sorting systematically varies with the policy environment (e.g. lower redistributive taxation should also provide incentives to search more intensely for a high ability partner) however, this is not true in our case (see the section that follows). The evidence from Clark [2014] is supportive of our mechanism.

In terms of income, the variables $(\alpha_1, \sigma_\varepsilon^2)$ will have a direct effect on inequality and social mobility (i.e. holding ability sorting fixed) and an indirect sorting effect.

Corollary 2 *The direct effect of α_1 on social mobility and on inequality is exacerbated by the sorting effect. The direct effect of σ_ε^2 on social mobility is exacerbated by the sorting effect, whereas the direct effect of σ_ε^2 on inequality is mitigated by the sorting effect.*

Intuitively, if we hold ability sorting fixed, a larger return to ability lowers social mobility and raises inequality. But it also facilitates stronger marital sorting on ability, and thereby raises the persistence and dispersion of ability, which in turn lowers social mobility and raises inequality further. Similarly, if we hold ability sorting fixed, a larger luck component raises social mobility and raises inequality. But it also weakens marital sorting on ability, and thereby lowers the persistence and dispersion of ability, which in turn raises social mobility further but also lowers inequality.

4.2 Institutional/Policy Environment

The institutional/policy environment is captured by our parameters α_2, β_1, and β_2. Recall that these describe the extent to which parental human capital inputs are redistributed, as well as the extent to which income is redistributed via taxation and departures from meritocracy. In terms of income, these parameters clearly have an impact on social mobility and inequality (see proposition 2). However, in the base model at least, they have no impact on the extent of *ability* sorting, persistence or dispersion.

Corollary 3 *The institutional/policy environment variables captured by the reduced-form parameters α_2, β_1, and β_2 have no effect on the sorting, persistence or dispersion of ability.*

This result highlights the difficulty in inferring (unobservable) changes in the sorting, persistence and dispersion of ability from (observable) changes in the sorting, persistence and dispersion of income. The evidence from Clark [2014] is supportive of this implication of our model.

4.3 Heritability Environment

The 'heritability' variables, b and σ_v^2, will clearly have a direct impact on the dispersion and persistence of the heritable characteristic (see proposition 1). But, less obviously, they will also have an effect on sorting and thus an indirect sorting effect on dispersion and persistence of ability via their effect on steady state belief precision.

Corollary 4 *The direct effect of b on the persistence and dispersion of ability is mitigated by the sorting effect. The direct effect of σ_v^2 on the persistence and dispersion of ability is exacerbated by the sorting effect.*

4.4 Summary

We have shown how the precision of beliefs is key to understanding the sorting, persistence and dispersion of ability as well as social mobility and inequality. We then showed which factors shape the precision of beliefs in the long run.

This exercise had little to say about income *levels* are affected by the presence of unobserved heritable characteristics because we took parental investment to be a fixed proportion of income. In the following section we endogenize parental investment.

The exercise also has little to say about sorting in the human capital dimension. Indeed, we have held this fixed throughout in order to isolate the impact on ability sorting. To demonstrate that nothing in the analysis hinges on perfect sorting on the human capital dimension we present an extension where an agent's human capital is not perfectly observed in the marriage market.

Finally, the conclusion that the institutional/policy environment does not matter arises because the relevant parental characteristics are assumed to be perfectly observed in the marriage market. We relax this assumption by supposing that parental investment has a stochastic impact on human capital. This feature breaks perfect sorting on parental investment and introduces a role for some policy variables.

5 Comparative Statics

In this section we ask how endogeneity of sorting affects the comparative statics of the model. In the first set of exercises, we use the parameter values shown in Table 1. The genetic contribution of the parents is represented by setting b close to 1, reflecting the low rate of mutation observed for most genes.

In Figure 3 we show the effect of increasing the share α_1 of skill in the production technology, from 0 to 1. Two series are shown in each panel: one represents the effect when the marital sorting adjusts according to the equilibrium of the model, while the other represents the results with the sorting held fixed at $\rho_\theta^{HW} = 0.5$. Figure 2 shows that the husband-wife correlation of ability ranges roughly from 0 to 0.9, implying that changes in α_1 have a significant capacity to influence sorting on genotype.

Figure 3 shows the impact of α_1 on related outcomes. We see that the intergenerational correlation of ability varies from around 0.5 to around 1. Importantly, this change in the intergenerational correlation is entirely due to the sorting effect: if

Table 1: Parameter Values

Parameter	Value	Parameter	Value
parental genetic effect b	0.95	α_2	0
ability shock σ_υ	0.5	β_1	1
productivity shock σ_ε	0.5	β_2	0

the husband-wife correlation were treated as fixed the value of α_1 would not affect the intergenerational correlation. A similar point is made for the standard deviation of ability. Turning to implications for income, we see that an increase in α_1 raises the intergenerational income correlation directly, but the sorting effect raises it even further. A similar point is made for inequality (standard deviation of income).

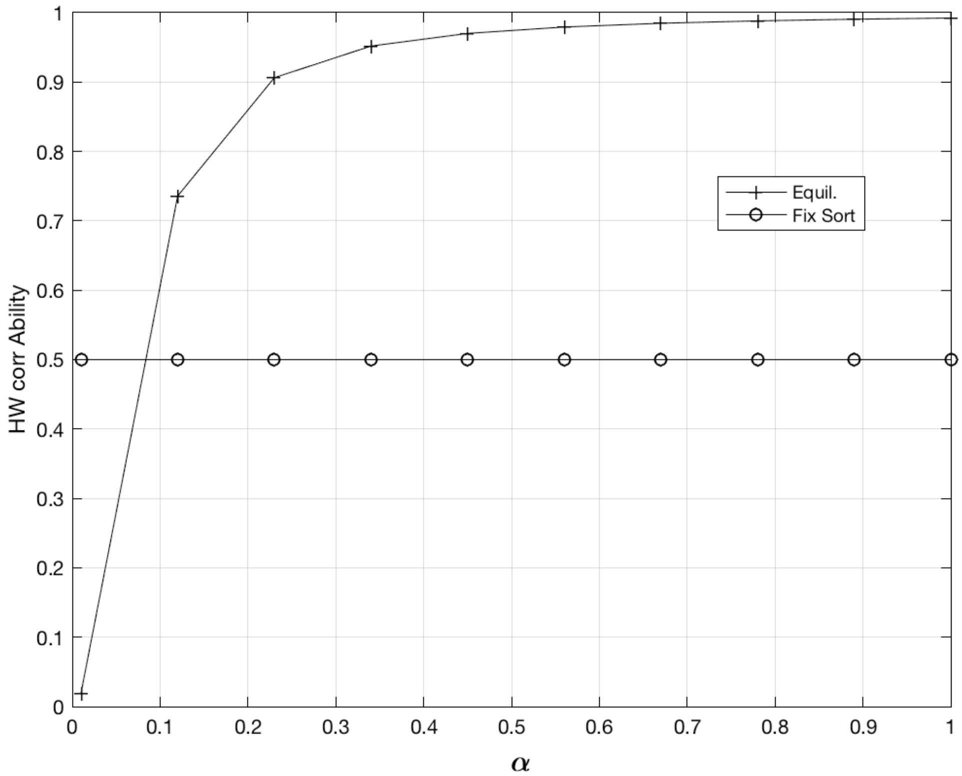

Figure 2: Technological change: An increase in α_1

6 Analyzing Multiple Generations

6.1 Multi-generational Correlations

What does the one-generation genotype correlation tell us about longer-run genotype correlations? We show that the true long-run genotype correlation is larger than that implied by a geometric extrapolation of the short-run correlation. This

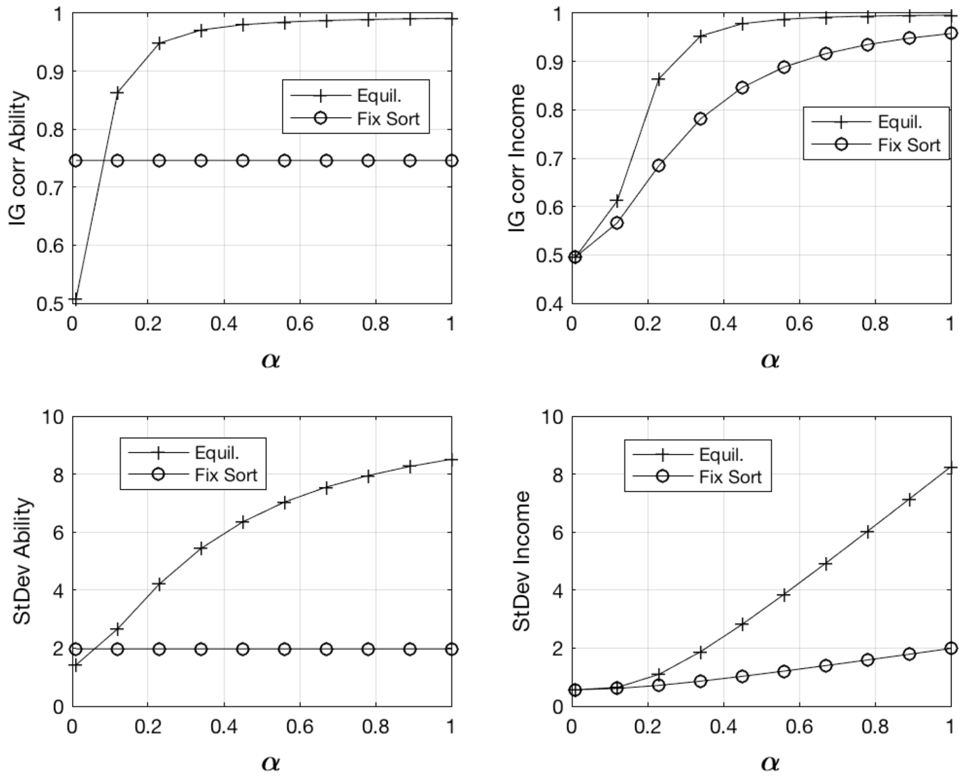

Figure 3: Technological change: An increase in α_1

is due entirely to imperfect genotype sorting. As such, the extent of this bias is endogenously determined in our setting.

The genotype correlation between family members $k \in \{2, 3, ..\}$ generations apart is given by:

$$\rho_{\theta,k}^{PC} \equiv \frac{\text{Cov}(\theta_{it}, \theta_{i,t-k})}{\sigma_\theta^2} = b^{k-1} \cdot \rho_\theta^{PC} = b^k \cdot \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}} \right]. \tag{29}$$

Thus the long correlation implied by extrapolating the short correlation geometrically understates the true long correlation:

$$\frac{(\rho_\theta^{PC})^k}{\rho_{\theta,k}^{PC}} = \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}} \right]^{k-1} \in (0, 1). \tag{30}$$

The extent of the bias increases in k as this ratio goes to zero as k increases. Furthermore this bias in endogenous in our setting, as the ratio is decreasing in γ.

We now turn to the same issue applied to incomes. To simplify the discussion, we set $\alpha_2 = \beta_2 = 0$ so that parental income has no direct effect on offspring income (as in Clark [2014]).[7] Here there is a similar bias, but it is not due to imperfect

[7] See the appendix for a generalized treatment of these claims.

genotype sorting. Rather, as stressed by Clark [2014] and others, it is due to income being driven by a persistent underlying variable (genotype). In this case we get:

$$\rho_{y,k}^{PC} = b^k \cdot \left[\frac{\alpha_1^2\sigma_\theta^2}{\sigma_y^2}\right] = b^k \cdot \left[\frac{\alpha_1^2\sigma_\theta^2}{\alpha_1^2\sigma_\theta^2 + \sigma_\varepsilon^2}\right], \tag{31}$$

so that the ratio of the implied value based on a single-generation correlation to the actual value is

$$\frac{(\rho_y^{PC})^k}{\rho_{y,k}^{PC}} = \left[\frac{\alpha_1^2\sigma_\theta^2}{\alpha_1^2\sigma_\theta^2 + \sigma_\varepsilon^2}\right]^k, \tag{32}$$

which goes to zero as k gets larger. Again, the contribution in this regard is to show that the extent of this bias depends on the (endogenous) strength of ability sorting (via the term σ_θ^2). In particular, a greater precision of beliefs increases the extent to which long correlations are under-stated by an extrapolation of short correlations.

6.2 Role of Luck and Family Status (Grandparents)

In this section we show how our analysis introduces new forces driving persistence of economic fortune. In short, the expected genotype of one's offspring, conditional on one's own genotype, is increasing in one's economic luck and in inherited family status. It is also increasing in the genotype of parents, grandparents and all previous generations. Even conditional on these genotypes, it is also increasing in the economic fortunes of parents, grandparents and all previous generations. These results do not violate the biological reality that only parental genotypes (and not phenotypes) matter for offspring genotypes. The key of course is that an agent's offspring depends on the genotype of their eventual spouse, and the quality of spouse that the agent can attract will be affected by the variables above. This effect cannot arise in models that take husband-wife genotype correlation as exogenous. Such models imply that the distribution of spousal genotypes depends only on one's own genotype, and importantly, not on one's phenotype.

To make these points as cleanly as possible, consider the expected genotype of agent i's offspring. This is given by:

$$\begin{aligned}
\mathbb{E}[\theta_{i,t+1} \mid \theta_{i,t}, \phi_{it}] &= \frac{b}{2} \cdot \theta_{i,t} + \frac{b}{2} \cdot \mathbb{E}[\theta_{i',t} \mid \theta_{i,t}, \phi_{it}] \\
&= \frac{b}{2} \cdot \theta_{i,t} + \frac{b}{2} \cdot \phi_{it} \\
&= \frac{b}{2} \cdot \theta_{i,t} + \frac{b}{2} \cdot [\lambda \cdot \bar{\phi}_{it} + (1-\lambda) \cdot s_{it}] \\
&= \frac{b}{2} \cdot \theta_{i,t} + \frac{b}{2} \cdot [\lambda \cdot \bar{\phi}_{it} + (1-\lambda) \cdot [\theta_{it} + \xi_{it}]] \\
&= \left[\frac{b}{2}(2-\lambda)\right] \cdot \theta_{i,t} + \left[\frac{b}{2} \cdot \lambda\right] \cdot \bar{\phi}_{it} + \left[\frac{b}{2} \cdot (1-\lambda)\right] \cdot \xi_{it}.
\end{aligned}$$

That is, the expected genotype of one's offspring depends on three factors: (i) own genotype, θ_{it}, (ii) inherited family status, $\bar{\phi}_{it}$, and (iii) own economic luck, ξ_{it}. Part (i) is obvious. Part (iii) indicates that economic luck affects the biology of offspring. Part (ii) indicates a role for grandparents and prior generations. For instance, the economic luck of grandparents will affect the status inherited by parents and thus the status inherited by agents. The same is true for the genetic luck of grandparents.

These results are relevant for understanding some additional incentives for parental investment that our analysis introduces. If offspring ability is affected by parental luck, then grandparents have an incentive to 'manufacture' economic luck by investing in their offspring (i.e. the parents). Although such investment cannot influence the genotype of their offspring it will influence the genotype of grandchildren (since, by the above logic, the investment will help one's child attract a higher genotype partner). We now turn to this issue.

7 Optimal Parental Investment

We now allow each household to make human capital investments in offspring optimally. We allow a limited form of asymmetric information whereby parents are better informed than the public about their investment. As a result, parents have two motivations for investing: a standard one of raising the income-generating capacity of offspring (Becker and Tomes [1986, 1979]), and a novel one of manipulating the market's assessment of their offspring's ability. By raising the market's assessment, offspring are able to secure partners with higher expected ability. Since matching is assortative on human capital and parental income, this has no impact on offspring income. However, it *will* have an impact on the income of grandchildren (and subsequent generations) because it raises their expected ability.

If family i makes a human capital investment given by a proportion z_{it} of their income, then their log consumption is

$$c_{it} = \ln(1 - z_{it}) + y_{it}. \tag{33}$$

This captures the cost of investing. The first benefit from investing has to do with the fact that investment raises the expected income of offspring. Recalling (2), their offspring will have a human capital of:

$$x_{i,t+1} = \alpha_0' + \alpha_1 \cdot \theta_{i,t+1} + \alpha_2 \cdot [\ln z_{it} + y_{it}] + \varepsilon_{i,t+1},$$

so that the expected income of offspring is therefore:

$$\mathbb{E}_t[y_{i,t+1}] = \pi_0' + \pi_1 \cdot \mathbb{E}_t[\theta_{i,t+1}] + \pi_2 \cdot y_{i,t} + \beta_1 \alpha_2 \cdot \ln z_{it}. \tag{34}$$

The second benefit of investing is that it will raise the status of offspring. As previously, the market observes human capital and parental income. But the market also has rational expectations about the investment that was made by agents' parents. If the market expects an investment share of z_{it}^*, then the relevant signal given an actual investment of z_{it} is:

$$s_{i,t+1} \equiv \frac{x_{i,t+1} - \alpha_0' - \alpha_2 \cdot y_{i,t}}{\alpha_1} = \theta_{i,t+1} + \xi_{i,t+1} + \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*).$$

The signal translates into status according to (24). That is:

$$\begin{aligned}
\mathbb{E}_t[\phi_{i,t+1}] &= \frac{\sigma_\xi^2}{\sigma_\xi^2 + \bar{\gamma}} \cdot \mathbb{E}_t[\bar{\phi}_{i,t+1}] + \frac{\bar{\gamma}}{\sigma_\xi^2 + \bar{\gamma}} \cdot \mathbb{E}_t[s_{i,t+1}] \\
&= \frac{\sigma_\xi^2}{\sigma_\xi^2 + \bar{\gamma}} \cdot b\phi_{it} + \frac{\bar{\gamma}}{\sigma_\xi^2 + \bar{\gamma}} \cdot \left[\mathbb{E}_t[\theta_{i,t+1}] + \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*)) \right] \\
&= b\phi_{it} + \frac{\bar{\gamma}}{\sigma_\xi^2 + \bar{\gamma}} \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*) \\
&= b\phi_{it} + \frac{\gamma}{\sigma_\xi^2} \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*)
\end{aligned} \tag{35}$$

where $\bar{\gamma} = (b^2/2) \cdot \gamma + \sigma_v^2$ is the steady state variance of prior beliefs.

Recall that family i's payoffs are given by

$$U_{it} = c_{it} + \mathbb{E}_t \left[\sum_{\tau=1}^{\infty} \delta^\tau c_{i,t+\tau} \right].$$

In the appendix we show that under the optimal investment strategy, the expectation term above is linear in $\mathbb{E}_t[y_{i,t+1}]$ and $\mathbb{E}_t[\phi_{i,t+1}]$. From (34) and (34) we see that these expectations are linear in $\ln z_{it}$. This, along with (33), tells us that family i's investment problem boils down to a simple problem of the form:

$$\max_{z_{it}\in[0,1]} \left\{\ln(1-z_{it}) + \zeta_1 \cdot \ln z_{it} + \zeta_2 \cdot \ln z_{it}\right\}.$$

This expression allows a clear view of the relevant forces at play. The first term is the cost of investment whereas the second and third term are the two sources of benefit. Specifically, ζ_1 captures the standard motivation to invest based on raising the earning capacity of offspring, whereas ζ_2 captures the novel motivation to invest based on raising offspring status.

Proposition 5 *All families optimally invest the same fraction of their income:*

$$z_{it}^* = z^* = \frac{\zeta_1 + \zeta_2}{1 + \zeta_1 + \zeta_2},$$

where

$$\zeta_1 \equiv \frac{\delta\beta_1\alpha_2}{1-\delta[\beta_1\alpha_2+\beta_2]}$$
$$\zeta_2 \equiv \zeta_1 \cdot \frac{\delta b}{1-\delta b} \cdot \frac{\gamma}{\sigma_\xi^2}.$$

The term ζ_1 captures the sort of incentives analyzed in standard models such as Becker and Tomes [1986, 1979]. The new force that we identify here is the ζ_2 term, and in particular the final component, γ/σ_ξ^2. This is the weight that the market places on an agent's performance (as captured by the signal) relative to family background (as captured by prior beliefs informed only by parental status). Since higher parental investment is associated theoretically and empirically with economic development, the analysis suggests a new mechanism through which economic development is hindered in societies where family background plays a central concern in the marriage market. Intuitively, in such cases it is difficult to shift the market's beliefs about offspring ability when the market places little weight on offspring performance relative to the prior. Recalling that γ is a function of σ_ξ^2, one can show that the ratio γ/σ_ξ^2 is increasing in σ_ξ^2. Thus an increase in the return to ability α_1 or a decrease in the importance of luck σ_ε^2 will reduce the weight the market places on individual performance in the long run, and will thus dampen incentives to invest. Also of note is the fact that the policy parameters continue to have no impact on the 'status-based' incentive to invest (although they will of course have an effect on the standard 'income-based' incentive to invest–greater redistributive taxation lowers the incentive to invest, etc.).

8 Extensions

8.1 Imperfect Sorting on Human Capital

By allowing perfect sorting on human capital and parental income, we are able to focus on the equilibrium sorting of ability. In this section we consider a version in which there is imperfect sorting on human capital. We show that the main insights are not exclusive to the perfect sorting setting. Further, for empirical purposes, it may also be helpful to evaluate how the parameters map into spousal human capital correlations. In particular, it will allow us to examine the nature of the relationship between (observable) spousal human capital correlations and (unobservable) spousal ability correlations.

We now consider an extension in which the human capital of agents in the marriage market is not perfectly observed (even by the agent themselves). Instead, everyone observes a noisy signal of human capital, generated according to:

$$\hat{x}_{it} \equiv x_{it} + \nu_{it} \tag{36}$$

where $\nu_{it} \sim N(0, \sigma_\nu^2)$. This signal of human capital is then used to update prior beliefs, $\bar{\psi}_{it}$, to form *interim* beliefs, $\hat{\psi}_{it}$. Marriage forms on the basis of interim beliefs. After the formation of marriages, human capital x_{it} is observed and posterior beliefs, ψ_{it} are formed. These posterior beliefs then form the basis of the prior beliefs inherited by the next generation.

Since

$$\hat{x}_{it} = \alpha_0 + \alpha_1 \theta_{it} + \alpha_2 \cdot y_{i,t-1} + \varepsilon_{it} + \nu_{it}, \tag{37}$$

it follows that the relevant signal is:

$$\hat{s}_{it} \equiv \frac{\hat{x}_{it} - \alpha_0 - \alpha_2 \cdot y_{i,t-1}}{\alpha_1} = \theta_{it} + \frac{\varepsilon_{it} + \nu_{it}}{\alpha_1}. \tag{38}$$

The error component of this signal is

$$\hat{\xi}_{it} \equiv \frac{\varepsilon_{it} + \nu_{it}}{\alpha_1} \tag{39}$$

which has a variance of

$$\sigma_{\hat{\xi}}^2 = \frac{\sigma_\varepsilon^2 + \sigma_\nu^2}{\alpha_1^2}. \tag{40}$$

Essentially the same updating procedure as in the base model applies. Indeed, since human capital x_{it} is observed after marriages are formed, the market updates beliefs on the basis of x_{it} since the signal $\hat{x}_{it}$ does not provide any additional information about ability over-and-above that provided by x_{it}. As such, belief updating occurs as in the baseline model without observation noise. That is, the variance of posterior beliefs converges to γ in the steady state. However, it is the variance of *interim* beliefs that matters for the strength of sorting on human capital. Similar arguments to the base case apply so that the steady state variance of interim beliefs is:

$$\hat{\gamma} \equiv \frac{\sigma^2_{\hat{\xi}}[\frac{b^2}{2} \cdot \gamma + \sigma^2_{\upsilon}]}{\sigma^2_{\hat{\xi}} + \frac{b^2}{2} \cdot \gamma + \sigma^2_{\upsilon}}. \tag{41}$$

Since $\sigma^2_{\hat{\xi}} > \sigma^2_{\xi}$ whenever there is income noise ($\sigma^2_{\nu} > 0$), it naturally follows that $\hat{\gamma} > \gamma$ in such cases. The expressions for spousal ability correlation, ability variance and intergenerational persistence of ability are the same as those derived before with γ replaced with $\hat{\gamma}$. Thus the new element introduced by imperfectly observed income is captured by $\sigma^2_{\hat{\xi}}$. Thus nothing of substance changes in terms of ability sorting–it just becomes noisier, although the institutional/policy parameters still exert no influence on the strength of ability sorting or persistence.

8.2 Imperfect Sorting on Parental Investment

In order for policy variables to matter (i.e. to affect the sorting on genes or the intergenerational correlation of ability), the parental contribution must not be observed perfectly. The most transparent way to model this (since it preserves symmetric imperfect information) is to suppose that parental contributions are a stochastic function of parental investment. That is, allocating a proportion z of income to human capital investment translates into an effective (log) contribution of

$$h_{it} = \ln z + y_{it} + \varepsilon^h_{it} \tag{42}$$

where $\varepsilon^h_{it} \sim N(0, \sigma^2_{\varepsilon^h})$. Thus human capital of offspring is therefore:

$$x_{it} = \alpha_0 + \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{i,t-1} + \alpha_2 \cdot \varepsilon^h_{it} + \varepsilon_{it}, \tag{43}$$

where, as before, $\alpha_0 \equiv \alpha'_0 + \alpha_2 \cdot z$. By defining $\varepsilon^x_{it} \equiv \alpha_2 \cdot \varepsilon^h_{it} + \varepsilon_{it}$ as 'aggregate' luck, we can express human capital in a generalized form:

$$x_{it} = \alpha_0 + \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{i,t-1} + \varepsilon^x_{it}. \tag{44}$$

This generalization is useful because our results need only be adjusted by replacing ε_{it} with ε^x_{it}. In particular, when considering $\sigma^2_{\varepsilon^h} > 0$, the variance of human capital luck generalizes beyond σ^2_{ε} to $\sigma^2_{\varepsilon^x} \equiv \sigma^2_{\varepsilon} + \alpha^2_2 \cdot \sigma^2_{\varepsilon^h}$.

This generalization has four important implications. First, we see that now a policy parameter, α_2, will matter for ability sorting. Recall that α_2 captures the strength with which parental investment translates into offspring human capital, and reflects technology but also policy aimed at redistributing parental inputs (equality of opportunity arises when $\alpha_2 = 0$). Changes in α_2 will matter for ability sorting because such changes will impact the steady state precision of beliefs.

Second, the impact of this policy parameter is counterintuitive. A greater equality of opportunity (a lower α_2) reduces signal noise, thereby making beliefs more precise in the steady state. This facilitates *stronger* ability sorting and persistence. Intuitively, some component of the signal noise is due to the randomness associated with the return on parental inputs–when parental inputs are less important (due to a greater equality of opportunity) this component of the noise diminishes and underlying ability is better revealed.

Third, the impact of a greater equality of opportunity (lower α_2) on social mobility becomes non-monotonic in general. A greater equality of opportunity will raise social mobility *holding 'ability mobility' fixed*, but will reduce ability mobility. Note too that while an decrease in α_2 will raise the persistence of ability, it will *lower* the persistence of income. Thus, movements toward greater social mobility in incomes may coincide with lower social 'ability' mobility.

Fourth, changes in a society's ability mobility will not be reliably reflected in changes in income mobility. Greater income mobility will in fact coincide with *less* ability mobility, if driven by greater equality of opportunity. On the other hand, greater income mobility will coincide with *greater* ability mobility, if driven by other parameters (e.g. the return to ability, α_1).

9 Conclusions

We developed a new model of marital matching in the presence of incomplete information about the genotype of potential mates. When choosing a mate, singles in our model care about the economic success of their descendants, which is influenced by unobservable genetic factors. Genetic inheritance in our model is based on the textbook Additive Genetic Model (AGM) which implies, inter alia, that both parents matter equally for the inheritance of complex traits such as income and educational attainment, and so the rate of intergenerational transmission depends on the degree of marital assortment by genotype.

The key contribution of our paper was to propose a tractable way of integrating incomplete information about genes into the equilibrium matching. This required some drastic simplifications: we based our analysis on a structural model with log-linear functional forms and Gaussian shocks, and only considered equilibria with perfect segregation on observables. Bayesian updating results in an endogenous ranking of potential spouses by family status, a scalar variable that follows a simple and intuitive law of motion. This implies that social mobility may be greatly reduced by the use of family background to infer the genotype of potential mates.

Our practical results are as follows. First, in our equilibrium, long-run social mobility is only weakly linked to short-run mobility. This means that social mobility estimates based on parent-child correlations of education or income may severely underestimate the impact of distant ancestors on current socio-economic status. Second, to the extent that political upheaval and economic reform do not affect the precision of genetic information derived from family background, our model would lead us to expect the impact of distant ancestors to be robust to such changes. Clark(2014) provides empirical evidence for both of these features.

The third result is that meritocratic reforms, to the extent that they increase the correlation between success and the genetically transmitted component of ability, could therefore enable stronger marital assortment on genes, and have the unintended consequence of increasing stratification. For instance, progressive redistribution of education investments may, by shifting weight away from imperfectly observed private inputs, improve the information flow in the matching market, paradoxically reinforcing the inter-generational rigidity of economic inequality.

These results suggest two interpretations of the robustness of long-run stratification to economic policies and regimes. The first is that such changes do not affect the information flow in the matching market. The second is that the equalizing tendency of reforms are offset by the information effects. We conduct a computational analysis to explore these effects. We found that even with the information structure rigged to produce strong sorting effects, the quantitative effects will be small, so the pattern of long-run stability of ancestor effects remains consistent with the premise of the paper, and with Clark's findings. –sorting matters –grandparents

References

S. Rao Aiyagari, Jeremy Greenwood, and Nezih Guner. On the state of the union. *Journal of Political Economy*, 108(2):213–244, 2000.

Axel Anderson. A dynamic generalization of becker's assortative matching result. *Journal of Economic Theory*, 159(PA):290–310, 2015. URL https://EconPapers.repec.org/RePEc:eee:jetheo:v:159:y:2015:i:pa:p:290-310.

Axel Anderson and Lones Smith. Dynamic matching and evolving reputations. *The Review of Economic Studies*, 77(1):3–29, 2010. doi: 10.1111/j.1467-937X.2009.00567.x. URL +http://dx.doi.org/10.1111/j.1467-937X.2009.00567.x.

Daniel J Barth, Nicholas W Papageorge, and Kevin Thom. Genetic ability, wealth, and financial decision-making. Working paper, Johns Hopkins University, 2017.

Gary Becker and Nigel Tomes. Human capital and rise and fall of families. *Journal of Labor Economics*, 4:S1–S39, 1986.

Gary S. Becker and Nigel Tomes. An equilibrium theory of the distribution of income and intergenerational mobility. *Journal of Political Economy*, 87(6):1153–1189, 1979.

Jere Behrman and Paul Taubman. Intergenerational earnings mobility in the united states: Some estimates and a test of becker's intergenerational endowments model. *The Review of Economics and Statistics*, 67(1):144–51, 1985. URL https://EconPapers.repec.org/RePEc:tpr:restat:v:67:y:1985:i:1:p:144-51.

Roland Benabou. Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70 (2):481–517, 2002. ISSN 1468-0262. doi: 10.1111/1468-0262.00293. URL http://dx.doi.org/10.1111/1468-0262.00293.

Hector Chade and Jan Eeckhout. Stochastic sorting. Technical report, University College London, 2017.

Gregory Clark. *The Son Also Rises: Surnames and the History of Social Mobility*. Princeton University Press, Princeton,N.J., 2014.

Dalton Conley, Thomas Laidley, Daniel W. Belsky, Jason M. Fletcher, Jason D. Boardman, , and Benjamin W. Domingue. Assortative mating and differential fertility by phenotype and genotype across the 20th century. *PNAS*, 113(24): 6647–6652, May 2016.

Miles Corak. Income inequality, equality of opportunity, and intergenerational mobility. *Journal of Economic Perspectives*, 27(3):79–102, 2013. URL https://EconPapers.repec.org/RePEc:aea:jecper:v:27:y:2013:i:3:p:79-102.

Marcus W. Feldman, Sarah P. Otto, and Freddy Bugge Christiansen. Genes, culture and inequality. In Kenneth Arrow, Samuel Bowles, and Steven Durlauf, editors, *Meritocracy and Economic Inequality*, pages 61–85. Princeton University Press, 2000.

Michael Kremer. How much does sorting increase inequality? *Quarterly Journal of Economics*, 113(4):1137–1167, 1998.

Ilse Lindenlaub. Sorting multidimensional types: Theory and application. Working paper, Yale University, 2014.

Aysu Okbay, Jonathan P. Beauchamp, and Mark Alan Fontana. Genome-wide association study identifies 74 loci associated with educational attainment. *Nature Letters*, 533(7604):539–542, 2016.

Kay Phillip, David W. Fulke, Gregory Caeyr, and Craig T. Nagoshi. Direct marital assortment for cognitive and personality variables. *Behavior Genetics*, 1987.

Tinca J C Polderman, Beben Benyamin, and Christiaan A de Leeuw. Meta-analysis of the heritability of human traits based on fifty years of twin studies. *Nature Genetics*, 47:702–709, 2015.

Matthew R. Robinson, Aaron Kleinman, Mariaelisa Graff, and Anna A. E. Vinkhuyzen. Genetic evidence of assortative mating in humans. *Nature Human Behaviour*, 1:16, January 2017.

Bruce Sacerdote. How large are the effects from changes in family environment? a study of korean american adoptees. *The Quarterly Journal of Economics*, 122 (1):119–157, 2007. URL https://EconPapers.repec.org/RePEc:oup:qjecon:v:122:y:2007:i:1:p:119-157.

Paul J. Zak and Kwang Woo Park. Population genetics and economic growth. *Journal of Bioeconomics*, 4(1):1–38, Jan 2002. ISSN 1573-6989. doi: 10.1023/A:1020604724888. URL https://doi.org/10.1023/A:1020604724888.

A Structural Model

In this section we develop the structural model that underlies the reduced-form equations describing human capital and income (i.e. equations (2), (3) and (4)).

A.1 Productivity

Let X_{it} denote the productivity (human capital) of a single agent of family i of generation t. We assume that X_{it} is determined by a function of ability G_{it} , the effective investment $H_{i,t-1}$, and i.i.d. luck ε_{it}:

$$X_{it} = A_2 G_{it}^{\alpha} \cdot H_{i,t-1}^{\chi_2} \cdot \exp(\varepsilon_{it})$$

, where A_2, α and χ_2 are parameters with positive values, and $\varepsilon_{it} \sim N(0, \sigma_\epsilon)$. We assume that ability is determined by the genotype according to the additive genetic model. Let the polygenic score be denoted θ_{it} ; then ability is given by:

$$G_{it} \equiv A_1 \cdot \exp(\theta_{it})^{\chi_1}$$

where A_1 and χ_1 are parameters with positive values.

The effective input is given in turn by a stochastic function of the private parental investment $Z_{i,t-1}$ and a public investment $\hat{P}_{t-1}$. The effective input is :

$$H_{i,t-1} \equiv \left[Z_{i,t-1} \exp \epsilon_{it}^{z}\right]^{1-\sigma} \left[\hat{P}_{t-1} \exp \epsilon_{it}^{p}\right]^{\sigma}$$

, where $\left(\sigma, \hat{P}_{t-1}\right)$ are positive parameters representing a redistribution scheme that is set exogenously by policy-makers. With this functional form, similar to Benabou [2002], the degree of progressivity of redistribution is determined by σ.

Let the standard deviations of the stochastic components of the private and public investment be denoted by σ_{ϵ}^{z} and σ_{ϵ}^{p} , respectively. The key assumption is that while $X_{it}, \hat{P}_{t-1}$ and $Z_{i,t-1}$ are publicly observed, the idiosyncratic shocks $\{\varepsilon_{it}, \varepsilon_{it}^{z}, \varepsilon_{it}^{p}\}$ are not. [8] . In the baseline version of the model, the investment shocks are not present: the standard deviations are set to zero. In this case the ability to infer genotype from X_{it} is limited by σ_{ϵ}.

We now derive the reduced-form parameters. Let $y_{i,t-1}$ be the log of household after-tax income, and suppose that the parental investment equals a fraction of z of household income:

$$Z_{i,t-1} = zY_{i,t-1}$$

[9]. In log form, the effective investment is:

$$\log H_{i,t-1} = \log A_2 + (1-\sigma)\left[\log z + y_{i,t-1} + \varepsilon_{it}^{z}\right] + \sigma\left[\log \hat{P}_{t-1} + \varepsilon_{it}^{p}\right].$$

Thus the log of human capital equals

$$x_{it} = \log A_2 + \alpha\left[\log A_1 + \chi_1\theta_{it}\right] + \chi_2\left[(1-\sigma)\left[\log z + y_{i,t-1} + \varepsilon_{it}^{z}\right] + \sigma\left[\log \hat{P}_{t-1} + \varepsilon_{it}^{p}\right]\right] + \epsilon_{it}$$

which we can write as

$$x_{it} = \alpha_0 + \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{i,t-1} + \alpha_2 \cdot \ln z_{i,t-1} + \hat{\varepsilon}_{it}$$

where the error term is given by:

$$\hat{\varepsilon}_{it} \equiv \alpha_2 (1-\sigma)\,\varepsilon_{it}^{z} + \chi_2\sigma\epsilon_{it}^{p} + \epsilon_{it}$$

[8] This is functionally equivalent to the family observing the shock realizations only after the input decisions and the realizations not being publicly verifiable.

[9] In this paper we restrict attention to cases where the optimal investment rule implies that each parent invests the same fraction z of income.

and as the coefficients by:

$$\alpha_0 \equiv \log A_2 + \alpha \log A_1 + \chi_2 \left[(1-\sigma) \log z\right] + \chi_2 \sigma \log \hat{P}_{t-1}$$

$$\alpha_1 \equiv \alpha \chi_1$$
$$\alpha_2 \equiv \chi_2 (1-\sigma)$$

. Thus to accommodate noisy investment in the reduced-form model is simple: we replace σ_ϵ by the standard deviation of the sum of the shocks $\hat{\sigma}_\epsilon \equiv \alpha_2 (1-\sigma) \sigma_\epsilon^z + \chi_2 \sigma_\epsilon^p + \sigma_\epsilon$.

We suppose that public investment (education spending) is financed by redistributing private expenditure. The balanced-budget condition is that total investment spending is unaffected by redistribution:

$$\int Z_{i,t-1}^{1-\sigma} \hat{P}_{t-1} di = \int Z_{i,t-1} di$$

, which implies

$$\hat{P}_{t-1} = \left[\frac{\int Z_{i,t-1} di}{\int Z_{i,t-1}^{1-\sigma} di}\right]^{1/\sigma}$$

.

A.2 Pre-Tax Income

Output in the model is given by a symmetric function of the human capital of both spouses:

$$Q_{it} \equiv X_{it}^{1/2} \cdot X_{i't}^{1/2}$$

. Household pre-tax income depends on household output and on a symmetric function of the income of the householders' parents:

$$Y_{it}^p = Q_{it}^\mu \cdot \bar{Y}_{it}^{1-\mu} \hat{Y}_t \tag{45}$$

where

$$\bar{Y}_{it} \equiv Y_{i,t-1}^{1/2} \cdot Y_{i',t-1}^{1/2}, \tag{46}$$

equals the geometric mean of post-tax household income of the parents.

Thus we can write pre-tax income Y_{it}^p in logs as :

$$y_{it}^p = \ln \hat{Y}_t + \mu \cdot [x_{i,t} + x_{i',t}]/2 + (1-\mu) \cdot [y_{i,t-1} + y_{i',t-1}]/2 \tag{47}$$

. To ensure that the resource constraint is satisfied requires that total output equal total pre-tax income:

$$\int Q_{it}di = \int Y^p_{it}di = \int Q^{\mu}_{it} \cdot \bar{Y}^{1-\mu}_{it}\hat{Y}_t di$$

, and hence the parental-influence parameter must satisfy:

$$\hat{Y}_t = \frac{\int Q_{it}di}{\int Q^{\mu}_{it} \cdot \bar{Y}^{1-\mu}_{it} di}$$

. In contrast to the role of parental income in determining human-capital investments, the role of parental income here is unproductive; it may represent parental influence in securing rents for their children. When $\mu = 1$, then income depends on that of the parents only via the effect of parental investment on productivity as described in the previous section.

A.3 Post-Tax Income

Post-tax income is determined by a balanced-budget redistribution scheme $\left(\tau, \hat{T}_t\right)$:

$$Y_{it} = [Y^p_{it}]^{1-\tau} \left[\hat{T}_t\right]^{\tau}$$

Thus we can write post-tax income in logs as

$$\begin{aligned} y_{it} &= (1-\tau)\, y^p_{it} + \tau \ln \hat{T}_t \\ &= (1-\tau) \left[\ln \hat{Y}_t + \mu \cdot [x_{i,t} + x_{i',t}]/2 + (1-\mu) \cdot [y_{i,t-1} + y_{i',t-1}]/2\right] + \tau \ln \hat{T}_t \end{aligned}$$

which, in the reduced form model, becomes:

$$y_{it} = \beta_{0,t} + \beta_1 \cdot [x_{i,t} + x_{i',t}]/2 + \beta_2 \cdot [y_{i,t-1} + y_{i',t-1}]/2, \tag{48}$$

$$\beta_{0,t} \equiv (1-\tau) \ln \hat{Y}_t + \tau \ln \hat{T}_t \tag{49}$$

$$\beta_1 \equiv (1-\tau)\, \mu \tag{50}$$

$$\beta_2 \equiv (1-\tau)\,(1-\mu)\,. \tag{51}$$

. Budget balance requires that total post-tax income equal total pre-tax income:

$$\int Y^p_{it}di = \int Y_{it}di = \int [Y^p_{it}]^{1-\tau} \left[\hat{T}_t\right]^{\tau} di$$

, which implies that:

$$\int Q_{it}di = \left[\hat{T}_t\right]^{\tau} \int \left[Q_{it}^{\mu} \cdot \bar{Y}_{it}^{1-\mu}\hat{Y}_t\right]^{1-\tau} di$$
$$= \left[\hat{T}_t\right]^{\tau} \left[\hat{Y}_t\right]^{1-\tau} \int \left[Q_{it}^{\mu} \cdot \bar{Y}_{it}^{1-\mu}\right]^{1-\tau} di$$

. The parameter value that balances the budget is given by

$$\hat{T}_t = \left[\frac{\int Q_{it}di}{\left[\hat{Y}_t\right]^{1-\tau} \int \left[Q_{it}^{\mu} \cdot \bar{Y}_{it}^{1-\mu}\right]^{1-\tau} di}\right]^{1/\tau} \tag{52}$$

.

B Segregation Equilibrium

For an agent in the marriage market, let their expected genotype be denoted $\phi_{it} \equiv \int \theta d\Psi_{it}(\theta)$. Then the matching equilibrium (segregation) conditions imply:

$$\phi_{it} = \phi_{i't}$$
$$y_{i,t-1} = y_{i',t-1}$$
$$x_{i,t} = x_{i',t}$$

That is

$$y_{i,t} = \beta_0 + \beta_1 \cdot x_{i,t} + \beta_2 \cdot y_{i,t-1}$$
$$= \beta_0 + \beta_1 \cdot [\alpha_0 + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot y_{i,t-1} + \varepsilon_{i,t}] + \beta_2 \cdot y_{i,t-1}$$
$$= \pi_0 + \pi_1 \cdot \theta_{i,t} + \pi_2 \cdot y_{i,t-1} + \varepsilon_{i,t}^{\varepsilon}$$

where

$$\pi_0 \equiv \beta_0 + \beta_1\alpha_0 \tag{53}$$
$$\pi_1 \equiv \beta_1\alpha_1 \tag{54}$$
$$\pi_2 \equiv \beta_1\alpha_2 + \beta_2 \tag{55}$$
$$\varepsilon_{it}^{\pi} \equiv \beta_1 \cdot \varepsilon_{it}. \tag{56}$$

Furthermore the ability transmission equation (along with segregation) gives

$$\mathbb{E}_t[\theta_{i,t+1}] = b \cdot \phi_{it}.$$

Thus we have

$$\begin{aligned}\mathbb{E}_t[y_{i,t+1}] &= \pi_0 + \pi_1 \cdot \mathbb{E}_t[\theta_{i,t+1}] + \pi_2 \cdot y_{it} \\ &= \pi_0 + \pi_1 \cdot b \cdot \phi_{it} + \pi_2 \cdot y_{it}\end{aligned}$$

We now can derive the expected type and income $\tau \geq 1$ periods into the future as a function of current values gives:

$$\begin{aligned}\mathbb{E}_t[\theta_{i,t+\tau}] &= b^\tau \cdot \phi_{it} \\ \mathbb{E}_t[y_{i,t+\tau}] &= \pi_0 + \pi_1 \cdot \mathbb{E}_t[\theta_{i,t+\tau}] + \pi_2 \cdot \mathbb{E}_t[y_{i\tau}] \\ &= \pi_0 + \pi_1 \cdot b^\tau \cdot \phi_{it} + \pi_2 \cdot \mathbb{E}_t[y_{i\tau}] \\ &= \left[\sum_{s=1}^{\tau} \pi_0 \cdot \pi_2^{\tau-s}\right] + \pi_2^\tau \cdot y_{it} + \left[\pi_1 \cdot \sum_{s=1}^{\tau} b^s \cdot \pi_2^{\tau-s}\right] \cdot \phi_{it},\end{aligned}$$

which again is linear in y_{it} and ϕ_{it}. In summary, the linear setting that we have imposed means that, when evaluating the attractiveness of a potential partner, the only relevant feature of the distribution describing beliefs about their genotype is its expectation.[10] In any case, to see that segregation is a matching equilibrium, note that the expected present value of future dynastic income is given by

$$V(y_{it}, \phi_{it}) \equiv y_{i,t} + \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \delta^\tau y_{i,t+\tau}\right] \tag{57}$$

$$= \varphi_0 + \varphi_1 \cdot y_{it} + \varphi_2 \cdot \phi_{it}, \tag{58}$$

where

$$\begin{aligned}\varphi_0 &\equiv \sum_{\tau=1}^{\infty} \delta^\tau \left[\sum_{s=1}^{\tau} \pi_0 \cdot \pi_2^{\tau-s}\right] = \frac{\pi_0 \delta^2}{(1-\delta\pi_2)(1-\delta)} \\ \varphi_1 &\equiv 1 + \sum_{\tau=1}^{\infty} \delta^\tau \cdot \pi_2^\tau = \frac{1}{1-\delta\pi_2} \\ \varphi_2 &\equiv \pi_1 \cdot \sum_{\tau=1}^{\infty} \delta^\tau \cdot \left[\sum_{s=1}^{\tau} b^s \cdot \pi_2^{\tau-s}\right] \\ &= \pi_1 \cdot \sum_{\tau=1}^{\infty} (\delta\pi_2)^\tau \cdot \left[\sum_{s=1}^{\tau} \left(\frac{b}{\pi_2}\right)^s\right] = \frac{\delta b \cdot \pi_1}{(1-\delta b)(1-\delta\pi_2)}\end{aligned}$$

We have shown that agents' payoffs (U_{it}) differ from V only by a constant, so agents evaluate the attractiveness of partners according to V. That is, when the male from i and the female from i' consider matching, they anticipate a value (up

[10] Of course, the full distribution matters when it comes to computing statistics such as the dispersion, sorting and persistence of genotypes.

to a constant) of:

$$y_{it} + \delta \cdot \mathbb{E}_t \left[V(y_{i,t+1}, \phi_{i,t+1}) \right]$$

which (ignoring constants) is:

$$(1 + \delta\varphi_1) \cdot y_{it} + \delta\varphi_2 \cdot \mathbb{E}_t \left[\phi_{i,t+1} \right]$$

which (ignoring constants) is:

$$(1 + \delta\varphi_1) \cdot [\beta_1 \cdot [x_{it} + x_{i't}]/2 + \beta_2 \cdot [y_{i,t-1} + y_{i',t-1}]/2] + \delta\varphi_2 \cdot b \cdot [\phi_{it} + \phi_{i't}]/2.$$

In other words, the 'quality' of each agent in the marriage market is captured by a simple index aggregating their observed traits:

$$q_{it} \equiv [(1 + \delta\varphi_1)\beta_1/2] \cdot x_{it} + [(1 + \delta\varphi_1)\beta_2/2] \cdot y_{i,t-1} + [\delta\varphi_2 b/2] \cdot \phi_{it}.$$

Stability and market clearing in the period t marriage market requires segregation on q_{it}, which is indeed achieved by segregation on human capital, parental income and beliefs. That is, if an agent of i were to strictly prefer to marry an agent from j to their assigned partner under segregation, i' (i.e. a family i' such that $q_{i't} = q_{it}$), then it must be that $q_{i't} < q_{jt}$. But then this implies $q_{it} < q_{jt} = q_{j't}$, so that j would strictly prefer to *not* match with i over their assigned partner under segregation j'. Thus, segregation constitutes a matching equilibrium.

C Optimal Investment, Infinite Horizon

Now suppose that each household invests in children optimally–i.e. endogenize z. Under the conjecture that agents optimally choose the same z, denoted z^*, we have

$$\begin{aligned} U_{it} &= c_{it} + \mathbb{E}_t[\sum_{\tau=1}^{\infty} \delta^\tau c_{i,t+\tau}] \\ &= \ln(1 - z_{it}) + y_{it} + \mathbb{E}_t[\sum_{\tau=1}^{\infty} \delta^\tau y_{i,t+\tau}] + \sum_{\tau=1}^{\infty} \delta^\tau (1 - z^*). \end{aligned}$$

Since the choice of z_{it} will not affect y_{it} or the final term (a constant), a household's objective can we written as

$$\begin{aligned} &\ln(1 - z_{it}) + \delta \cdot \mathbb{E}_t[V(y_{i,t+1}, \phi_{i,t+1})] \\ =&\ln(1 - z_{it}) + \delta \cdot \mathbb{E}_t[\varphi_0 + \varphi_1 \cdot y_{i,t+1} + \varphi_2 \cdot \phi_{i,t+1}]. \end{aligned}$$

Again ignoring constants (φ_0), the household's objective is

$$\ln(1-z_{it}) + \varphi_1\delta \cdot \mathbb{E}_t[y_{i,t+1}] + \varphi_2\delta \cdot \mathbb{E}_t[\phi_{i,t+1}].$$

The two expectations represent the two motives for investment. The first is as attempt to raise the income of descendants by raising the income of offspring. The second is an attempt to raise the income of descendants by raising the expected type of grandchildren by attracting a higher-type spouse for offspring. These two terms can be determined as follows:

$$\begin{aligned}
\mathbb{E}_t[y_{i,t+1}] &= \pi_0 + \pi_1 \cdot y_{i,t} + \pi_2 \cdot \mathbb{E}_t[\theta_{i,t+1}] + \beta_1\alpha_2 \cdot \ln z_{it} \\
\mathbb{E}_t[\phi_{i,t+1}] &= \lambda b \cdot \phi_{it} + (1-\lambda) \cdot \left[\theta_{i,t+1} + \xi_{i,t+1} + \frac{\beta_1\alpha_2 \cdot (\ln z_{it} - \ln z^*)}{\alpha_1}\right]
\end{aligned}$$

Once again ignoring constants, this problem boils down to maximizing

$$\begin{aligned}
&\ln(1-z_{it}) + \{\delta \cdot \varphi_1 \cdot \beta_1\alpha_2\} \cdot \ln z_{it} + \left\{\delta \cdot \varphi_2 \cdot (1-\lambda)\frac{\beta_1\alpha_2}{\alpha_1}\right\} \cdot \ln z_{it} \\
=&\ln(1-z_{it}) + \left\{\frac{\delta\beta_1\alpha_2}{1-\delta[\beta_1\alpha_2+\beta_2]}\right\} \cdot \ln z_{it} + \left\{\frac{(\delta\beta_1)^2 b\alpha_2}{(1-\delta b)(1-\delta[\beta_1\alpha_2+\beta_2])} \cdot (1-\lambda)\right\} \cdot \ln z_{it} \\
=&\ln(1-z_{it}) + \left\{\frac{\delta\beta_1\alpha_2}{1-\delta[\beta_1\alpha_2+\beta_2]}\right\} \cdot \ln z_{it} + \left\{\frac{\delta\beta_1\alpha_2}{1-\delta[\beta_1\alpha_2+\beta_2]} \cdot \frac{\beta_1\delta b}{1-\delta b} \cdot (1-\lambda)\right\} \cdot \ln z_{it}
\end{aligned}$$

The maximizer of this (with respect to z_{it}) is easily seen to be that stated in the proposition.

D Deriving Correlations

D.1 Genotype

The relevant equations are reproduced here:

$$\theta_{it} = \frac{b}{2} \cdot \theta_{i,t-1} + \frac{b}{2} \cdot \theta_{i',t-1} + \upsilon_{it} \tag{59}$$

$$\theta_{it} = \phi_{i,t} + \varepsilon_{it}^{\gamma} \tag{60}$$

$$\phi_{i,t} = \phi_{i',t} \tag{61}$$

Notice that in the steady state $\bar{\theta} \equiv E[\theta_{it}] = 0$ and $\bar{\phi} \equiv E[\phi_{i,t}] = 0$. Thus this is a system of mean-zero random variables. Recalling that if r_1 and r_2 are mean-zero random variable then $\text{Cov}(r_1, r_2) = \mathbb{E}[r_1 r_2]$, we can use (59) to get:

$$\text{Cov}(\theta_{it}, r_{it}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, r_{it}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, r_{it}) + \text{Cov}(\upsilon_{i,t}, r_{it}) \tag{62}$$

where r_{it} is any mean-zero random variable. Thus, we have the following system:

$$\mathrm{Cov}(\theta_{it},\theta_{it}) = \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i,t-1},\theta_{it}) + \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i',t-1},\theta_{it}) + \sigma_v^2 \tag{63}$$

$$\mathrm{Cov}(\theta_{it},\theta_{i,t-1}) = \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i,t-1},\theta_{i,t-1}) + \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i',t-1},\theta_{i,t-1}) \tag{64}$$

$$\mathrm{Cov}(\theta_{it},\theta_{i',t-1}) = \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i,t-1},\theta_{i',t-1}) + \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i',t-1},\theta_{i',t-1}) \tag{65}$$

Using the steady state conditions $\mathrm{Cov}(\theta_{it},\theta_{it}) = \mathrm{Cov}(\theta_{i,t-1},\theta_{i,t-1}) = \mathrm{Cov}(\theta_{i',t-1},\theta_{i',t-1}) = \sigma_\theta^2$ and $\mathrm{Cov}(\theta_{i,t-1},\theta_{i',t-1}) = \mathrm{Cov}(\theta_{i,t},\theta_{i',t})$ simplifies this to:

$$\sigma_\theta^2 = \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i,t-1},\theta_{it}) + \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i',t-1},\theta_{it}) + \sigma_v^2 \tag{66}$$

$$\mathrm{Cov}(\theta_{it},\theta_{i,t-1}) = \frac{b}{2}\cdot\sigma_\theta^2 + \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i',t},\theta_{i,t}) \tag{67}$$

$$\mathrm{Cov}(\theta_{it},\theta_{i',t-1}) = \frac{b}{2}\cdot\mathrm{Cov}(\theta_{i,t},\theta_{i',t}) + \frac{b}{2}\cdot\sigma_\theta^2 \tag{68}$$

Eliminating $\mathrm{Cov}(\theta_{it},\theta_{i',t-1})$ gives

$$\left[1-\left(\frac{b}{2}\right)^2\right]\cdot\sigma_\theta^2 = \left(\frac{b}{2}\right)\cdot\mathrm{Cov}(\theta_{i,t-1},\theta_{it}) + \left(\frac{b}{2}\right)^2\cdot\mathrm{Cov}(\theta_{i,t},\theta_{i',t}) + \sigma_v^2 \tag{69}$$

$$\left(\frac{b}{2}\right)\mathrm{Cov}(\theta_{it},\theta_{i,t-1}) = \left(\frac{b}{2}\right)^2\cdot\sigma_\theta^2 + \left(\frac{b}{2}\right)^2\cdot\mathrm{Cov}(\theta_{i',t},\theta_{i,t}). \tag{70}$$

These tell us the genotype variance and the intergenerational genotype covariance as a function of the spousal covariance. Conditional on the the spousal correlation, this system yields:

$$\sigma_\theta^2 = \frac{\sigma_v^2}{1-2\left(\frac{b}{2}\right)^2\left(1+\rho(\theta_{i',t},\theta_{i,t})\right)} \tag{71}$$

$$\tag{72}$$

Now we perform the same exercise using (60):

$$\mathrm{Cov}(\theta_{it},\theta_{i't}) = \mathrm{Cov}(\phi_{it},\theta_{i't}) \tag{73}$$

$$\mathrm{Cov}(\theta_{it},\phi_{i't}) = \mathrm{Cov}(\phi_{it},\phi_{i't}) \tag{74}$$

$$\mathrm{Cov}(\theta_{it},\theta_{it}) = \mathrm{Cov}(\phi_{it},\theta_{it}) + \gamma \tag{75}$$

$$\mathrm{Cov}(\theta_{it},\phi_{it}) = \mathrm{Cov}(\phi_{it},\phi_{it}). \tag{76}$$

Using $\text{Cov}(\theta_{i't}, \phi_{it}) = \text{Cov}(\theta_{it}, \phi_{i't})$ gives

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \text{Cov}(\phi_{it}, \phi_{i't}) \tag{77}$$

$$\text{Cov}(\theta_{it}, \theta_{it}) = \text{Cov}(\phi_{it}, \phi_{it}) + \gamma. \tag{78}$$

The segregation-on-status equation (61) gives $\text{Cov}(\phi_{it}, \phi_{i't}) = \text{Cov}(\phi_{it}, \phi_{it})$. Therefore

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \text{Cov}(\theta_{it}, \theta_{it}) - \gamma. \tag{79}$$

Thus the steady state covariances of interest are given as the solution to (71), (70), and (79). The solution is

$$\sigma_\theta^2 = \frac{\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2}}{1 - b^2} \tag{80}$$

$$\text{Cov}(\theta_{it}, \theta_{i,t-1}) = \frac{b\sigma_\upsilon^2 - \gamma \cdot \frac{b}{2}}{1 - b^2} \tag{81}$$

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \frac{\sigma_\upsilon^2 - \gamma \cdot (1 - \frac{b^2}{2})}{1 - b^2}. \tag{82}$$

It therefore follows that the correlations of interest are:

$$\rho_\theta^{PC} \equiv \frac{\text{Cov}(\theta_{it}, \theta_{i,t-1})}{\sigma_\theta^2} = \frac{b\sigma_\upsilon^2 - \gamma \cdot \frac{b}{2}}{\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2}} \tag{83}$$

$$\rho_\theta^{HW} \equiv \frac{\text{Cov}(\theta_{it}, \theta_{i't})}{\sigma_\theta^2} = \frac{\sigma_\upsilon^2 - \gamma \cdot (1 - \frac{b^2}{2})}{\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2}}. \tag{84}$$

The above analysis also reveals that the genotype correlation between family members $k \in \{2, 3, ..\}$ generations apart is given by:

$$\rho_{\theta,k}^{PC} \equiv \frac{\text{Cov}(\theta_{it}, \theta_{i,t-k})}{\sigma_\theta^2} = b^{k-1} \cdot \rho_\theta^{PC} = b^k \cdot \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}}\right]. \tag{85}$$

Thus the long correlation implied by extrapolating the short correlation understates the true long correlation:

$$\frac{(\rho_\theta^{PC})^k}{\rho_{\theta,k}^{PC}} = \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}}\right]^{k-1} \in (0, 1). \tag{86}$$

The extent of the bias increases in k as this ratio goes to zero as k increases. Furthermore this bias in endogenous in our setting, as the ratio is decreasing in γ.

D.2 Income

Notice that in the steady state $\bar{\theta} \equiv E[\theta] = 0$, $\bar{x} \equiv E[x] = \frac{\alpha_0}{1-\alpha_2}$, and $\bar{y} \equiv E[y] = \frac{\beta_0+\beta_1\bar{x}}{1-\beta_2}$. For what follows, take x and y to be their de-meaned counterparts (to save on notation). In steady state with segregation we have

$$x_{it} = \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{i,t-1} + \varepsilon_{it} \tag{87}$$

$$y_{it} = \beta_1 \cdot [x_{it} + x_{i't}]/2 + \beta_2 \cdot [y_{i,t-1} + y_{i',t-1}]/2 \tag{88}$$

$$x_{it} = x_{i't} \tag{89}$$

$$y_{i,t-1} = y_{i',t-1} \tag{90}$$

Combining these gives

$$y_{it} = \pi_1 \cdot \theta_{it} + \pi_2 \cdot y_{i,t-1} + \beta_1 \cdot \varepsilon_{it}. \tag{91}$$

Using a method identical to that for genotype, we get the following system:

$$\text{Cov}(y_{it}, y_{it}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{it}) + \pi_2 \cdot \text{Cov}(y_{i,t-1}, y_{it}) + \beta_1^2 \cdot \sigma_\varepsilon^2 \tag{92}$$

$$\text{Cov}(y_{it}, \theta_{it}) = \pi_1 \cdot \text{Cov}(\theta_{it}, \theta_{it}) + \pi_2 \cdot \text{Cov}(y_{i,t-1}, \theta_{it}) \tag{93}$$

$$\text{Cov}(y_{it}, y_{i,t-1}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t-1}) + \pi_2 \cdot \text{Cov}(y_{i,t-1}, y_{i,t-1}). \tag{94}$$

From (59), (90), and $\text{Cov}(\theta_{i',t-1}, y_{i',t-1}) = \text{Cov}(\theta_{i,t-1}, y_{i,t-1})$ we have:

$$\text{Cov}(\theta_{it}, y_{i,t-1}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, y_{i,t-1}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, y_{i,t-1}) \tag{95}$$

$$= b \cdot \text{Cov}(\theta_{i,t-1}, y_{i,t-1}). \tag{96}$$

Applying the steady state conditions gives the following system:

$$\text{Cov}(y_{it}, y_{it}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{it}) + \pi_2 \cdot \text{Cov}(y_{i,t-1}, y_{it}) + \beta_1^2 \cdot \sigma_\varepsilon^2 \tag{97}$$

$$\text{Cov}(y_{it}, \theta_{it}) = \pi_1 \cdot \text{Cov}(\theta_{it}, \theta_{it}) + \pi_2 \cdot \text{Cov}(y_{i,t-1}, \theta_{it}) \tag{98}$$

$$\text{Cov}(y_{it}, y_{i,t-1}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t-1}) + \pi_2 \cdot \text{Cov}(y_{i,t}, y_{i,t}) \tag{99}$$

$$\text{Cov}(\theta_{it}, y_{i,t-1}) = b \cdot \text{Cov}(\theta_{i,t}, y_{i,t}). \tag{100}$$

Solving gives us the two covariances of interest:

$$\text{Cov}(y_{it}, y_{it}) \equiv \sigma_y^2 = \frac{\left(\frac{1+b\pi_2}{1-b\pi_2}\right)\pi_1^2 \cdot \sigma_\theta^2 + \beta_1^2\sigma_\varepsilon^2}{1-\pi_2^2} \tag{101}$$

$$\text{Cov}(y_{it}, y_{i,t-1}) = \frac{\left(\frac{b+\pi_2}{1-b\pi_2}\right)\pi_1^2 \cdot \sigma_\theta^2 + \pi_2 \cdot \beta_1^2\sigma_\varepsilon^2}{1-\pi_2^2}, \tag{102}$$

where σ_θ^2 was derived above in the genotype section. The correlation of interest is:

$$\rho_y^{PC} \equiv \frac{\mathrm{Cov}(y_{it}, y_{i,t-1})}{\mathrm{Cov}(y_{it}, y_{it})} = \frac{\left(\frac{b+\pi_2}{1-b\pi_2}\right)\pi_1^2 \cdot \sigma_\theta^2 + \pi_2 \cdot \beta_1^2 \sigma_\varepsilon^2}{\left(\frac{1+b\pi_2}{1-b\pi_2}\right)\pi_1^2 \cdot \sigma_\theta^2 + \beta_1^2 \sigma_\varepsilon^2} \tag{103}$$

$$= \frac{\left(\frac{b+\pi_2}{1-b\pi_2}\right)\alpha_1^2 \cdot \sigma_\theta^2 + \pi_2 \cdot \sigma_\varepsilon^2}{\left(\frac{1+b\pi_2}{1-b\pi_2}\right)\alpha_1^2 \cdot \sigma_\theta^2 + \sigma_\varepsilon^2} \tag{104}$$

Here we see that income would be persistent *even if income did not depend on parental income.* That is,

$$\rho_y^{PC}{}_{|\pi_2=0} = b \cdot \frac{\alpha_1^2 \cdot \sigma_\theta^2}{\alpha_1^2 \cdot \sigma_\theta^2 + \sigma_\varepsilon^2} \tag{105}$$

$$= \frac{b}{1 + \frac{\sigma_\varepsilon^2}{\alpha_1^2 \cdot \sigma_\theta^2}} = \frac{b}{1 + \frac{\sigma_\varepsilon^2 (1-b^2)}{\alpha_1^2 \cdot (\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2})}} \tag{106}$$

This is decreasing in γ, implying that the sorting channel *magnifies* the direct impact of α_1 (positive) and σ_ε^2 (negative). This is true more generally.

We also see that ρ_y^{PC} depends on π_2 (increasing presumably) whereas ρ_θ^{PC} was independent of π_2. Thus, policy that affects the sensitivity of human capital to parental inputs (relative to public inputs, say) or meritocracy in general, will have an effect on income mobility but will have no impact on ability mobility. As such, (i) changes in persistence of observed characteristics need not be informative about changes in the persistence of unobserved characteristics, and (ii) the effect of such policy will be limited by the fact that income will persist even if parental income has no direct effect on income.

D.2.1 Longer-Term Income Mobility

To work out $\mathrm{Cov}(y_{it}, y_{i,t-k})$ for $k = 2, 3, ...$ note:

$$\mathrm{Cov}(y_{it}, y_{i,t-k}) = \pi_1 \cdot \mathrm{Cov}(\theta_{it}, y_{i,t-k}) + \pi_2 \cdot \mathrm{Cov}(y_{i,t-1}, y_{i,t-k}) \tag{107}$$

$$= \pi_1 \cdot \mathrm{Cov}(\theta_{it}, y_{i,t-k}) + \pi_2 \cdot \mathrm{Cov}(y_{i,t}, y_{i,t-(k-1)}) \tag{108}$$

and

$$\mathrm{Cov}(\theta_{it}, y_{i,t-k}) = b \cdot \mathrm{Cov}(\theta_{i,t-1}, y_{i,t-k}) \tag{109}$$

$$= b \cdot \mathrm{Cov}(\theta_{i,t}, y_{i,t-(k-1)}) \tag{110}$$

$$= b^k \cdot \mathrm{Cov}(\theta_{i,t}, y_{i,t}) \tag{111}$$

$$= b^k \cdot \frac{\pi_1}{1 - b\pi_2} \cdot \sigma_\theta^2. \tag{112}$$

Thus, letting $\rho_{y,k}^{PC} \equiv \text{Cov}(y_{it}, y_{i,t-k})/\sigma_y^2$ we have

$$\rho_{y,k}^{PC} = b^k \cdot \left[\frac{\pi_1^2}{1 - b\pi_2} \cdot \frac{\sigma_\theta^2}{\sigma_y^2} \right] + \pi_2 \cdot \rho_{y,k-1}^{PC}. \tag{113}$$

Solving explicitly:

$$\rho_{y,k}^{PC} = \left[\frac{\pi_1^2}{1 - b\pi_2} \cdot \frac{\sigma_\theta^2}{\sigma_y^2} \right] \cdot \left[\sum_{s=2}^{k} b^s \cdot \pi_2^{k-s} \right] + \pi_2^{k-1} \cdot \rho_y^{PC}. \tag{114}$$

This can be used to show Clark's point that long correlations are under-stated by a simple extrapolation of short correlations. Taking $\beta_1 = 1$ and $\beta_2 = \alpha_2 = 0$ gives

$$\rho_{y,k}^{PC} = b^k \cdot \left[\frac{\alpha_1^2 \sigma_\theta^2}{\sigma_y^2} \right] = b^k \cdot \left[\frac{\alpha_1^2 \sigma_\theta^2}{\alpha_1^2 \sigma_\theta^2 + \sigma_\varepsilon^2} \right], \tag{115}$$

so that the ratio of the implied value based on a single-generation correlation to the actual value is

$$\frac{(\rho_y^{PC})^k}{\rho_{y,k}^{PC}} = \left[\frac{\alpha_1^2 \sigma_\theta^2}{\alpha_1^2 \sigma_\theta^2 + \sigma_\varepsilon^2} \right]^k, \tag{116}$$

which goes to zero as k gets larger. Our contribution in this regard is to show that the extent of this bias depends on the (endogenous) strength of ability sorting (via the term σ_θ^2). In particular, a greater precision of beliefs increases the extent to which long correlations are under-stated by an extrapolation of short correlations.