Peer Effects and the Promise of Social Mobility: A Model of Human Capital Investment*

Chris Bidner†

October 23, 2008

(First Version: February 2006)

Abstract

I analyze a model of human capital development in the presence of peer effects. Parents invest in their child, and this investment conveys a positive externality upon the childs peers. Parents also acquire wealth, which i) finances consumption, and ii) determines a childs peer group. I show how the freedom to compete for desirable peers exacerbates the natural underinvestment problem. The analysis thereby produces a general equilibrium framework in which the inefficiencies displayed in a rat-race interact with those stressed in the multi-tasking literature. I consider an extension in which both wealth and parental investment are observed with noise.

Keywords: Peer Effects, Premarital Investment, Matching, Human Capital
JEL Codes: D13, C78, D62, J24, D82

*This work is based on Chapter 3 of my dissertation from the University of British Columbia. Thanks to participants in the 2006 Canadian Economics Association Meetings (Concordia), the 2006 NEUDC Conference (Cornell), and the Development, Institutions, Environment, and Trade lunch at UBC. Thanks are especially due to Ken Jackson, Patrick Francois, and Mike Peters for valuable discussions. All errors are mine.

†School of Economics, University of New South Wales. *email*: `c.bidner@unsw.edu.au`

1 Motivation and Introduction

The determinants of human capital formation are, for various reasons, important to understand. Economists have recognized that such determinants include not only the level of resources that are devoted to the process, but also the social context in which this investment takes place. For example, there is now a large literature on 'peer effects' that analyzes and attempts to quantify the notion that an individual's outcomes are sensitive to the particular individuals that they interact with (see Durlauf (2004) for a comprehensive survey). This paper presents a model of human capital development in which parents allocate resources mindful of the existence of peer effects. A central concern is the efficiency of such resource allocations.

Peer effects are modeled in a simple, direct manner: parents make investments in their child's human capital and this spills over to the child's peers. If peers were fixed, then there is a natural underinvestment problem since no parent takes into account the fact that their investment benefits others. However, peers are not fixed: parents face 'the promise of social mobility' in the sense that they have the capacity to undertake costly actions that place their child among desirable peers. In particular, a family's (endogenous) wealth determines the type of peer group that their child will interact with. This is modeled as a marriage problem with observable wealth, but can be thought of as representing a competitive market process whereby wealth determines which families can afford to live in which neighborhoods (or attend which schools). Finally, the model captures the feature that acquiring wealth places a demand on family resources, thereby raising the cost of parental investments.

Most existing studies that model human capital development in the presence of peer effects are concerned with the efficiency of equilibrium segregation across neighborhoods. Prominent examples include Benabou (1996a), Durlauf (1996), and de Bartolome (1990). Although these studies are concerned with the efficiency of parental location choices, they generally trivialize the parental investment choice.[1] In contrast, the efficiency of such investment choices are the central issue here.

A related literature abstracts from explicit peer effects and instead focuses on 'fiscal spillovers': the effect of neighborhood composition on local public finance decisions,

[1]Although a parents' human capital acts as an input in the production of their childs' human capital, it is not a choice variable in Benabou (1996a) (it is an exogenous type). Human capital is produced with neighborhood-wide educational inputs (per capita) in de Bartolome (1990), and, in a similar vein, Durlauf (1996) assumes that all individuals in a neighborhood receive the same investment.

such as educational expenditures (e.g. Benabou (1996b) and Fernandez and Rogerson (1996)). There are two relevant points to be made here. First, the standard public goods problem (under-provision) is readily overcome when contributions are monetary, since it is relatively simple for local governments to establish and enforce the suitable contracts, e.g. imposing suitable tax rates. Indeed, this is the essence of Tiebout competition. Such mechanisms are not available when the contributions take the form of parental investment, implying that the under-investment problem remains. Second, for any given tax rate, families prefer to be surrounded by wealthier families since they generate greater tax revenue. Families also wish to be surrounded by wealthy families in the model presented here, but for a very different reason: wealth signals that such families have also made high parental investments. This perspective may help reconcile the puzzling coexistence of i) the fact that parents have a concern about which school their child attends, and ii) a general disagreement in the empirical literature as to whether school resources have a significant impact on outcomes.

The model developed here extends the literature on competitive matching with pre-match investments by analyzing multiple pre-match investments. This is an important extension since the literature stresses two distinct roles of pre-match investments. First, an investment has a *surplus-generating role* when it serves to increase an agent's value as a potential partner - i.e. to generate surplus within a match. Second, an investment has a *matching role* when it serves as a means through which more desirable partners can be attracted.[2] There is a class of models have a single investment that plays both roles simultaneously (e.g. Bidner (2008a), Peters (2007b), Peters (2007a), Peters and Siow (2002) and Cole, Mailath, and Postelwaite (2001)).[3] A second class of models employ a single investment in the matching role only (e.g. Bidner (2008b), Hoppe, Moldovanu, and Sela (2005), Damiano and Li (2007), and Rege

[2]The only other paper that I am aware of that incorporates multiple pre-match investments is Han (2005). In that paper, firms choose both a workplace characteristic and a wage payment (workers choose a single productive characteristic). Although the firm makes multiple investments, both of the investments are observed, and therefore both simultaneously play the surplus-generating and matching role. In contrast, the present paper uses multiple investments to distinguish the roles.

[3]These model can be further classified according to the significance of agents' types. In Peters (2007b), Peters (2007a), Peters and Siow (2002) and Cole, Mailath, and Postelwaite (2001), types determine the cost of investment (much like in signaling models) but do not affect an agent's value as a partner, which only depends on their investment (making signaling uninteresting). In Bidner (2008a), types determine the productivity of investment and therefore influence an agent's value as a partner (as in signaling), but do not affect investment costs (making signaling infeasible).

(2007)). Finally, a large and varied class of models uses a single investment to focus on the surplus-generating role only (e.g. non-cooperative models of public good contribution, hold-up, etc., as well as competitive models such as the Kremer (1993) O-Ring Theory). Conceptually, these roles are distinct. As such, interesting insights may be overlooked if we either ignore one of the roles, or try to impose both roles on a single investment. Section 6 below sheds more light on the issue, as it considers an extension of the model in which both investments are observed with noise.

The central message delivered by the model is that competition for peers exacerbates the inherent inefficiency associated with parental investment externalities. There are essentially two components to this. First is the fact that families devote too many resources to acquiring wealth in order to compete for better peers. Modern treatments of this phenomenon explicitly incorporate matching (e.g. Hoppe, Moldovanu, and Sela (2005) and Rege (2007)), but the general 'rat-race' phenomenon has long been recognized (e.g. Akerlof (1976), and Frank (1985)). The model presented here places this phenomenon within a 'general equilibrium' setting because the objects that agents are competing for - the parental investment embodied in peers - is itself endogenous. This leads to the second aspect: competition for peers consumes parental resources, which makes parental investments themselves more costly. This mechanism is reminiscent of the adverse effects of high-powered incentives stressed in the literature on multi-tasking (e.g. Holmstrom and Milgrom (1991)). Again, the model places this mechanism within a 'general equilibrium' context because incentives are only high-powered because of the possibility of interaction with others.

The conclusion that competition for peers is detrimental is in direct contrast to the positive conclusions drawn from models in which a single investment plays both a surplus-generating role and a matching role. In such models, the desire to attract better partners provides an added impetus to invest - as it does in this model - but, the fact that there is only one investment automatically implies that the under-investment problem is, at least in part, resolved.[4] Despite the dramatic difference in the conclusions reached, the models produce many observationally-equivalent outcomes. For instance, positive assortative matching (on wealth, parental investment, and type) is predicted by both models. However, the models are empirically distinguishable, at least in principal, because they differ on the variables that *cause* positive matching. For instance, models with a single investment predict that it is the child's

[4]In Bidner (2008a), the added impetus actually leads to the reverse problem - over-investment.

human capital that allows for a better match, whereas the model here predicts that it is wealth.

The basics of the model are laid out in Section 2. Essentially, families observe their type, acquire wealth and make parental investments, then compete for a desirable partner (peer) in the matching market. The equilibrium concept is defined following a description of the matching market. Various general results, including existence, uniqueness, and some welfare properties, are presented in Section 3. Following this, two illustrations are presented. The first, in Section 4, is quite simple and is designed to demonstrate i) how to calculate equilibria, and ii) some strong welfare dominance properties. The second illustration, in Section 5, is more detailed, and demonstrates how to derive equilibria when simple closed-form solutions are not available. Furthermore, the illustration provides the background for an extension in Section 6. The extension examines a situation in which both wealth and parental investment are observed (with noise) in the matching market. I demonstrate that many of the results are robust in this dimension, and that additional insight is obtained as i) outcomes depend on the distribution of types, and ii) special cases are obtained as the different noise levels are manipulated.

Although the model is motivated by peer/neighborhood effects, I believe the mechanisms highlighted by the model are applicable to a variety of situations that share the essential features - e.g. analysis of labour and marriage markets.

2 Model

2.1 Fundamentals

A family consists of one adult and one child, and is indexed by $i \in [0, 1]$. Each adult is endowed with an ability, θ_i, where θ_i is continuously distributed on $\Theta \equiv [\underline{\theta}, \overline{\theta}]$ according to Ψ, which is assumed to have a positive and bounded density $\psi \in (0, \infty)$ on Θ.

Adults have preferences defined over two outcomes: consumption and the human capital of their child. Consumption is financed by wealth, and wealth is determined by an investment that the adult makes in their productivity. If an adult makes x units of investment in their productivity, then this allows them to consume an amount that produces utility according to $f(x)$, where f is a twice differentiable function with $f_x > 0$, $f_{xx} < 0$, and $\lim_{x \to 0} f_x(x) = \infty$.

The process of earning income and consuming does not involve any form of interaction with other families. In contrast, the development of a child's human capital is, in part, a social phenomenon. In particular, child i not only benefits from parental investments made by adult i, but also benefits from parental investments made by the parents of their peers. To focus ideas, suppose that children socialize in pairs.[5] If adult i makes y units of parental investment and their child socializes with a child whose parents make y' units of parental investment, then the human capital of child i is given by $h(y, y')$. The function h is twice differentiable with $h_y, h_{y'} > 0$ and $h_{yy} \leq 0$. I assume that interaction is weakly complementary in the sense that $h_{yy'} \geq 0$. That is, the marginal product of parental investment in non-decreasing in the level of parental investment made by their partner. This property of h is important in matching problems because it determines the efficient matching pattern (Becker (1973)). Finally, I make the regularity assumption that $h_{yy}(y, y) + h_{yy'}(y, y) \leq 0$. This says that the marginal product of parental investment is non-increasing when evaluated at a point in which both members of the match make the same investment. The assumption ensures that a well-defined social optimum exists.

To fix ideas, it will be convenient to assume that h belongs to the class of generalized CES functions:

$$h(y, y') = \left[(1 - \phi) \cdot q(y)^{\rho} + \phi \cdot q(y')^{\rho}\right]^{\frac{1}{\rho}},$$

where $\rho \in (-\infty, 1]$, $\phi \in [0, 1]$, and q is an increasing, twice continuously differentiable, concave function.[6]

Even when q is chosen to be the identity function, $q(z) = z$, the CES specification is flexible enough to capture linear ($\rho = 1$), Cobb-Douglas ($\rho \to 0$), and Leontief ($\rho \to -\infty$) specifications. The parameter ϕ captures the degree to which a child's human capital is sensitive to the parental investment embodied in their peers.

An adult's ability, θ, determines the costs incured in making both types of investment. In particular, if an adult of ability θ chooses the investment bundle (x, y), then their total investment is $T = x + y$, which has an associated cost of $c(T, \theta)$, where c is a twice continuously differentiable function where i) the marginal cost is positive,

[5]This assumption is made for simplicity. Allowing for any finite number of agents per group is not problematic if we assume that all an agent cares about is the *average* of the investments made by others in the group.

[6]Note that q being C^2 implies that both q' and q'' are continuous functions, the monotonicity of q implies that there is no $\hat{y} < \infty$ such that $\lim_{y \to \hat{y}} q'(y) = 0$. Together, these imply that $-q''(y)/[q'(y)]^2$ is continuous on $(0, \infty)$.

except at zero: $c_T(0,\theta)=0$ and $c_T(T,\theta)>0$ for $T>0$, ii) the marginal cost is strictly increasing: $c_{TT}>0$, and most importantly iii) the marginal cost is decreasing in ability: $c_{T\theta}(\cdot)<0$.

The assumption that investment costs depend on the total investment, and not the composition, can be motivated by interpreting x as expenditures on consumption, and y as expenditures on a child's human capital. These expenditures are financed from wealth, which is acquired from labor income. A parent's labor income depends on their labor supply, as well as their wage. The wage equals their marginal productivity, which is their ability. Adults are endowed with a unit of time that is divided between work and leisure. By allocating t units of time to working, the adult obtains a leisure payoff of $\ell(1-t)$, and can allocate $T=t\theta$ units to the two types of expenditures. Thus, if we assume that $\ell'(1)=0$, we can interpret the cost of investment as the opportunity cost of leisure: $c(T,\theta)=-\ell(1-(T/\theta))$. Despite this interpretation, the functional forms adopted in the illustrations will be chosen for their analytical simplicity.

Finally, in order to analyze the effect of altruism, I assume that the adults' objective function incorporates a weighted sum of consumption and child human capital. In particular, if an adult chooses the investment bundle (x,y) and is matched with a family that chooses an investment bundle $(\cdot,y')$, then the adult's total payoff is:

$$V\left(x,y,y',\theta\right)\equiv(1-\alpha)\cdot f(x)+\alpha\cdot h(y,y')-c(x+y,\theta), \tag{1}$$

where $\alpha\in(0,1]$ parameterizes altruism. To ensure that parents choose a positive amount to invest in their child's human capital - that is, to make the model interesting - I assume that $\lim_{y\to 0}V_y\left(x,y,y',\theta\right)>0$ for all values of (x,y'). This is automatically satisfied when either $\rho<1$ or $\alpha=1$. In all other cases, the assumption is satisfied if $\lim_{y\to 0}q'(y)=\infty$.

2.2 Structure

Each adult clearly has an interest in who their child socializes with. The mechanism through which a child is assigned a peer is intended to capture the feature that parents are somewhat able to influence the quality of their match through the choice of investments. In particular, the model will unfold in two stages as follows:

1. Adults observe their ability, θ, and choose their investment bundle (x,y).

2. Families enter a matching market in which they match with another family.

The matching market is modeled in a somewhat reduced form manner, and is intended to capture the salient features of a frictionless matching game with a large, even, number of agents. The key element is that only agents' wealth, x, is observed in the matching market (this is relaxed in Section 6 below). It is convenient to assume that agents are able to 'hide' any amount of their wealth at some arbitrarily small, but positive, cost. We think of each agent, having observed the wealth of all agents, proposing to - and receiving propositions from - other agents. Any agent that goes unmatched gets a payoff equivalent to being matched with a family that makes zero investment in their child's human capital. Thus, all agents prefer to be matched than to remain unmatched. The matching market is said to be in equilibrium once all further mutually agreeable propositions are exhausted. I impose certain conditions on the matching market that are intended to capture the the essential properties of this equilibrium allocation as the number of agents becomes infinitely large. This is discussed in more detail in the following section.

2.3 Equilibrium

The economy is in equilibrium when i) agents make their investments optimally, given a conjecture about the matching market, and ii) equilibrium in the matching market, given the pattern of investments that are made, does not contradict the agents' initial conjecture. I consider two classes of equilibria; *pooling* and *separating*. Rather than introducing an all-encompassing definition of equilibrium at this point, I offer specialized definitions for each of these classes in their respective sections.

2.3.1 Pooling Equilibrium

In a pooling equilibrium, all families have the same observed wealth, say x^P. Since all families appear identical in the matching market, matching must effectively be random. Furthermore, any unilateral deviation from a wealth of x^P will not change a family's matching prospects since they will still be matched with some family (and all families appear identical). This implies that no family can be hiding wealth in a pooling equilibrium, since hiding wealth entails a small cost.

Given that other families invest according to $(x^P, y^P(\theta))$, each agent faces the following optimization problem:

$$\max_{x,y} \left\{(1-\alpha)\cdot f(x) + \alpha \cdot H(y) - c(x+y,\theta)\right\}, \quad (2)$$

where

$$H(y) \equiv \int h(y, y(z))d\Psi(z).$$

The profile $(x^P, y^P(\theta))$ is a pooling equilibrium if the solution to (2) is $\{x^P, y(\theta)\}$, for all $\theta \in \Theta$. Importantly, the optimal value of x must be x^P for all families. For this reason, pooling equilibria will often fail to exist.

Proposition 1. *A pooling equilibrium does not exist if there is imperfect altruism ($\alpha < 1$).*

In light of this, one must take care when commenting on the trade-off emphasized in the literature: that signaling is wasteful but facilitates efficient matching patterns. In Hoppe, Molodovanu, and Sela (2005), this trade-off is motivated by making a comparison across equilibria - however, once the investment yields a private benefit in addition to any signaling aspect, the equilibrium without investment fails to exist. Of course, one could always motivate the trade-off by comparing equilibrium outcomes to a benchmark in which the 'signal' is also hidden. I analyze a model in which both types of investment are observed with noise in Section 6 below.

One last point to note is that, since the marginal benefit to parental investment is non-decreasing in the investment of others, pooling equilibria will generally not be unique (the parental investments, not wealth, will differ).

2.3.2 Separating Equilibrium

Separating equilibria have the property that the parental investment made by each family is perfectly revealed by their observed wealth level. For this to occur, families must have no incentive to 'hide' part of their wealth from the matching market. This in turn requires that families of different types must optimally choose different wealth levels. I look for equilibria in which wealth is a differentiable and strictly monotone function of type.

The pair of functions, $\{x(\theta), y(\theta)\}$, are *candidate equilibrium investment functions* if $x(\cdot)$ is a differentiable, strictly monotone function. If agents invested according to these candidate functions, then observing a family with a wealth of z in the matching market reveals that the family is of type $x^{-1}(z)$, and therefore has made a parental investment of $\mu(z) \equiv y(x^{-1}(z))$. Since all families prefer to match with those that have higher values of $\mu(z)$, it follows that the only stable matching is positive assortative on $\mu(z)$.

Let X be the set of investments that arise in equilibrium (the image of $x(\theta)$). Agents recognize that if they enter the matching market with a wealth of $z \in X$, then they will be matched with any family that has made a parental investment of $\mu(z)$. On the other hand, if they enter with a wealth of $z \notin X$ then, since they match with some family, they get matched with a family that has made some parental investment in $\{\mu(z') \mid z' \in X\}$. In light of this, we can interpret μ as a *matching market return function*.

Separating behaviour is only consistent with μ being non-decreasing (at least on X). Suppose to the contrary that for some pair of observed equilibrium wealth levels, (x, x'), where $x < x'$, we had $\mu(x) > \mu(x')$. Those with an observed wealth of x' would be better off hiding part of their wealth and displaying a wealth of x to the matching market (since the associated cost is arbitrarily small). This would then contradict the fact that x' is a wealth level observed in equilibrium.

To summarize, we say that the function μ is *consistent* with the candidate investment functions $\{x(\theta), y(\theta)\}$ if i) μ is non-decreasing on X, and ii)

$$\mu(z) \begin{cases} = y(x^{-1}(z)) & \text{if } z \in X \\ \in \{y(x^{-1}(z)) \mid z \in X\} & \text{otherwise.} \end{cases} \tag{3}$$

The first of these conditions can also be expressed as:

$$\mu(x(\theta)) = y(\theta), \text{ for all } \theta \in \Theta. \tag{4}$$

Taking the non-decreasing return function as given, families invest optimally. In particular, given μ, investments are *optimal* if

$$\{x(\theta), y(\theta)\} \in \arg\max_{x,y} \{V(x, y, \mu(x), \theta)\} \tag{5}$$

for all $\theta \in \Theta$.

Putting this all together, a separating equilibrium is defined as follows.

Definition 1. *A separating equilibrium is a pair of candidate investment functions, $\{x(\cdot), y(\cdot)\}$, and a matching market return function, $\mu(\cdot)$, such that:*

1. *$\{x(\cdot), y(\cdot)\}$ are optimal given $\mu(\cdot)$, and*

2. *$\mu(\cdot)$ is consistent with $\{x(\cdot), y(\cdot)\}$.*

One central property of a separating equilibrium is that families are perfectly segregated along the parental investment dimension. This, by itself, does not imply that families are also perfectly segregated along the wealth dimension. That is, if parental investment were non-monotonic in type, then two different types will make the same parental investment (and therefore can be matched together), yet have different wealth levels. This possibility is ruled out by the following.

Result 1. *Parental investment is a weakly increasing, differentiable function of type in a separating equilibrium.*

This comes from the observation i) μ being non-decreasing implies that if wealth is increasing (decreasing) in type then parental investment is weakly increasing (decreasing) in type, and ii) that total investment is increasing in type (see Appendix). Differentiability follows from the assumed differentiability of $x(\cdot)$ and an inspection of the equation that implicitly defines optimal parental investments. Since both $x(\theta)$ and $y(\theta)$ are differentiable in a separating equilibrium, condition (4) implies that so too is μ. This fact is used in deriving the equilibrium return function, but before doing so, it is useful at this point to establish a set of benchmark investment levels since they will also feature in the derivation.

2.3.3 Some Benchmarks

To begin, suppose that each family were *exogenously* matched with a family of the same type. The optimal investments in this setting are called the *Nash* investments (since families take their partner as given). Some insight into the role played by 'the promise of social mobility' can be obtained by comparing the Nash investments to the equilibrium investments. The Nash investments, $x^N(\theta)$ and $y^N(\theta)$, satisfy the following:

$$\left\{x^N(\theta), y^N(\theta)\right\} \in \arg\max_{x,y} \; V(x, y, y^N(\theta); \theta). \tag{6}$$

When investing in this way, families do not take into account that their parental investment benefits their partner. To formalize this, we can define the *Efficient* investments, $x^*(\theta)$ and $y^*(\theta)$, as those that satisfy:

$$\{x^*(\theta), y^*(\theta)\} \in \arg\max_{x,y} \; V(x, y, y; \theta). \tag{7}$$

Given the positive externality, the following result is not particularly surprising.

Result 2. *Nash investments are not efficient. In particular,* $y^N(\theta) < y^*(\theta)$ *and* $x^N(\theta) \geq x^*(\theta)$.

Nash wealth is (weakly) greater than the efficient wealth level since the Nash parental investment is lower, which lowers the cost of making the wealth enhancing investment. The reason that the inequality is weak is that both wealth levels may be zero (when $\alpha = 1$).

Result 3. *Both the Nash- and Efficient Investments are independent of* ρ.

This follows from the observation that for any $z > 0$, we have $h(z, z) = q(z)$, $h_y(z, z) = (1 - \phi) \cdot q'(z)$, and $h_{y'}(z, z) = \phi \cdot q'(z)$ for all values of ρ. Although this is a special feature of the generalized CES form imposed on h, it will be useful below.

2.3.4 Deriving the Equilibrium

The equilibrium is derived in two steps. First, the matching market return function is derived. Once we verify i) that this function is increasing on X, and ii) that appropriate values for off-equilibrium investments can be found, the second step involves using the first-order conditions to derive the optimal parental investment associated with given wealth level. These two curves are plotted in the same space, and their intersection characterizes equilibrium investments.

To begin, consider the problem of deriving the matching market return function. Parents make choose their investment bundle, (x, y), taking μ as given. Since wealth is a strictly increasing function, almost all families will optimally make interior wealth investments. This, together with the observation that μ is differentiable in equilibrium, implies that optimal investments are characterized by the first-order conditions:

$$\begin{aligned}(1-\alpha) \cdot f_x\left(x(\theta)\right) + \alpha \cdot h_{y'}\left(y(\theta), y(\theta)\right) \cdot \mu_x(x(\theta)) \\ = c_T\left(x(\theta) + y(\theta), \theta\right)\end{aligned} \tag{8}$$

$$\alpha \cdot h_y\left(y(\theta), y(\theta)\right) = c_T\left(x(\theta) + y(\theta), \theta\right) \tag{9}$$

Equating the left-hand sides of these, and using (4), we get the following differential equation:

$$\mu_x(x) = \frac{\alpha \cdot h_y\left(\mu, \mu\right) - (1-\alpha) \cdot f_x\left(x\right)}{\alpha \cdot h_{y'}\left(\mu, \mu\right)} \equiv \Gamma(\mu, x) \tag{10}$$

In order to pin down μ, we need an initial condition. To obtain this, we turn our attention to the fact that we need suitable off-equilibrium values for μ. All we need is that for all $z \notin X$, $\mu(z) = y(\theta)$ for some θ. Given that y is non-decreasing in θ we can safely set $\mu(z) = y(\underline{\theta})$ for all $z \notin X$ (if the objective is to ensure that no family has an incentive to deviate to an off-equilibrium investment). Now consider the problem faced by families of the lowest type. In equilibrium they are always matched with a family that invests $y(\underline{\theta})$, and by the above argument, can do no worse by deviating to anything else. Therefore, their equilibrium investments coincide with those that would be made had they taken their partner as fixed. This gives us the initial condition that the lowest types make their Nash investments: $\{\mu_0, x_0\} = \left\{y^N(\underline{\theta}), x^N(\underline{\theta})\right\}$. This, combined with (10), defines an initial values problem.

The solution to the initial values problem represents a candidate equilibrium return function, which we need to verify is strictly increasing on X. To do this, we sketch out the direction field associated with Γ. That is, for any given point in (μ, x) space, we know that the slope of $\mu(x)$ equals $\Gamma(x, \mu)$. The essential features of this process are illustrated in Figure 1. To begin, consider the case in which $\alpha < 1$, and consider the set of points such that $\Gamma = 0$. Such points are described by the implicit function, $N(x)$, which satisfies:

$$(1 - \alpha) \cdot f_x(x) = \alpha \cdot h_y(N(x), N(x)). \tag{11}$$

It is straightforward to verify that $N(x)$ is a strictly increasing function, as depicted. At points to the left of $N(x)$, we have $\Gamma < 0$ and at points to the right of $N(x)$, we have $\Gamma > 0$. Thus, any solution to the initial values problem will be downward-sloping to the left of $N(x)$, flat at $N(x)$, and upward sloping to the right of $N(x)$, as depicted. The particular solution depends on the initial condition, but note that the initial condition lies on $N(x)$ since (11) is a consequence of the first-order conditions associated with the Nash investments. The blue line depicts a solution to the initial values problem, and it is straightforward to see that it must be strictly increasing on $x \geq x_0$. Values of x below x_0 do not arise in equilibrium, so μ need not be governed by Γ in this region. The dashed blue line is one possibility for what μ looks like below x_0: it captures a situation in which agents realize that cutting their investment below x_0 means that they must match with the least desirable family (who makes a parental investment of μ_0). When $\alpha < 1$, note that the initial condition will be strictly interior. Since $\Gamma(x, \mu)$ is continuously differentiable at all $x > 0$ and $\mu > 0$, the fundamental theorem of differential equations can be applied to demonstrate that a solution exists, and is unique. If $\alpha = 1$, then $\Gamma > 0$ on the entire space so that we can be sure any solution is strictly increasing, and the initial condition lies on the μ axis. Existence and uniqueness of the solution to the initial values problem is immediate when $\alpha = 1$, since we can simply integrate to get: $\mu(x) = [(1-\phi)\cdot\phi^{-1}]\cdot x + y^N(\underline{\theta})$.

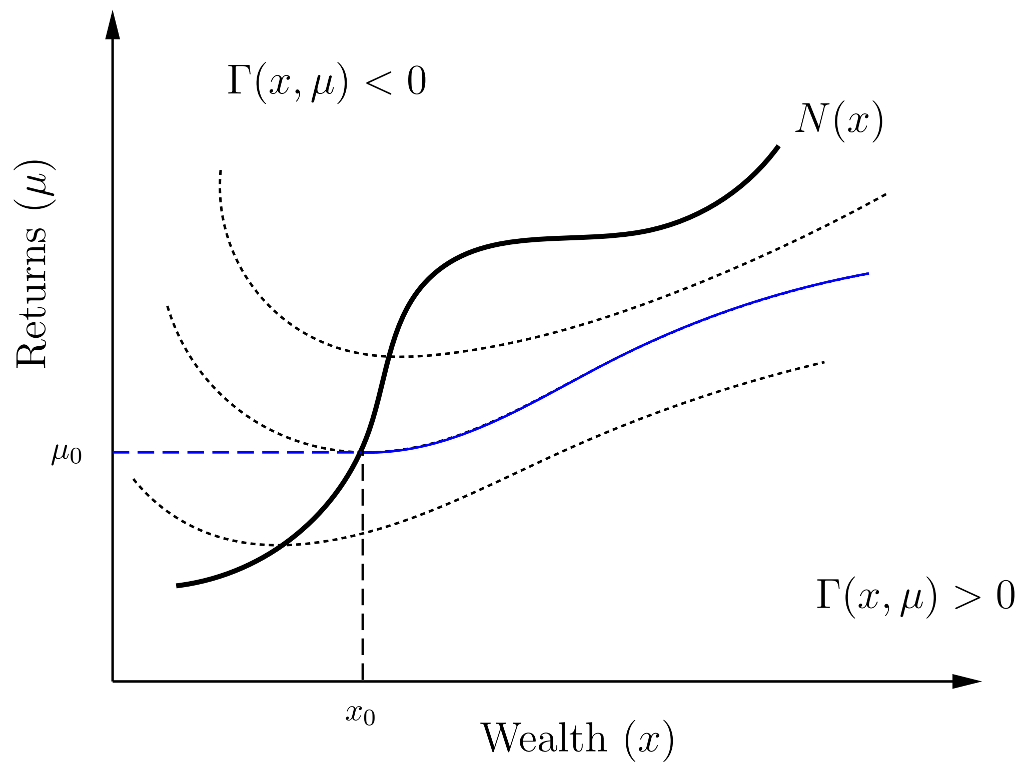

Figure 1: Properties of μ: A Sketch of the Direction Field associated with $\Gamma(x,\mu)$

Result 4. *The initial values problem has a solution in which μ is non-decreasing on X, and this solution is unique.*

Notice that μ is completely independent of i) the distribution of types, and ii) any cost parameters. Furthermore, note that once the generalized CES form is applied, we have:

$$\Gamma(x,\mu) = \frac{1-\phi}{\phi} - \frac{1}{\phi}\cdot\frac{1-\alpha}{\alpha}\cdot\frac{f'(x)}{q'(\mu)}, \tag{12}$$

which is independent of ρ. Since the Nash investments are also independent of ρ, we have the following.

Result 5. *The solution to the initial values problem is independent of ρ.*

The result relies on the CES form, however, it indicates that 'complementarity' (as captured by ρ) is not of first-order significance.[7] What is important is the degree of spillovers, ϕ.

Once we have the return function, the equilibrium investment functions can be derived from another implication of the first-order conditions. In particular, if the equilibrium wealth for a family of type θ is x, then their optimal parental investment is given by $\hat{y}(x, \theta)$, where $\hat{y}(x, \theta)$ satisfies:

$$\alpha \cdot h_y \left(\hat{y}(x, \theta), \hat{y}(x, \theta) \right) = c_T \left(x + \hat{y}(x, \theta), \theta \right).$$

The function $\hat{y}(x, \theta)$ is decreasing in x and increasing in θ. Figure 2 depicts $\hat{y}(x, \theta)$ in (y, x) space for three different values of θ. The equilibrium return function derived above is superimposed on this space also, since consistency of beliefs, condition (4), requires that $\mu(x(\theta)) = y(\theta)$. The equilibrium investments therefore are given by the points of intersection of $\hat{y}(x, \theta)$ and $\mu(x)$, as depicted. The fact that $\hat{y}(x, \theta)$ never starts below the Nash parental investment (and starts strictly above it when $\alpha < 1$), and equals zero for some finite wealth level, implies that the curves cross exactly once.

The final step in establishing the existence and uniqueness of a separating equilibrium is to show that that the first-order necessary conditions are also sufficient. To do this, I show that the objective function is globally concave when evaluated using a candidate return function (see the Appendix). To conclude this section, we therefore have the following.

Proposition 2. *A separating equilibrium exists, and it is (essentially) unique. Moreover, the separating equilibrium is independent of ρ.*

The qualification 'essentially' reflects the fact that one could specify alternative off-equilibrium values for μ that would not disrupt investment behaviour.

3 Analysis

3.1 Efficiency

A theme common to papers that study this kind of environment in full-information settings (e.g. Peters and Siow (2002) and Cole, Mailath, and Postlewaite (2001)) is that

[7] The general condition required of h is as follows. Suppose that h depends on parameters, ξ, so that we can write $h = h(y, y'; \xi)$. For the separating equilibrium to be independent of ξ, we need $h_y(y, y; \xi)$ and $h_{y'}(y, y; \xi)$ to both be independent of ξ for all y.

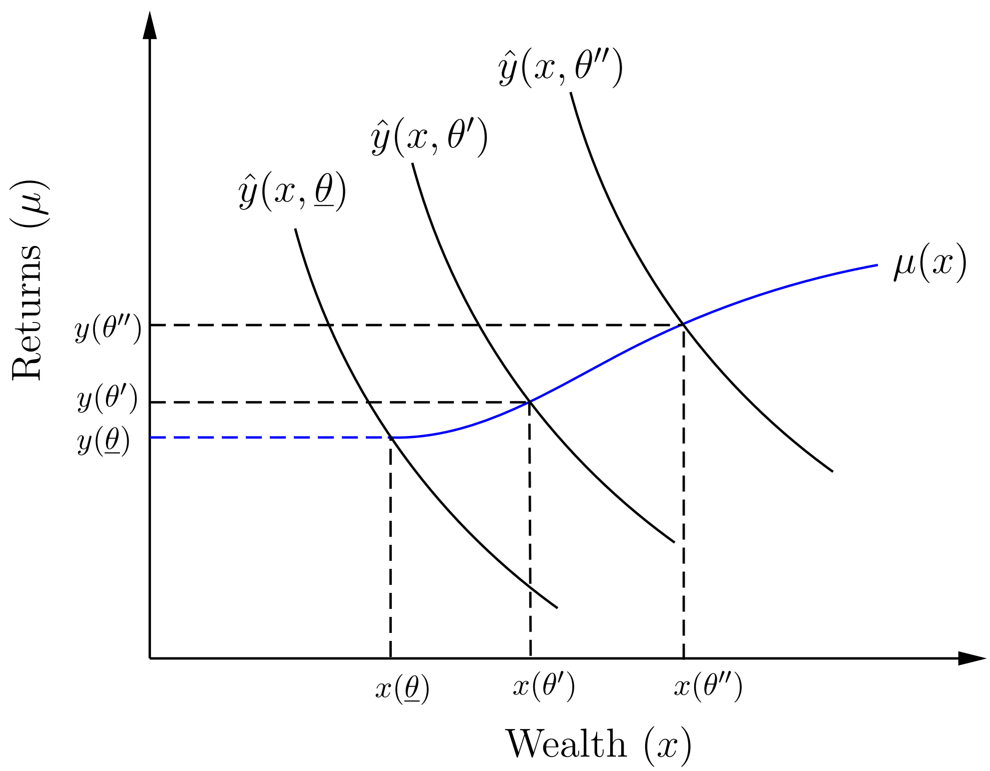

Figure 2: Derivation of Equilibrium Investments

the competition for partners helps remedy the inefficiency surrounding the externality associated with parental investment. This is not the case here.

Proposition 3. *Investments are inefficient in the separating equilibrium. In particular, wealth is weakly greater and parental investments are weakly lower than the corresponding Nash investments.*

A geometric proof is offered in Figure 3. In (x, y) space, the figure shows the locus of

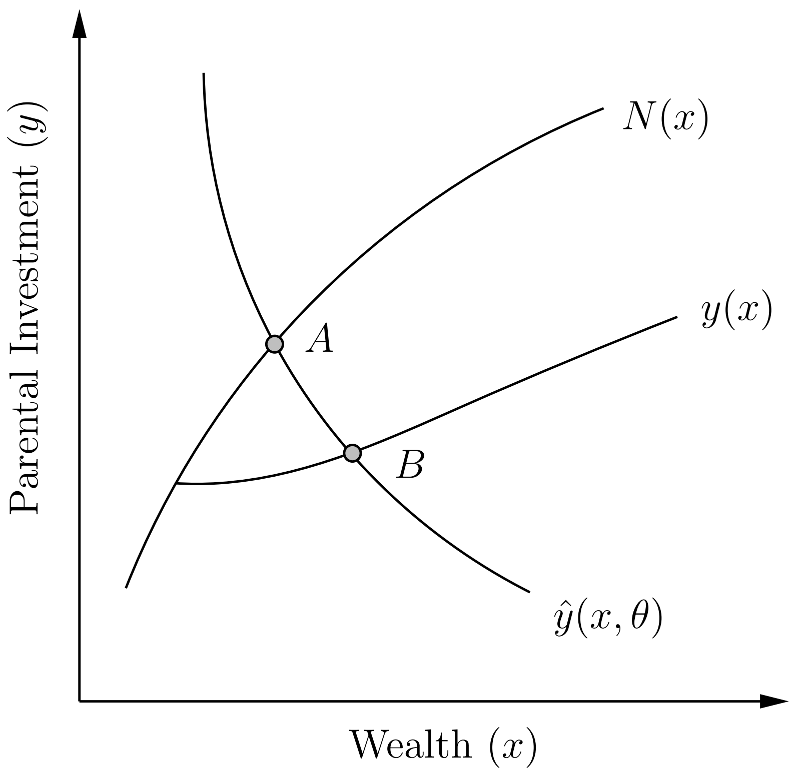

Figure 3: Geometric Proof of Proposition 3

Nash investments, $N(x)$, and equilibrium investments, $y(x)$, in the case of $\alpha < 1$. The locus of Nash investments is implicitly defined by $(1 - \alpha) \cdot f_x(x) = \alpha \cdot [h_y(N(x), N(x))]$ as described above. The equilibrium investments are given by $y(x) = \mu(x)$ as derived above. In addition to these, the first-order condition that describes the optimal choice of y given any particular x, is plotted. This is given by $\alpha \cdot h_y(\hat{y}(x, \theta), \hat{y}(x, \theta)) = c_T(x + \hat{y}(x, \theta), \theta)$. The figure clearly shows that the equilibrium investments (point B) are 'southeast' of the Nash investments (point A). When $\alpha = 1$, the locus of Nash investments coincides with the y axis. The $\hat{y}(x, \theta)$ curve is still well-defined, and the conclusion remains.

3.2 Total Investment

If, in a separating equilibrium, incentives to invest in wealth are too great, and incentives to make parental investments too small, then what about total investment? Let $T^E(\theta) \equiv x(\theta)+y(\theta)$ denote the total investment made by a type θ family in a separating equilibrium. Let $T^*(\theta)$ and $T^N(\theta)$ represent the total investment in the efficient and Nash cases, respectively.

Proposition 4. *For all $\theta \in \Theta$, $T^N(\theta) \leq \min\{T^E(\theta), T^*(\theta)\}$. If q is linear, then $T^N(\theta) = T^E(\theta) < T^*(\theta)$.*

The second illustration described below assumes a form in which q is strictly concave, and derives an equilibrium in which $T^N < T^E = T^*$. Thus, we can be sure that the pattern of inequalities identified in the result do not hold *for all* q.

3.3 An Alternative Benchmark: Random Matching

Equilibrium outcomes are compared to the Nash outcomes, which can be interpreted as the investments that would arise if agents matched on type (rather than wealth). Another reasonable benchmark may be the investments that arise if agents were randomly matched. This corresponds to a setting in which all of the agents' characteristics are hidden. In certain cases, the two benchmarks display the same (aggregate) qualities.

Proposition 5. *If $\rho = 1$, then investments in the 'random matching' benchmark are identical to the investments in the Nash benchmark. Average welfare is the same across the benchmarks, although the lower types prefer random matching, and higher types prefer Nash.*

This follows simply from the observation that h is additively separable if $\rho = 1$, which in turn implies that the Nash investment is independent of the type of partner that a family is matched with (and therefore to any mixture, including random matching). The latter part of the proposition simply reflects the fact lower types get a better quality partner on average under random matching.

To make some progress with more general results, assume $\alpha = 1$ for clarity. Equilibrium with random matching requires that each type equalizes the marginal cost of parental investment with the *expected* marginal return.[8] Suppose that all families made the Nash investments. For lower types, the expected marginal return is higher than the marginal return with perfect segregation (since they are matched with higher types on average), whereas the opposite is true for higher types. Thus, lower types have incentives to invest more than their Nash level, and higher types have incentives to invest less. To supplement this intuition, one can show that parental investment can not be decreasing in ρ for all types.[9] Furthermore, optimal investments with random matching will be sensitive to the distribution of types. Explicit solutions are difficult to obtain when $\rho < 1$, however, a Cobb-Douglas ($\rho \to 0$) example is solved in the Appendix.

4 Simple Illustration

The purpose of this section is to provide a simple demonstration of how to calculate and analyze equilibria. I assume perfect altruism, which implies that both pooling and separating equilibria exist. Comparing welfare across these equilibria will be a central concern.

Assume perfect altruism ($\alpha = 1$), and let the human capital and cost functions take the following forms:

$$h(y, y') = (1 - \phi) \cdot y + \phi \cdot y'$$
$$c(x + y, \theta) = \frac{1}{2\theta} (x + y)^2 .$$

Under this specification parents derive no intrinsic value from wealth and parental investments are substitutes where $\phi > 0$ captures the degree to which there are spillovers in parental investments. In terms of the generalized CES formulation, q is the identity function and $\rho = 1$.

4.1 Efficient Investments

Since h is separable, the definition of the efficient investments is independent of the actual matching pattern. The efficient parental investment (wealth is efficiently zero)

[8] For $\rho < 1$, the marginal return to parental investment will depend on who the family happens to be matched with.

[9] This is shown by treating investments as a function of ρ, and totally differentiating the first order condition.

satisfies:

$$y^*(\theta) = \arg\max\ y - c(y, \theta),$$

implying that $y^*(\theta) = \theta$. If a family matches with a family of type θ', then the family's welfare is:

$$W^*(\theta, \theta') = \frac{1}{2} \cdot \theta + \phi\left(\theta' - \theta\right).$$

Thus, average welfare is:

$$W^* \equiv \int_\Theta W^*(\theta) d\Psi(\theta) = \frac{1}{2} \cdot \mathbb{E}\left[\theta\right],$$

since $\mathbb{E}[\mathbb{E}[\theta' \mid \theta]] = \mathbb{E}[\theta]$ for *any* matching pattern.

4.2 Pooling Equilibrium

There is a pooling equilibrium under this specification, since having no wealth is a best response to all other families having no wealth. In this equilibrium matching is random because all families appear equally attractive. An implication of this is that each family recognizes that increasing their wealth above zero will not affect the expected parental investment made by their match. The objective facing families is:

$$\max_{x,y}\ (1 - \phi) \cdot y + \phi \cdot \mathbb{E}[y'] - c(x + y, \theta).$$

There is clearly no incentive to acquire wealth, so that $x^P(\theta) = 0$. The optimal level of parental investment is $y^P(\theta) = (1 - \phi) \cdot \theta$. If a family of type θ ends up being matched with a family of type θ', then the family's payoff is:

$$W^P(\theta, \theta') = \frac{(1 - \phi)^2}{2} \cdot \theta + \phi \cdot (1 - \phi) \cdot \theta',$$

and the expected welfare of a family of type θ in the pooling equilibrium is therefore:

$$W^P\left(\theta\right) = \frac{(1 - \phi)^2}{2} \cdot \theta + \phi \cdot (1 - \phi) \cdot \mathbb{E}\left[\theta\right],$$

since once again $\mathbb{E}[\mathbb{E}[\theta' \mid \theta]] = \mathbb{E}[\theta]$. Average welfare is therefore:

$$W^P \equiv \int_\Theta W^P(\theta) = (1 - \phi^2) \cdot \frac{1}{2} \cdot \mathbb{E}\left[\theta\right].$$

This is less than W^* as expected, and is monotonically declining in ϕ.

4.3 Separating Equilibrium

Given that all agents believe that a wealth of x will lead to finding a match that has $\mu(x)$ units of parental investment, agent i's objective function is:

$$V\left(x, y, \mu(x), \theta\right) = (1-\phi)\cdot y + \phi \cdot \mu(x) - c(x+y, \theta).$$

The first-order conditions are:

$$\phi \cdot \mu_x(x(\theta)) = (1-\phi) = \frac{y(\theta)+x(\theta)}{\theta}.$$

The first equality (i.e. equating the investments' marginal returns) gives us a particularly simple differential equation:

$$\mu_x(x) = \frac{1-\phi}{\phi}.$$

Integrating both sides gives us the equilibrium return function:

$$\mu(x) = \frac{1-\phi}{\phi}\cdot x + y_0.$$

Since $x^N(\underline{\theta}) = x^P(\underline{\theta}) = 0$, the value of y_0 is given by $y^N(\underline{\theta}) = y^P(\underline{\theta}) = (1-\phi)\cdot\underline{\theta}$, so that we have:

$$\mu(x) = \frac{1-\phi}{\phi}\cdot x + (1-\phi)\cdot\underline{\theta}. \tag{13}$$

The second equation we need, $\hat{y}(x,\theta)$, is found by using the second equality in the first-order conditions:

$$\hat{y}(x,\theta) + x = (1-\phi)\cdot\theta. \tag{14}$$

Since consistency requires $\mu(x(\theta)) = y(\theta)$, we can determine the equilibrium investments by using (13) and (14):

$$\begin{aligned} y^S(\theta) &= (1-\phi)^2\cdot\theta + \phi\cdot(1-\phi)\cdot\underline{\theta} \\ x^S(\theta) &= \phi\cdot(1-\phi)\cdot\left[\theta - \underline{\theta}\right]. \end{aligned}$$

The equilibrium welfare for a type θ family is:

$$W^S(\theta,\theta') = W^S(\theta) = \frac{(1-\phi)^2}{2}\cdot\theta + \phi\cdot(1-\phi)\cdot\underline{\theta},$$

since $\theta' = \theta$ in equilibrium (due to positive assortative matching). Average welfare is:

$$W^S \equiv \int_\Theta W^S(\theta) = \frac{(1-\phi)^2}{2}\cdot\mathbb{E}\left[\theta\right] + \phi\cdot(1-\phi)\cdot\underline{\theta}.$$

4.4 Results

A comparison of the expressions for average welfare reveals that $W^S \leq W^P \leq W^*$, where the inequalities hold with equality if and only if there are no spillovers ($\phi = 0$). The difference between welfare in either equilibrium and the first-best welfare grows monotonically in spillovers. These results could be anticipated given the general results discussed in the previous section, however there are stronger welfare results generated in this setting.

First, *all agents* find that the expected payoff is greater in the pooling equilibrium than in the separating equilibrium.

Result 6. *The pooling equilibrium 'strictly ex-ante Pareto dominates' the separating equilibrium:* $W^S(\theta) < W^P(\theta)$ *for all* $\theta \in \Theta$.

A brief inspection of the relevant expressions reveals that $W^P(\theta) - W^S(\theta) = \phi(1 - \phi) \cdot [\mathbb{E}(\theta) - \underline{\theta}]$, which is clearly positive. There is an even stronger result than this. All agents get a greater payoff in the pooling equilibrium than in the separating equilibrium, regardless of which family they end up being matched with in the pooling equilibrium.

Result 7. *The pooling equilibrium 'ex-post Pareto dominates' the separating equilibrium:* $W^S(\theta) \leq \min_{\theta'}\{W^P(\theta, \theta')\}$ *for all* $\theta \in \Theta$.

One may conjecture that this result reflects the fact that although some agents get lower-quality matches, they have lower investment costs. This is the logic behind the analogous result in standard signaling models. This is not the case here, because it is straightforward to verify that total investment for a family of type θ equals $(1 - \phi) \cdot \theta$ in both the pooling and separating equilibria.[10] Since investment costs are the same across equilibria, the following must apply.

Result 8. *The realized human capital level any given child realizes in the pooling equilibrium is never less than their human capital level in a separating equilibrium.*

To verify this, note that if a child with parents of ability θ is matched with a family of ability θ' in a pooling equilibrium, then their human capital level is $(1 - \phi) \cdot [(1 -$

[10]This is largely a consequence of y entering in a linear manner, since the first-order condition for parental investment (in both equilibria) is $c(T, \theta) = (1 - \phi)$, which automatically pins down T for each θ. With imperfect altruism ($\alpha < 1$), such linear specifications must be abandoned if we are to ensure that optimal parental investments are interior (for all possible types).

$\phi) \cdot \theta + \phi \cdot \theta']$, whereas their human capital level in the separating equilibrium is $(1-\phi) \cdot [(1-\phi) \cdot \theta + \phi \cdot \underline{\theta}]$. The difference between these, $(1-\phi)\phi \cdot [\theta' - \underline{\theta}]$, is also clearly never negative.

To summarize, the pooling equilibrium welfare-dominates the separating equilibrium in a very strong sense (under the specification considered here). To paraphrase Result 7, the very lowest payoff that a family can receive in the pooling equilibrium is never lower than the best payoff that they can receive in the separating equilibrium. Unlike standard signaling models, this has nothing to do with different investment costs across the equilibria, but rather, arises from the fact that the pooling equilibrium does not encourage a diversion of family resources away from parental investment.

Given this strong welfare dominance, it seems quite reasonable to conjecture that the vast majority of societies will be characterized by norms consistent with the pooling equilibrium. For instance, there would be no impetus to acquire wealth in order to secure a desirable environment for a child's development, and there would be little segregation - at least along wealth lines. This is hardly an accurate description of most modern societies. These observations can be reconciled by the fact that the existence of pooling equilibria is much less robust than is the existence of separating equilibria. Indeed, if consumption is valued at all then the pooling equilibrium will not exist. Intuitively, all families would 'naturally' have different wealth levels, reflecting differences in endowed abilities. Further, those with high wealth levels would also be the ones with high investments in human capital. Thus, it is relatively easy to identify the desirable families, but each family recognizes that in order to convince a desirable family to match with them, they must 'masquerade' as a high-wealth family. Any pooling behaviour therefore unravels. The following section analyzes a model with imperfect altruism - and therefore, with no pooling equilibrium.

5 An Extended Illustration

The point of this section is to derive equilibrium variables for a more detailed economy. The functional forms are chosen so that the extension considered in the section is more readily analyzed. I assume non-perfect altruism, which implies that pooling equilibria fail to exist. This allows me to focus on the separating equilibrium, which in turn allows me to analyze the effect of spillovers, altruism, and the distribution of types.

The assumed functional forms are as follows:

$$
\begin{aligned}
f(x) &= \ln x \\
h(y, y') &= (1-\phi) \cdot \ln y + \phi \cdot \ln y' \\
c(x+y, \theta) &= \frac{1}{\theta} \frac{(x+y)^{1+\eta}}{1+\eta},
\end{aligned}
$$

where $\alpha \in (0,1)$ is the altruism parameter, $\phi \in [0,1]$ is the spillover parameter, and $\eta \geq 0$ describes the degree to which there is 'crowding out effects'. The functional form for h is obtained from the generalized form by letting q be the natural log and setting $\rho = 1$.

To begin, the symmetric Nash investments are given by:

$$
\begin{aligned}
x^N(\theta) &= \left[\frac{(1-\alpha)}{[1-\alpha\phi]^{\frac{\eta}{1+\eta}}} \right] \cdot \theta^{\frac{1}{1+\eta}}, \\
y^N(\theta) &= \left[\frac{\alpha(1-\phi)}{[1-\alpha\phi]^{\frac{\eta}{1+\eta}}} \right] \cdot \theta^{\frac{1}{1+\eta}}.
\end{aligned}
$$

Note that i) total investment equals $[(1-\alpha\phi)\theta]^{[1/(1+\eta)]}$, which is decreasing in both altruism and spillovers, however ii) the relative amount allocated to parental investment, $y^N(\theta)/x^N(\theta)$, is equal to $\alpha(1-\phi)/(1-\alpha)$, which is increasing in altruism and decreasing in spillovers.

Since the symmetric Nash investments coincide with the investments that would be made if matching were random, the fact that $x^N(\theta)$ is strictly increasing in θ implies that there can not be a pooling equilibrium. Out of interest, the efficient investments are given by:

$$
\begin{aligned}
x^*(\theta) &= (1-\alpha) \cdot \theta^{\frac{1}{1+\eta}}, \\
y^*(\theta) &= \alpha \cdot \theta^{\frac{1}{1+\eta}}.
\end{aligned}
$$

In order to calculate the separating equilibrium, we begin by deriving the initial values problem.

$$
\begin{aligned}
\mu'(x) &= \frac{1}{\phi} \left[(1-\phi) - \frac{1-\alpha}{\alpha} \frac{\mu(x)}{x} \right] \\
\mu\left(x^N(\underline{\theta})\right) &= y^N(\underline{\theta}).
\end{aligned}
$$

This is an ordinary linear differential equation which has the following solution:

$$
\mu(x) = Z \cdot x^{-\frac{1-\alpha}{\alpha\phi}} + \delta \cdot x, \tag{15}
$$

where

$$\delta \equiv \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)}$$

and Z adjusts so that the initial condition is satisfied. In other words, Z satisfies:

$$y^N(\underline{\theta}) = Z \cdot x^N(\underline{\theta})^{-\frac{1-\alpha}{\alpha\phi}} + \delta \cdot x^N(\underline{\theta}).$$

Re-arranging to get Z, then substituting into (15), we get:

$$\mu(x) = \left(\frac{x^N(\underline{\theta})}{x}\right)^{\frac{1-\alpha}{\alpha\phi}} \left[y^N(\underline{\theta}) - \delta \cdot x^N(\underline{\theta})\right] + \delta \cdot x \tag{16}$$

This gives us the equilibrium matching return function. The other equation we need, $\hat{y}(x,\theta)$, is given by the first-order condition for the optimal choice of y:

$$\frac{\alpha \cdot (1-\phi)}{\hat{y}(x,\theta)} = \frac{1}{\theta}\left[x + \hat{y}(x,\theta)\right]^{\eta}. \tag{17}$$

Using the fact that $y(\theta) = \mu(x(\theta))$, equation (16) describes parental investment as an increasing function of x (for $x \geq x^N(\underline{\theta})$), whereas (17) describes y as a decreasing function of x. When plotted in (x,y) space, the two curves intersect exactly once, in a manner similar to that depicted in Figure 2 above. The effect of altruism and spillovers on investment can be determined by manipulating the two curves. However, explicit solutions are readily computed for the case in which $\underline{\theta} = 0$:

$$\begin{aligned}
\mu(x) &= \left[\frac{\alpha(1-\phi)}{1-\alpha(1-\phi)}\right] \cdot x \\
x(\theta) &= [1-\alpha(1-\phi)] \cdot \theta^{\frac{1}{1+\eta}} \\
y(\theta) &= \alpha(1-\phi) \cdot \theta^{\frac{1}{1+\eta}}
\end{aligned}$$

Note that total equilibrium investment in this case is $\theta^{[1/(1+\eta)]}$, which is the same as the total efficient investment. This is not general - it only holds when $\underline{\theta} = 0$ (otherwise total efficient investment is greater). The existence of spillovers, however, distorts the composition of this investment. Welfare for a type θ family is:

$$\begin{aligned}
W(\theta) = \frac{1}{1+\eta} \cdot [\ln\theta - 1] + \\
&(1-\alpha) \cdot \ln(1-\alpha(1-\phi)) + \alpha \cdot \ln(\alpha(1-\phi)).
\end{aligned}$$

The first term is independent of both spillovers and altruism, and the remaining terms are independent of type and cost externalities. The difference between equilibrium welfare and efficient welfare therefore depends only on the latter term. This term is plotted as a function of α for various values of ϕ in Figure 4. The case in which $\phi = 0$ corresponds to the efficient welfare. The figure shows how equilibrium welfare departs from efficient welfare as altruism increases, and this occurs to a greater extent when spillovers are greater.

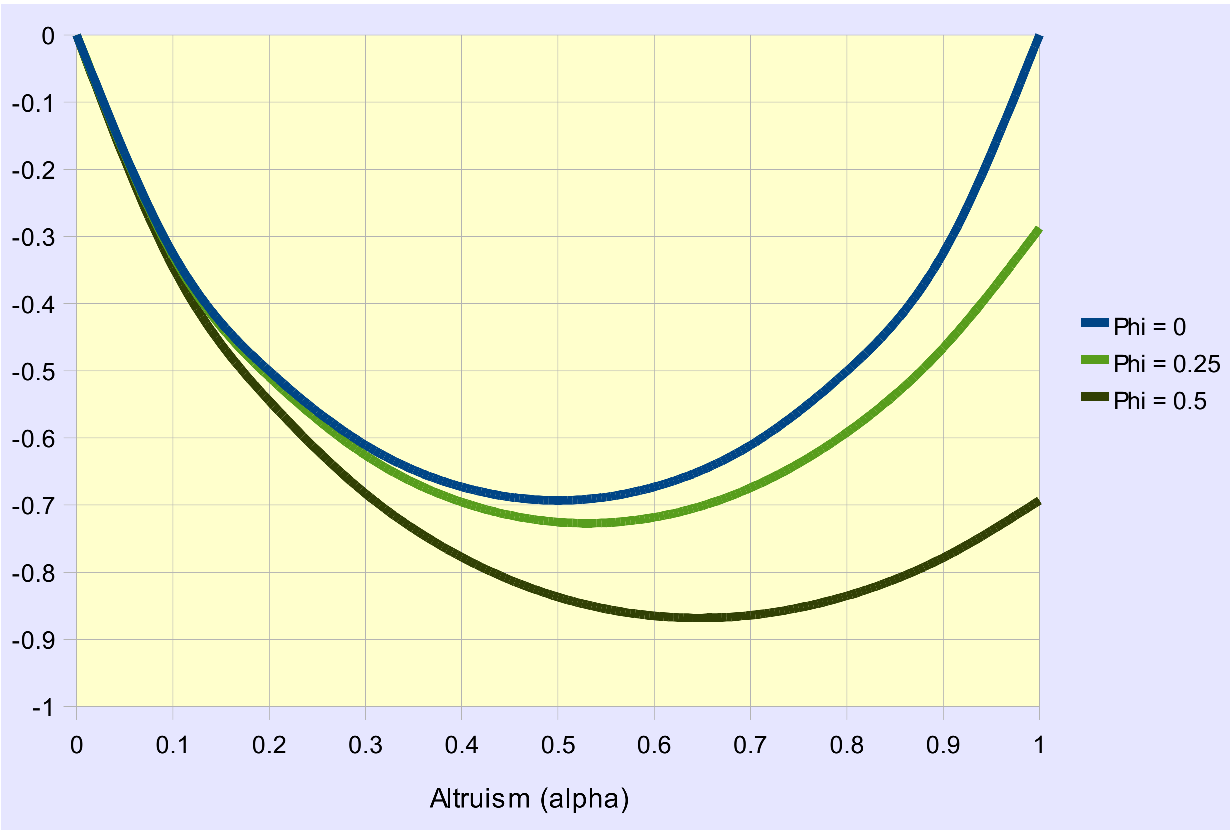

Figure 4: Welfare: The Effect of Altruism and Spillovers

A striking (general) feature of the above analysis is that the only relevant aspect of the distribution of types is the level of the lowest ability. In particular, the equilibrium investments of a type θ agent approaches the Nash investments as $\underline{\theta}$ approaches θ. That is, all agents obtain a higher payoff as the lowest ability is raised.

The fact that the equilibrium is insensitive to other qualities of the distribution of types, such as mean and variance, does not seem plausible. The following section demonstrates that this is a consequence of the assumption that wealth is perfectly observed, whereas parental investment is imperfectly observed (but not that parental investment in unobserved).

6 Imperfectly Observed Investments

The model so far has worked with the seemingly extreme assumption that wealth is perfectly observed, whereas parental investments are unobserved. This section makes an attempt at relaxing this assumption by supposing that both wealth and parental investments are observed with some noise. In this setting the distribution of abilities will become relevant since this information will be incorporated into the process of Bayesian updating. To make some progress, fairly particular functional forms are imposed. First, assume that the distribution of types is log-normal:

$$\ln \theta_i = \ln \overline{\theta} + \varepsilon_i^{\theta}, \;\; \varepsilon_i^{\theta} \sim N\left[0, \sigma_\theta^2\right]. \tag{18}$$

Assume also that investments are observed with noise as follows:

$$\ln \tilde{x}_i = \ln x_i + \varepsilon_i^{x}, \;\; \varepsilon_i^{x} \sim N\left[0, \sigma_x^2\right] \tag{19}$$

$$\ln \tilde{y}_i = \ln y_i + \varepsilon_i^{y}, \;\; \varepsilon_i^{y} \sim N\left[0, \sigma_y^2\right]. \tag{20}$$

The log structure ensures that all random variables are positive. If the investment functions happen to be of the form: $y(\theta) = \beta_y \theta^{\gamma}$ and $x(\theta) = \beta_x \theta^{\gamma}$, then it turns out (see the Appendix for a derivation) that:

$$\mu(\tilde{x}_i, \tilde{y}_i) \equiv \mathbb{E}\left[\ln y_i \mid \tilde{x}_i, \tilde{y}_i\right] = \lambda_x \cdot \ln \tilde{x}_i + \lambda_y \cdot \ln \tilde{y}_i + \text{constants}, \tag{21}$$

where

$$\lambda_x \equiv \frac{\sigma_y^2 \gamma^2 \sigma_\theta^2}{\left[\sigma_y^2 + \sigma_x^2\right] \gamma^2 \sigma_\theta^2 + \sigma_y^2 \sigma_x^2} \tag{22}$$

$$\lambda_y \equiv \frac{\sigma_x^2 \gamma^2 \sigma_\theta^2}{\left[\sigma_y^2 + \sigma_x^2\right] \gamma^2 \sigma_\theta^2 + \sigma_y^2 \sigma_x^2} \tag{23}$$

These expressions appear messy, but they are quite intuitive. For a fixed $\gamma^2 \sigma_\theta^2$, λ_x is increasing in σ_y^2 and decreasing in σ_x^2: the more noisy the signal of y relative to x the more weight that one should put on the signal of x. The same intuition applies for λ_y. For a fixed σ_x^2 and σ_y^2, both λ_x and λ_y are increasing in $\gamma^2 \sigma_\theta$: as the distribution of types becomes more dispersed, then more weight should be placed on the signals, and less on one's prior.

Since μ is increasing in both arguments, all families find those with high values of μ more attractive, and as such, families will match assortitatively on μ. Note that when there is no noise on wealth (i.e. $\sigma_x^2 = 0$) but an arbitrarily small amount of noise on parenting investment (i.e. $\sigma_y^2 > 0$) families match assortitatively on wealth (i.e. $\lambda_x = 1$ and $\lambda_y = 0$).

This equilibrium matching pattern implies that if family i is matched with family j in equilibrium, then $\mu(\tilde{x}_j, \tilde{y}_j) = \mu(\tilde{x}_i, \tilde{y}_i)$, so that once the noise on the signals are realized, agent i's expected utility is:

$$v(x_i, y_i; \theta) = (1-\alpha) \cdot \ln x_i + \alpha(1-\phi) \cdot \ln y_i + \alpha\phi \cdot \mu(\tilde{x}_i, \tilde{y}_i) - c(x_i + y_i; \theta). \tag{24}$$

It is straightforward to show that this implies the ex-ante expected utility can be written as:

$$V(x_i, y_i; \theta) = \xi \cdot \ln x_i + \zeta \cdot \ln y_i - c(x_i + y_i; \theta) + \text{constants}, \tag{25}$$

where

$$\xi \equiv (1-\alpha) + \alpha\phi\lambda_x, \qquad \text{and} \qquad \zeta \equiv \alpha(1-\phi) + \alpha\phi\lambda_y. \tag{26}$$

Maximizing this with respect to x and y produces the equilibrium investment functions, which are indeed of the form conjectured, where:

$$\beta_y = \frac{\zeta}{[\zeta + \xi]^{\frac{\eta}{1+\eta}}} = \frac{\alpha\left[1 - \phi(1-\lambda_y)\right]}{\left[1 - \alpha\phi\left[1 - \lambda_x - \lambda_y\right]\right]^{\frac{\eta}{1+\eta}}} \tag{27}$$

$$\beta_x = \frac{\xi}{[\zeta + \xi]^{\frac{\eta}{1+\eta}}} = \frac{1 - \alpha(1-\phi\lambda_x)}{\left[1 - \alpha\phi\left[1 - \lambda_x - \lambda_y\right]\right]^{\frac{\eta}{1+\eta}}} \tag{28}$$

$$\gamma = \frac{1}{1+\eta}. \tag{29}$$

To verify this result, consider what happens as $\sigma_y^2 \to \infty$. In this case $\lambda_x \to 1$ and $\lambda_y \to 0$, which means that $\beta_y \to [\alpha(1-\phi)]$ and $\beta_x \to [1 - \alpha(1-\phi)]$, as derived above for the case in which $\underline{\theta} = 0$. This is true for any finite σ_x^2, which implies the above analysis does not at all rely on wealth being perfectly observed (as long as parenting effort is not observed).

On the other hand, if wealth is perfectly observed and parenting investments are imperfectly observed (i.e. $\sigma_x^2 = 0$ and $\sigma_y^2 > 0$), then we end up with the same results as those derived in the case in which parenting effort is not observed at all (since $\lambda_x = 1$ and $\lambda_y = 0$ in this case). Note that this holds for an arbitrarily small amount of noise contained in the signal of parenting effort.

In summary, the qualitative results from the above analysis (in which wealth is perfectly observed and parental investment is unobserved) carries through to cases in which i) wealth is imperfectly observed and parenting effort is unobserved, and ii) wealth is perfectly observed and parenting effort is (arbitrarily) imperfectly observed.

The opposite case - in which parental investments are better observed than wealth - is, in the limit, reminiscent of Peters and Siow (2002). Investments do indeed approach the efficient investments in the limit (since $\lambda_y \to 1$ and $\lambda_x \to 0$, implying $\beta_y \to \alpha$ and $\beta_x \to (1 - \alpha)$).

It may also be of interest to note that if both investments are imperfectly observed, then the equilibrium investments depend upon the distribution of abilities. In particular, investments approach the Nash investments as the distribution of types becomes degenerate around the mean (i.e. both λ_x and λ_y go to zero as σ_θ^2 goes to zero). Intuitively, equilibrium behaviour approaches Nash behaviour as the population becomes more homogeneous.

7 Conclusions

The paper has aimed to illuminate the consequences of peer effects in a model of human capital development in the presence of the promise of social mobility. The key features of the model are i) parents care about the human capital of their child, as well as consumption, ii) human capital depends on parental investment and peer group spillovers, iii) peer groups are endogenously determined on the basis of wealth, and iv) that parental investment and wealth accumulation place competing demands on parents' resources. The central message is that competition for desirable peer groups can induce detrimental incentives to undertake (socially productive) parental investment.

I have shown that pooling equilibria generally have superior welfare properties to separating equilibria, but do not exist in general. A unique separating equilibrium always exists, and delivers an average welfare lower than that associated with Nash outcomes (which are themselves inefficient). One surprising result is that the separating equilibrium is invariant to the CES substitution parameter. Various properties of equilibrium are illustrated through the employment of functional forms, and an extension is provided in which both investments are observed with noise. Although specialized, the extension demonstrates how the distribution of types matters and provides a connection between the base model and related models in the literature.

I hope the model proves useful as a starting point for research on pre-match investment with multiple investments. For instance, I see value in generalizing the extension presented in Section 6, and in embedding the model within a dynamic search framework.

References

AKERLOF, G. A. (1976): "The Economics of Caste and of the Rat Race and Other Woeful Tales," *The Quarterly Journal of Economics*, 90(4), 599–617.

BECKER, G. S. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81(4), 813–46.

BENABOU, R. (1996a): "Equity and Efficiency in Human Capital Investment: The Local Connection," *Review of Economic Studies*, 63(2), 237–64.

——— (1996b): "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance," *American Economic Review*, 86(3), 584–609.

BIDNER, C. (2008a): "A Spillover-Based Theory of Credentialism," Working paper, University of British Columbia.

——— (2008b): "Two-Sided Noisy Signaling," Working paper, University of British Columbia.

COLE, H. L., G. J. MAILATH, AND A. POSTLEWAITE (2001): "Efficient Non-Contractible Investments in Large Economies," *Journal of Economic Theory*, 101(2), 333–373.

DAMIANO, E., AND H. LI (2007): "Price discrimination and efficient matching," *Economic Theory*, 30(2), 243–263.

DE BARTOLOME, C. A. M. (1990): "Equilibrium and Inefficiency in a Community Model with Peer Group Effects," *Journal of Political Economy*, 98(1), 110–33.

DURLAUF, S. N. (1996): "A Theory of Persistent Income Inequality," *Journal of Economic Growth*, 1(1), 75–93.

DURLAUF, S. N. (2004): "Neighborhood Effects," in *Handbook of Regional and Urban Economics*, ed. by J. V. Henderson, and J. F. Thisse, vol. 4, pp. 2173–2242. Elsevier.

FERNANDEZ, R., AND R. ROGERSON (1996): "Income Distribution, Communities, and the Quality of Public Education," *The Quarterly Journal of Economics*, 111(1), 135–64.

FRANK, R. H. (1985): "The Demand for Unobservable and Other Nonpositional Goods," *American Economic Review*, 75(1), 101–16.

HAN, S. (2005): "Competitive Investments and Matching: Hedonic Pricing Problems," Mimeo, McMaster University.

HOLMSTROM, B., AND P. MILGROM (1991): "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design," *Journal of Law, Economics and Organization*, 7(0), 24–52.

HOPPE, H. C., B. MOLDOVANU, AND A. SELA (2005): "The Theory of Assortative Matching Based on Costly Signals," Discussion Papers 85, SFB/TR 15 Governance and the Efficiency of Economic Systems, Free University of Berlin, Humboldt University of Berlin, University of Bonn, University.

KREMER, M. (1993): "The O-Ring Theory of Economic Development," *The Quarterly Journal of Economics*, 108(3), 551–75.

PETERS, M. (2007a): "A Non-Cooperative Approach to Hedonic Equilibrium," mimeo, University of British Columbia.

——— (2007b): "The pre-marital investment game," *Journal of Economic Theory*, 137(1), 186–213.

PETERS, M., AND A. SIOW (2002): "Competing Premarital Investments," *Journal of Political Economy*, 110(3), 592–608.

REGE, M. (2007): "Why do people care about social status?," *Journal of Economic Behavior and Organization*, Forthcoming.

Appendix

A Complementarities: Separating Equilibrium vs Random Matching

Consider an alternative benchmark in which all characteristics are hidden, thereby requiring that partners are randomly assigned. For simplicity, assume perfect altruism ($\alpha = 1$). This allows us to focus on incentives to make parental investments in isolation (since wealth will optimally be zero for all families). In contrast to the illustrations used in the text, this illustration assumes that $\rho < 1$. In particular, as $\rho \to 0$, h approaches the Cobb-Douglas form:

$$h = q(y)^{1-\phi} \cdot q(y')^{\phi}.$$

I use this functional form, with q being the identity function, and assume that costs are given by $c(y, \theta) = (1/2) \cdot y^2/\theta$.

There is always an equilibrium in which all agents invest zero. There is also a more interesting equilibrium in which positive investments are made. The first-order condition, once re-arranged, is:

$$(1-\phi) \cdot \int [y^R(\tau)]^{\phi} d\Psi(\tau) = \frac{[y^R(\theta)]^{1+\phi}}{\theta}.$$

The left-side is a constant (from the perspective of a particular family). If this is denoted by A, then equating this to the right side, we have:

$$y^R(\theta) = A^{\frac{1}{1+\phi}} \cdot \theta^{\frac{1}{1+\phi}}.$$

Using this form in the left side, we have that the value of A satisfies:

$$A = \phi \cdot \int A^{\frac{\phi}{1+\phi}} \cdot \tau^{\frac{\phi}{1+\phi}} d\Psi(\tau),$$

thereby implying that:

$$A^{\frac{1}{1+\phi}} \equiv (1-\phi) \cdot \int \tau^{\frac{\phi}{1+\phi}} d\Psi(\tau).$$

The optimal investment under random matching is therefore:

$$y^R(\theta) = (1-\phi) \cdot \left[\int \tau^{\frac{\phi}{1+\phi}} d\Psi(\tau)\right] \cdot \theta^{\frac{1}{1+\phi}}.$$

It is straightforward to show that the Nash investments are $y^N(\theta) = (1-\phi)\cdot\theta$. Therefore, comparing these we get:

$$\frac{y^R(\theta)}{y^N(\theta)} = \int \left[\frac{\tau}{\theta}\right]^{\frac{\phi}{1+\phi}} d\Psi(\tau).$$

The term on the right is continuous and strictly decreasing in θ, strictly greater than one at $\theta = \underline{\theta}$, and strictly less than one at $\theta = \overline{\theta}$. Thus, there is a critical type $\theta' \in (\underline{\theta}, \overline{\theta})$ such that all families with $\theta < \theta'$ invest more than their Nash level, and all families with $\theta > \theta'$ invest less than their Nash level.

Although investments made by individual families can not be unambiguously ranked across the benchmarks, *average* investment can be. Average investment with random matching is

$$\mathbb{E}[y^R] = (1-\phi)\cdot\left[\int \tau^{\frac{\phi}{1+\phi}} d\Psi(\tau)\right]\cdot\left[\int \theta^{\frac{1}{1+\phi}} d\Psi(\theta)\right], \tag{30}$$

whereas average Nash investment is:

$$\mathbb{E}[y^N] = (1-\phi)\cdot\int \theta d\Psi(\tau).$$

Jensen's inequality implies that the average Nash investment is greater than the average investment with random matching, since:

$$\mathbb{E}[y^R] = (1-\phi)\cdot\mathbb{E}\left[\theta^{\frac{\phi}{1+\phi}}\right]\cdot\mathbb{E}\left[\theta^{\frac{1}{1+\phi}}\right] < (1-\phi)\cdot\mathbb{E}\left[\theta\right] = \mathbb{E}[y^N].$$

In terms of human capital, the random matching environment does even worse: not only is the average parental investment lower, matches are formed in a less efficient manner. Average human capital under random matching is:

$$\mathbb{E}[h^R] = \mathbb{E}[(y^R)^{1-\phi}]\cdot\mathbb{E}[(y^R)^{\phi}], \tag{31}$$

which, again by Jensen's inequality, is less than $\mathbb{E}[(y^R)]$. Average human capital in the Nash environment is simply $\mathbb{E}[(y^N)]$, which we have already established is greater than $\mathbb{E}[(y^R)]$. Thus, average human capital is greater in the Nash environment. Finally, since average Nash investment is less than average efficient investment, we also know that *average welfare in the Nash benchmark is greater than the average welfare in the random matching benchmark*.

A.1 Comparison with Separating Equilibrium

The fact that the separating equilibrium is independent of ρ means that we can use the equilibrium values derived in Section 4. Average welfare in the separating equilibrium is:

$$W^S = (1-\phi) \cdot \left[\frac{1-\phi}{2} \cdot \mathbb{E}[\theta] + \phi \cdot \underline{\theta} \right].$$

After a few algebra steps, the expected welfare in the random matching setting is:

$$W^R = (1-\phi) \cdot \frac{1+\phi}{2} \cdot \mathbb{E}\left[\theta^{\frac{\phi}{1+\phi}} \right]^2 \cdot \mathbb{E}\left[\theta^{\frac{1-\phi}{1+\phi}} \right].$$

Unlike the Nash benchmark, the welfare rank is ambiguous (in the sense that it depends on the distribution of types). Simple manipulation shows the following.

Proposition 6. *Welfare is greater in the separating equilibrium than under random matching if and only if:*

$$\frac{1-\phi}{1+\phi} \geq \frac{\mathbb{E}\left[\theta^{\frac{\phi}{1+\phi}} \right]^2 \cdot \mathbb{E}\left[\theta^{\frac{1-\phi}{1+\phi}} \right]}{\mathbb{E}\left[\theta \right]} - 2 \cdot \frac{\phi}{1+\phi} \cdot \frac{\underline{\theta}}{\mathbb{E}[\theta]}.$$

One unusual feature of this is that the rank depends on how low the lowest type is relative to the mean. The above condition becomes easier to satisfy as the gap between the lowest type and the mean shrinks. This reflects the fact that distortions in the separating equilibrium are made more severe as the lowest type falls.

To illustrate, suppose that types are log-normally distributed: $\ln \theta \sim N(m, \sigma^2)$, then it turns out that:

$$\begin{aligned} W^R &= \frac{(1-\phi)(1+\phi)}{2} \cdot \exp\left[m + \frac{\sigma^2}{2} \cdot \left[\frac{2\phi^2 + (1-\phi)^2}{(1+\phi)^2} \right] \right] \\ W^S &= \frac{(1-\phi)(1-\phi)}{2} \cdot \exp\left[m + \frac{\sigma^2}{2} \right], \end{aligned}$$

so that the separating equilibrium provides the greater welfare if and only if:

$$\frac{1-\phi}{1+\phi} \geq \exp\left(-\sigma^2 \cdot \frac{\phi(2-\phi)}{(1+\phi)^2} \right).$$

In other words, if:

$$\sigma^2 \geq S(\phi) \equiv \ln\left(\frac{1+\phi}{1-\phi} \right) \cdot \frac{(1+\phi)^2}{\phi(2-\phi)}.$$

The function $S(\phi)$ is strictly increasing on $[0,1]$ with $\lim_{\phi\to 0} S(\phi) = 1$ and $\lim_{\phi\to 1} S(\phi) = \infty$. Three main properties emerge. First, the mean of the distribution of log-types plays no role. Second, the separating equilibrium produces greater welfare than random matching for sufficiently large σ^2, whereas the opposite is true for sufficiently small σ^2. Third, random matching always produces a greater welfare for ϕ sufficiently close to one.

Of even greater significance is the possibility that average human capital under random matching is greater than in the separating equilibrium. This never occurs when human capital is treated as fixed, since the matching arrangement under the separating is more efficient. Average human capital in the separating equilibrium is:

$$\mathbb{E}[h^S] = (1-\phi)^2 \cdot \mathbb{E}[\theta] + \phi(1+\phi)\cdot \underline{\theta},$$

whereas average human capital under random matching is:

$$\mathbb{E}[h^R] = (1-\phi)\cdot \mathbb{E}[\theta^{\frac{\phi}{1+\phi}}]^2 \cdot \mathbb{E}[\theta^{\frac{1-\phi}{1+\phi}}]$$

For simplicity, let $\underline{\theta} = 0$ so that the average human capital level in the separating equilibrium is greater than under random matching if:

$$\mathbb{E}[\theta^{\frac{\phi}{1+\phi}}]^2 \cdot \mathbb{E}[\theta^{\frac{1-\phi}{1+\phi}}] \leq (1-\phi)\cdot \mathbb{E}[\theta].$$

Using the log-normal example again, this requires that:

$$\exp\left(m + \frac{\sigma^2}{2}\cdot\left[\frac{2\phi^2 + (1-\phi)^2}{(1+\phi)^2}\right]\right) \leq (1-\phi)\cdot \exp\left(m + \frac{\sigma^2}{2}\right),$$

or, once simplified:

$$\exp\left(-\sigma^2 \cdot \frac{\phi(2-\phi)}{(1+\phi)^2}\right) \leq (1-\phi).$$

In other words, if

$$\sigma^2 \geq S^*(\phi) \equiv \ln\left(\frac{1}{1-\phi}\right)\cdot \frac{(1+\phi)^2}{\phi(2-\phi)}.$$

The properties of $S^*(\phi)$ are similar to those of $S(\phi)$. Figure 5 depicts both $S(\phi)$ and $S^*(\phi)$.

When (ϕ, σ^2) lies in region A, average human capital is greater in the separating equilibrium. This is the standard result. When (ϕ, σ^2) lies in region C, we have that average human capital is actually greater under random matching: despite the fact

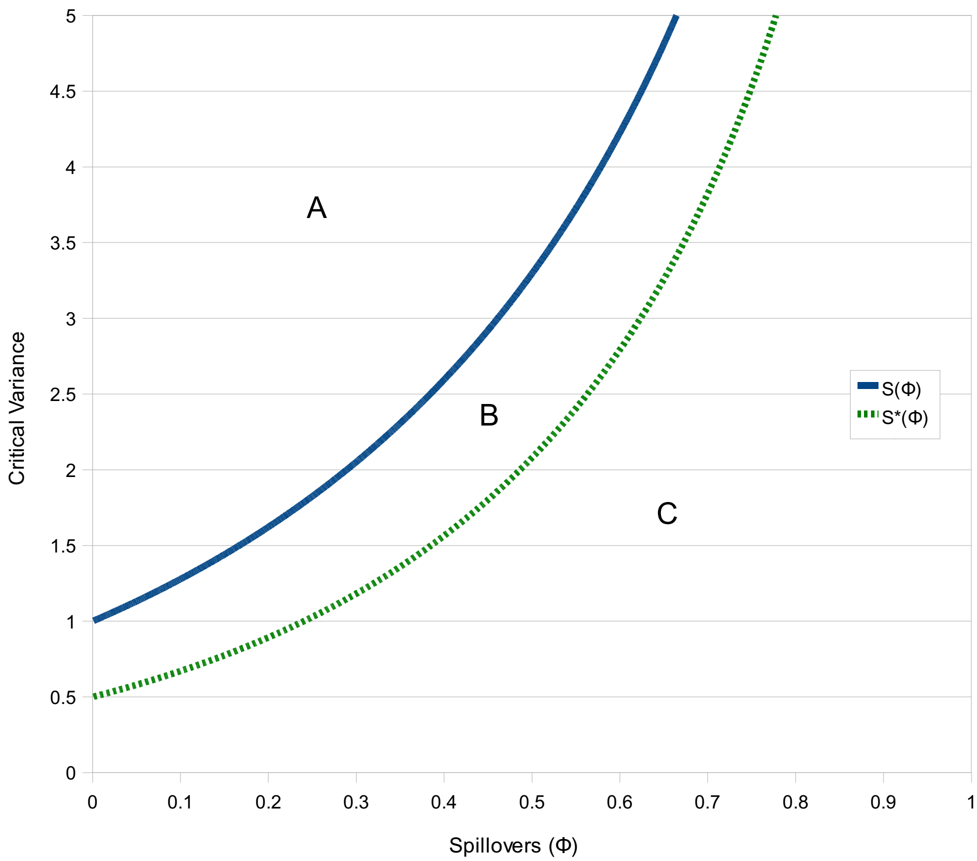

Figure 5: The Functions $S(\phi)$ and $S^{*}(\phi)$

that matches are formed less efficiently under random matching, parental investment is greater. In region B, average human capital is greater in the separating equilibrium but average welfare is greater under random matching (i.e. the superior matching pattern in the separating equilibrium does not compensate for the extra costs involved in achieving separation).

B Proofs

B.1 Proof of Proposition 1

Proof. Suppose to the contrary that a pooling equilibrium exists, and yet $\alpha < 1$. The first-order conditions (the assumptions on f and h, along with the fact that $\alpha \in (0,1)$, guarantee that the solution is interior) imply that:

$$(1-\alpha) \cdot f_x(x) = \alpha \cdot H_y(y) = c_T(x+y,\theta).$$

Since x is a constant across types in a pooling equilibrium (by definition), the first equality implies that y will also be a constant across types (since $H_y(y)$ is strictly increasing). The final inequality then implies that the marginal cost is constant across types, which is contradicted by the fact that $c_{T\theta}(\cdot) < 0$. □

B.2 Proof of Result 1

Proof. The first part comes from the condition that $\mu(x(\theta)) = y(\theta)$. If $x(\cdot)$ is increasing (decreasing), then $y(\cdot)$ is weakly increasing (decreasing). That total investment is increasing in type comes from noting that the payoff function can be expressed as $V(x,y,\mu(x),\theta) = U(x,y) - c(x+y,\theta)$, and applying a revealed preference argument to two different types, $\theta < \theta'$, gives:

$$U(x',y') - c(x'+y',\theta') \geq U(x,y) - c(x+y,\theta')$$

and

$$U(x,y) - c(x+y,\theta) \geq U(x',y') - c(x'+y',\theta).$$

Adding these, re-arranging, and defining $T = x+y$ and $T' = x'+y'$ gives

$$c(T',\theta) - c(T,\theta) \geq c(T',\theta') - c(T,\theta').$$

The fact that $c_{T\theta}(\cdot) > 0$ implies that this inequality can only hold if $T' > T$.

The assumptions on V ensure that optimal parental investment is positive, and, since $V(x, y, \mu, \theta)$ is differentiable (and concave) in y, optimal parental investment must be characterized by the first-order condition: $V_y(x(\theta), y(\theta), \mu(x(\theta)), \theta) = 0$. Again using the equilibrium condition that $\mu(x(\theta)) = y(\theta)$ indicates that $y(\theta)$ is implicitly defined by $V_y(x(\theta), y(\theta), y(\theta), \theta) = 0$. Since V_y is differentiable in all arguments and $x(\cdot)$ is differentiable by assumption, the derivative of $y(\cdot)$ exists and is given by $-[V_{yx}x'(\theta) + V_{y\theta}]/[V_{yy} + V_{yy'}]$ (the denominator is ensured to be non-zero by the regularity assumption that $h_{yy} + h_{yy'} \leq 0$. $\square$

B.3 Proof of Proposition 2

Proof. The proof demonstrates that the first-order conditions are sufficient for a maximum by showing that the objective function is globally concave when evaluated using a candidate return function, $\mu(x)$. That is, I prove that $v(x, y) \equiv (1 - \alpha) \cdot f(x) + \alpha \cdot h(y, \mu(x))$ is concave. I need to show that $v_{xx} \leq 0$, $v_{yy} \leq 0$, and $v_{xx}v_{yy} \geq v_{xy}^2$. These are given by:

$$v_{xx} = (1 - \alpha) \cdot f_{xx} + \alpha \cdot [h_{y'}\mu_{xx} + h_{y'y'}(\mu_x)^2] \leq 0 \tag{32}$$

$$v_{yy} = \alpha \cdot h_{yy} \leq 0 \tag{33}$$

$$v_{xx}v_{yy} - v_{xy}^2 = v_{xx}v_{yy} - (\alpha \cdot h_{yy'}\mu_x)^2 \geq 0, \tag{34}$$

for arbitrary values of $\{x, y\}$. It is immediate that (33) is satisfied. To make progress with (32), note that we can determine $h_{y'}\mu_{xx}$ as follows:

$$\begin{aligned} h_{y'}\mu_{xx} &= h_{y'} \cdot \frac{\partial}{\partial x}\{\Gamma(\mu(x), x)\} = h_{y'}[\Gamma_\mu \mu_x + \Gamma_x] \\ &= \left[\frac{1-\alpha}{\alpha}\frac{f_x}{q'(\mu)}q''(\mu)\right] \cdot \mu_x - \frac{1-\alpha}{\alpha} \cdot f_{xx}. \end{aligned}$$

Once substituted into (32), the condition becomes:

$$v_{xx} = \alpha \cdot \left[\frac{1-\alpha}{\alpha}\frac{f_x}{q'(\mu)}q''(\mu) \cdot \mu_x + h_{y'y'}(\mu_x)^2\right],$$

which is non-positive, since $q' > 0$, $q'' \leq 0$, $\mu_x \geq 0$, and $h_{y'y'} \leq 0$.

Turning to (34), after expanding and simplifying we end up with:

$$\begin{aligned} v_{xx}v_{yy} - v_{xy}^2 = \alpha^2 \cdot h_{yy} \cdot \mu_x \cdot \left[\frac{1-\alpha}{\alpha}f_x\frac{q''}{q'}\right] \\ + (\alpha \cdot \mu_x)^2 \cdot [h_{yy}h_{y'y'} - h_{yy'}^2], \end{aligned}$$

which is non-negative since it is the sum of two non-negative terms (the latter is non-negative since h is concave), and therefore (34) is also satisfied. I conclude that v is a concave function. Since $-c(x+y,\theta)$ is a also a concave function, the objective function is concave, and the first-order conditions are sufficient for a global maximum. □

B.4 Proof of Proposition 4

Proof. Consider the problem:

$$\max_{x,y}\{F(x,y)\} \quad \text{subject to } x+y \leq T, \tag{35}$$

where F is such that the 'y' solution is strictly positive: $y(T) > 0$. Let $F(T)$ be the associated maximum value function, and consider the problem:

$$\max_{T}\{F(T) - c(T,\theta)\}. \tag{36}$$

Consider two different functions, F and $\hat{F}$. The associated solutions, T^* and $\hat{T}^*$ will satisfy $T^* > \hat{T}^*$ if it happened to be the case that $F'(T) > \hat{F}'(T)$ for all T. Since T only enters the constraint in the original problem, the envelope theorem can be used to show that this holds if

$$F_y(x(T), y(T)) > \hat{F}_y(\hat{x}(T), \hat{y}(T)). \tag{37}$$

If we let F be the objective function facing the social planner, and $\hat{F}$ be the objective function facing an agent in equilibrium, then $F_y = \alpha \cdot q'(y(T))$ and $\hat{F}_y = \alpha \cdot (1-\phi) q'(\hat{y}(T))$. If q is linear, then $q'(y(T)) = q'(\hat{y}(T))$ and $\hat{F}_y(z')/F_y(z) = (1-\phi) < 1$ for any (z, z'), implying that $T^* > T$. By interpreting $\hat{F}$ as the objective function facing a Nash investor, the same expressions apply (since the marginal return to parental investment is the same in the Nash and equilibrium settings). This observation implies both i) that total efficient investment is strictly greater than total Nash investment, and ii) total Nash investment equals total equilibrium investment. □

C Derivation: Investments with Noise

The following uses the fact that if $x \sim N(y,a)$ and $y \sim N(b,d)$, then $y \mid x \sim N(\lambda x + (1-\lambda)b, v)$, where $\lambda \equiv d/(d+a)$ and $v \equiv ad/(a+d)$.

Since

$$\ln y_i = \ln \beta_y + \gamma \ln \theta_i = \ln \beta_y + \gamma \ln \overline{\theta} + \gamma \varepsilon_i^{\theta},$$

the prior is given by

$$\ln y \sim N(\ln \beta_y + \gamma \ln \overline{\theta}, \gamma^2 \sigma_\theta^2). \tag{38}$$

The structure of the noise implies that:

$$\ln \tilde{y} \sim N(\ln y, \sigma_y^2). \tag{39}$$

However, note that we also have:

$$\ln x_i = \ln \beta_x + \gamma \ln \theta_i = \ln(\beta_x/\beta_y) + \ln y_i.$$

By adding noise to this, we have:

$$\ln \tilde{x} + \ln(\beta_y/\beta_x) = \ln y_i + \varepsilon_i^x.$$

Therefore, we also have:

$$\ln \tilde{x} + \ln(\beta_y/\beta_x) \sim N(\ln y, \sigma_x^2). \tag{40}$$

Updating the prior (eqn (38)) with the information contained in the signal of parental investment (eqn (39)) gives:

$$\ln y \,|\tilde{y} \;\sim N\left(\lambda \cdot \ln \tilde{y} + (1-\lambda) \cdot A_1, \sigma_1^2\right), \tag{41}$$

where $A_1 \equiv [\ln \beta_y + \gamma \ln \overline{\theta}]$ is a constant and

$$\lambda \equiv \frac{\gamma^2 \sigma_\theta^2}{\gamma^2 \sigma_\theta^2 + \sigma_y^2} \qquad \text{and} \qquad \sigma_1^2 \equiv \frac{\gamma^2 \sigma_\theta^2 \sigma_y^2}{\gamma^2 \sigma_\theta^2 + \sigma_y^2}. \tag{42}$$

This posterior forms the new prior when using the information contained in the signal of wealth (eqn (40)). Using the same results, we have:

$$\ln y \,|\tilde{y}, \tilde{x} \;\sim N\left(\lambda' \cdot \ln \tilde{x} + (1-\lambda')\lambda \cdot \ln \tilde{y} + A_2, \sigma_2^2\right), \tag{43}$$

where $A_2 \equiv \lambda' \ln(\beta_y/\beta_x) + (1-\lambda')(1-\lambda) \cdot A_1$ is a constant and

$$\lambda' \equiv \frac{\sigma_1^2}{\sigma_1^2 + \sigma_x^2} \qquad \text{and} \qquad \sigma_2^2 \equiv \frac{\sigma_1^2 \sigma_x^2}{\sigma_1^2 + \sigma_x^2}. \tag{44}$$

Then, it follows that:

$$\mathbb{E}[\ln y \mid \tilde{x}, \tilde{y}] = \left[\frac{\sigma_1^2}{\sigma_1^2 + \sigma_x^2}\right] \cdot \ln \tilde{x} + \left[\frac{\sigma_x^2}{\sigma_1^2 + \sigma_x^2} \frac{\gamma^2 \sigma_\theta^2}{\gamma^2 \sigma_\theta^2 + \sigma_y^2}\right] \cdot \ln \tilde{y} + \text{constants}, \tag{45}$$

where, after simplification, the two bracketed coefficients correspond to λ_x and λ_y given in the text.