Strategic Shirking in Bilateral Trade

by

Christoph Lülfesmann*
Simon Fraser University

August 2006

Summary

This paper considers a version of the standard holdup model, in which a buyer and a seller can undertake relationship specific investments before conducting their transaction. In this setting, we identify a novel reason for contractual inefficiency. An investing party (here, the seller) may shirk for strategic reasons, in particular, exert an effort so low that subsequent trade becomes inefficient. We first show that under a fixed-price contract which would otherwise be optimal and induce trade, strategic shirking can arise irrespective of the precontracted trade price. We then establish that if strategic shirking arises under a fixed-price contract, no general mechanism exists which restores efficient trade. Finally, we show that the defection issue is more severe when the parties trade after the buyer’s valuation was realized, as compared to a scenario where the trade transaction is finalized in a state of uncertainty.

Keywords: Bilateral Trade, Hold-Up, Specific Investments, Shirking.
JEL-Classification: D23, H57, L51.

*Department of Economics, Simon Fraser University, 8888 University Drive, Burnaby, BC V5A 1S6, Canada. Email: cluelfes@sfu.ca. Phone: +1-604-291 5813. I thank several seminar audiences for their helpful comments and suggestions. All remaining errors are my own. Financial support by SSHRC is gratefully acknowledged.
1 Introduction

The literature on incomplete-contracting often studies a scenario where a buyer and a seller want to trade some good at some future date, and where they can expend noncontractible investments into their relationship (Grossman and Hart, 1986; Hart and Moore, 1988). While renegotiation always ensures an efficient trade outcome in this framework, implementing efficient investments can be a difficult task. A solution to this problem depends on the characteristics of investments, specifically, whether they are ‘selfish’ or ‘cooperative’ in nature. As shown by Edlin and Reichelstein (1996), a simple fixed-price contract generates an efficient outcome when investments are selfish, that is, when they do not inflict a direct externality on the trade partner.¹

This positive result does not extend to situations where investments entail a cooperative element, for example, in cases where a seller’s investment raises the buyer’s valuation for the good. For the polar case where investments are completely cooperative, Maskin and Moore (1999) and Che and Hausch (1999) have found that buyer and seller should not sign any long-term arrangement and rely on spot contracting instead. The result is thus the same as in the classic hold up literature which does not admit long-term contracts (Grout, 1984; Grossman and Hart, 1986; Williamson, 1985): in fear of exploitation by their partner, agents underinvest into their relationship. Che and Hausch also explore the intermediate scenario of hybrid

¹This is always true if only one party expends relationship-specific investments, and if ex-post bargaining results in some linear sharing rule (for example, the Nash-bargaining solution). Chung (1991), Denski and Sappington (1991), Aghion, Dewatripont and Rey (1994), Hermalin and Katz (1993) and Nöldeke and Schmidt (1995) have shown that option contracts or specific-performance contracts attain the first best even for two-sided investments if one party can be assigned the entire bargaining power. For monotonic sharing rules, Edlin and Reichelstein (1996) also attain the first best if the cost and valuation functions exhibit certain separability properties.
investments which affect both valuation and production costs. While the first best usually cannot be achieved in these circumstances, implementing the second-best effort requires long-term contracting, provided investments have are primarily selfish effect.\footnote{Cooperative (sometimes called cross-investments) or hybrid investments have also been investigated in MacLeod and Malcomson (1993), Aghion and Tirole (1994), Schmitz (2002), Roider (2004) and Lülfesmann (2005), among others.}

The present paper identifies a novel reason why contracting may be useless. Specifically, I show that a partner in a relationship may have an interest to destroy the benefits of trade under any long term contract for strategic reasons, by investing an amount so low that subsequent trade becomes inefficient. This is true even in situations where investments are primarily selfish in nature, and where in absence of this defection motive, an initial arrangement would make the trade relationship successful. To attain these results, we set up a natural variant of the standard bilateral trade model in which subsequent trade is efficient ex post only if the relationship specific investments are sufficiently large. This is a scenario often relevant in reality: for example, a buyer may refrain from ordering a custom made item if she is not satisfied with the design quality proposed by the seller. Similarly, a joint project may be stopped prematurely if one party (the buyer) did not work hard enough to identify cost reducing measures at the planning stage. These issues are often of great importance in defence procurement where the development of an advanced weapon system requires a substantial amount of investments in form of human and physical capital (Kovacic, 1991; Rogerson, 1994). If the defense contractor’s investment is insufficient in terms of quantity or quality, the government may not be willing to procure the system after the design phase.\footnote{An interesting case in this respect is the procurement of the U.S. Navy’s A-12 aircraft (see Washington Post, 1995). At the beginning of 1988, a team composed of McDonnell-Douglas and General Dynamics was awarded the 4.8 billion USD fixed-price contract for the development and
research, pharmaceutical companies frequently terminate projects with a Biotech
research firm after initial tests do not turn out to be sufficiently promising to war-
rant a completion of the project (Lerner and Merges, 1998; Schmidt, 2003). The
paper shows that in these and similar situations where non-contractible investments
are important, agents may shirk for strategic reasons even under the best available
contract. As a consequence, agents may deliberately prevent subsequent trade by
undertaking insufficient investment.

To address these issues, we study a simplified framework where trade is indivisible,
and where only the seller invests. The main focus is on the case where investments
are primarily selfish, i.e., where the dominant effect of investments is a reduction in
production costs. In a baseline scenario where subsequent trade is efficient regardless
of the investment level, the optimal ex-ante arrangement is shown to be a fixed-price
contract and the seller expends some second-best investment, say $e^*$. Thereafter,
we explore a slightly modified scenario in which a positive minimum investment is
needed to make subsequent trade efficient ex post.\textsuperscript{4}

If this threshold investment is smaller than $e^*$, it should not adversely affect the
outcome of the bilateral relationship. In particular, one might again expect a fixed-
price contract to implement $e^*$ and to induce trade in equilibrium. Unfortunately,
however, we show that this will often not be the case. Under a fixed-price con-
tact, the seller may now have an incentive to shirk strategically, in the sense that

\textsuperscript{4}Technically, this implies that the expected surplus from the relationship is not globally concave
in investments even if the valuation and cost functions are well behaved. See Section 2 below.
she voluntarily undertakes an investment strictly below the threshold level. The subsequent trade transaction then becomes inefficient and rational agents will implement a no-trade outcome. The occurrence of shirking is first demonstrated for a fixed-price contract. More generally, we then prove that if strategic shirking occurs under a fixed-price contract, there exists no general mechanism which lets trade be an equilibrium outcome. Hence, there may be no way to make subsequent trade valuable and the parties should not start their relationship from the outset.

To obtain an intuitive understanding, consider a deterministic setting and a fixed price contract under which the partners agree to trade the good at a preset price. If the seller’s cost reducing and quality enhancing investment turns out to be insufficient, this contract will be renegotiated to prevent the good from being inefficiently traded ex post. Starting from the default payoffs, the associated renegotiation gain is positive and, moreover, strictly decreasing in the seller’s investments. On the other hand, the seller’s pre-renegotiation (default) payoff from precontracted trade is clearly increasing in investments. The important point to see is that the combination of both effects may yield a local optimum at the investment stage in which the seller’s investment is positive but strictly smaller than the threshold level required for ex-post efficient trade. Moreover, the corresponding payoff may well exceed the payoff from the alternative strategy (the other local optimum) of not shirking and exerting the second-best investment $c^*$ under which no renegotiation arises and the trade transaction is completed successfully. Note that the outcome is completely irresponsible to changes in the contractual trade price. This price enters both alternative payoffs in exactly the same way and as an implication, it does not affect the seller’s relative payoff comparison. Shirking can thus occur under any fixed-price contract, provided the seller’s share of the renegotiation surplus is sufficiently large.
Under these conditions, the good is not traded in equilibrium and the relationship has no value.\footnote{Schmitz (2002) investigates a similar indivisible trade model in which the seller invests into the buyer’s valuation of the good, and where this valuation is private information. He shows that the interplay of moral hazard and hidden action renders the first best infeasible even if the trading problem is trivial, i.e., if ex-post trade is always efficient regardless of investments.}

The paper analyzes two variants of this basic setting, distinguished by the timing of the final trade decision. While in a first variant this decision is taken in knowledge of the buyer’s valuation, there is uncertainty in the second variant. We find that strategic defection which prevents subsequent trade can occur in both settings. However, the shirking problem is more severe under certainty as it arises over a significantly broader parameter range. This result has an intuitive explanation: when trading under uncertainty, shirking shuts down the transaction even in states where trade would actually be valuable, leading to a negative ‘renegotiation gain’ and diminishing the incentive to shirk. Interestingly, the trade partners may thus want to commit themselves to finalize the transaction in a situation of uncertainty, in order to mitigate strategic defection. This is in contrast to the standard setting, where trading under certainty is always efficient because it avoids ex-post inefficiencies.

The paper proceeds as follows. Section 2 sets up the model. In Section 3, trade takes place before the buyer learns her valuation. Section 4 considers trade under certainty and compares both scenarios. Section 5 briefly concludes.

\section{The Model}

Consider two risk-neutral parties, the seller $S$ and the buyer $B$, who can trade one unit of an indivisible good.\footnote{We confine attention to binary trade because it simplifies analysis and exposition considerably. As will become clear, though, the logic of our arguments immediately extends to settings with...} Prior to production, the seller expends relationship-
specific investments \( e \in \mathbb{R}_0^+ \). These investments have two effects: they decrease the seller’s production costs \( c(e) \), and raise the buyer’s valuation \( v(e, s) \) of the commodity. The stochastic variable \( s \in [s, \bar{s}] \) represents a state of the world that is distributed according to a continuous distribution function \( F(s) \) and realized after the seller expended his idiosyncratic investment, henceforth often called effort.\(^7\) Let \( t \) be a monetary transfer from \( B \) to \( S \) and \( q \in \{0, 1\} \) be an indicator variable for the trade decision. The parties’ utilities are

\[
U^S = t - qc(e) - \psi(e) \quad \text{and} \quad U^B = qv(e, s) - t, \tag{1}
\]

where the function \( \psi(e) \) represents the seller’s costs of exerting effort. Denote by \( V(e) = E_s v(e, s) \) the buyer’s expected valuation for given investments. We assume that \( v(e, s), c(e) \) and \( \psi(e) \) are twice continuously differentiable in \( e \). Also, \( v(e, s) \) is increasing in \( s \) and concave in investments, and \( c(e) \) and \( \psi(e) \) are convex in investments.\(^8\) The effort level \( e \) and the state of the world \( s \) are perfectly observed by either party, but non-verifiable. An enforcing party can, however, monitor whether trade occurs and whether \( S \) delivers the good (this assumption renders fixed-price contracts as well as revelation mechanisms feasible).

In what follows, two different scenarios will be explored. In Section 3, we assume that the parties have to take their trade decision under conditions of uncertainty, i.e., \emph{before} the state of the world \( s \) is revealed (scenario \( UC \)). This would reflect a situation where the commodity is an ‘experience good’ whose actual valuation is learned only after the purchase.\(^9\) Then, the maximal joint surplus ex post is defined

\(^7\)All subsequent results qualitatively apply for stochastic costs as well.

\(^8\)To ensure interior solutions, we additionally require that either \( v(\cdot) \) or \( -c(\cdot) \) are strictly concave in \( e \), or that alternatively \( \psi(\cdot) \) is strictly convex in \( e \).

\(^9\)The model is also compatible with a scenario where \( B \)'s valuation materializes only after he made an additional investment. For example, \( B \) could incur production costs if the item is an intermediary good, or marketing costs if he is a retailer.
by
\[ \phi^{uc}(e) = \max_{q \in \{0,1\}} q[V(e) - c(e)]. \tag{2} \]

Since trade is ex-post efficient whenever \( V(e) \geq c(e) \), we can define
\[ \bar{e} \equiv \inf \{ e \mid V(e) - c(e) \geq 0 \} \tag{3} \]

as the (unique) threshold investment below which the joint surplus from trade - gross of investment costs - is negative.

Similarly, define \( S^{uc}(e) = \phi^{uc}(e) - \psi(e) \) as the expected overall joint surplus from the relationship for given \( e \). Under our previous assumptions, \( S^{uc}(e) \) is strictly concave in \( e \) if \( \bar{e} = 0 \), i.e., if trade is ex-post efficient even if the seller does not undertake any investment. Conversely, for \( \bar{e} > 0 \) so that some minimum investment is required to render trade viable, \( S^{uc}(e) \) cannot be globally concave: for any \( e < \bar{e} \), \( \phi^{uc}(\cdot) = 0 \) so that \( S^{uc}(\cdot) \) is decreasing in \( e \), while \( S^{uc}(\cdot) > 0 \) for a nonempty range of investments \( e \geq \bar{e} \). We should note that in this latter case, the global-concavity assumption in Che and Hausch (1999) is violated (Assumption 4 in their paper).

In Section 4 below, we will then study a setting where the state of the world \( s \) has already been revealed at the date of trade, and the agents finalize their transaction under certainty (scenario \( C \)). In this case, the maximum joint surplus for given investments \( e \) and a given state \( s \) is again unique and defined as
\[ \phi^c(e, s) = \max_{q \in \{0,1\}} q[v(e, s) - c(e)]. \tag{4} \]

Trade should be realized whenever \( v(e, s) \geq c(e) \), while the partners should abolish the transaction otherwise. Let
\[ \bar{e} \equiv \inf \{ e \mid v(e, \bar{s}) - c(e) \geq 0 \} \tag{5} \]
be the threshold investment above which trade is ex-post efficient even in the least favorable state of the world, $s$. Notice that $\tilde{e} \geq \bar{e}$ because, at an investment level $\tilde{e}$ where the expected surplus from trade breaks even, the actual surplus is still negative in state $s$. The expected net surplus in this setting is $S^c(e) = \int_s \phi^c(e, s) dF(s) - \psi(e)$ and again strictly concave if $\tilde{e} = 0$. In contrast, for $\tilde{e} > 0$, trade may be inefficient for some nonempty subset of states. Since higher investments increase the likelihood of efficient trade for any $e < \tilde{e}$, global concavity of $S^c(e)$ is then not guaranteed even if the functions $v(\cdot), c(\cdot)$ and $\psi(\cdot)$ are well behaved.

To make both variants comparable, we will assume that trade in scenario $c$ is efficient in every state $s$, provided the seller undertakes first-best investments.\footnote{In other words, it is not optimal to sacrifice trade in a subset of states of the world in order to economize on effort costs. Technically, this assumption is satisfied if $e^{FB}$ as defined in (6) below exceeds $\tilde{e}$.} This is convenient because the first-best investments $e^{FB}$ then coincide in both scenarios, and are implicitly determined by the first-order condition (subscripts denote derivatives)

$$
\phi^c_e(e^{FB}) = \phi^c_{e}(e^{FB}) = V_e(e^{FB}) - c_e(e^{FB}) = \psi_e(e^{FB}).
$$

The sequence of events is as follows: at date 0, the parties can write an initial contract. While the exposition mostly focuses on non-contingent fixed-price contracts, general revelation mechanisms will be analyzed as well. Subsequently at date 1, $S$ can undertake his investment $e$. At the end of the game, the parties can trade at date 4 either before (scenario UC) or after (scenario C) the state of the world $s$ is revealed. In either case, $B$ and $S$ can play a revelation game at date 2 and revise their initial contract at date 3 before the final trade decision is taken. For concreteness, the outcome of renegotiation is described by the generalized Nash-bargaining solution, where the seller’s bargaining power is parameterized as $\gamma \in [0, 1]$. If the
parties finally agree on trade, $S$ produces the good and delivers it to the buyer; payoffs are then realized at date 4. Figure 1 below illustrates the timing.

![Figure 1: Sequence of Events](image)

We will say that $S$’s investments are *purely selfish* whenever $c_e(.) < 0$ and $v_e(.) = 0$, whereas investments are *purely cooperative* when $c_e = 0$ and $v_e(.) > 0$ for all $e$. Also, define

$$\tilde{\gamma} \equiv \sup \{ \gamma | \gamma v_e(e, s) + (1 - \gamma) c_e(e) \leq 0 \forall e, s \}$$

and

$$\hat{\gamma} \equiv \inf \{ \gamma | \gamma v_e(e, s) + (1 - \gamma) c_e(e) \geq 0 \forall e, s \}$$

as parameters which indicate the degree of cooperativeness.\footnote{The parameter $\hat{\gamma}$ corresponds to the measure $\alpha$ in Che and Hausch (1999).} Both measures are increasing in the degree of selfishness, and they assume a value of one if investments are purely selfish. Likewise, the measures decrease as investments become more cooperative, and are zero for purely cooperative investments. Note that, for a given tuple $(e, s)$, $\gamma v_e(e, s) + (1 - \gamma) c_e(e)$ is an increasing function of $\gamma$. Since $\tilde{\gamma}$ (respectively, $\hat{\gamma}$) represents the largest (smallest) bargaining parameter for which this expression is non-positive (non-negative) for arbitrary $(e, s)$, we must have $\tilde{\gamma} \geq \hat{\gamma}$.\footnote{The measures $\tilde{\gamma}$ and $\hat{\gamma}$ coincide if $v_e$ is independent of $s$, i.e., if the valuation function is either deterministic or separable between $e$ and $s$.} In what
follows, we will say that investments are *primarily selfish* if $\gamma \leq \hat{\gamma}$, while they are *primarily cooperative* if $\gamma \geq \hat{\gamma}$.\(^{13}\)

As stated earlier, we are concerned with situations where the seller can trigger a no-trade outcome by choosing a sufficiently small effort. In order to cast our setting into the existing literature, however, consider first a situation where $\bar{e} = \tilde{e} = 0$. Then, trade is always ex-post efficient irrespective of $e$ (and irrespective of $s$ in scenario $C$), and the surplus functions $S^{uc}(e)$ and $S^{c}(e)$ are strictly concave. We can now state the following Proposition which builds on existing results in the literature, most importantly, Edlin and Reichelstein (1996) and Che and Hausch (1999).\(^{14}\)

**Proposition 1.** Suppose $\bar{e} = \tilde{e} = 0$. In each scenario $UC$ and $C$, respectively, the first-best effort level $e^{FB}$ can be implemented if investments are purely selfish or if $\gamma = 1$. Otherwise, the seller underinvests under the optimal initial contract. More specifically,

1. if investments are primarily selfish ($\gamma \leq \hat{\gamma}$), a fixed-price contract implements the maximum attainable effort level $e^{*}$, which is implicitly determined by $-c_e(e^{*}) = \psi_e(e^{*})$;

2. if investments are primarily cooperative ($\gamma \geq \hat{\gamma}$), maximal investments prevail when the parties sign no initial contract, and bargain over trade only after the investment stage. The second-best effort level $e^{**}$ is then implicitly defined by

\(^{13}\)To straighten the exposition, we will disregard the intermediate range in between $\hat{\gamma}$ and $\tilde{\gamma}$ in what follows. In this range, investments are primarily selfish for a subset of states, and primarily cooperative for the complementary subset.

\(^{14}\)Note that in contrast to these papers which analyze variable quantities, we assume a binary trade decision. The only significant difference in results arises for the case of primarily selfish investments. The first best might then be achievable in the variable-quantity framework while it is unachievable in the current setting. Intuitively, a fixed-price contract can in the variable-trade setting prescribe a quantity in excess of the expected efficient quantity. While renegotiation ensures that the ex-post efficient quantity is actually traded, a large default quantity stimulates investment incentives and can even lead to overinvestments as long as investments are primarily selfish.
\[ \gamma [V_e(e^{**}) - c_e(e^{**})] = \psi_c(e^{**}). \]

**Proof:** see the Appendix.

The Proposition shows that if investments are primarily selfish, the optimal ex-ante arrangement is a simple fixed-price contract which obliges the seller to deliver the good in exchange for a monetary transfer \( t \). Under a fixed-price contract, the seller’s utility function becomes \( U^S = t - c(e) - \psi(e) \). Hence, \( S \) reaps the full marginal return from her investments in cost reduction, while \( B \) appropriates the increase in the valuation of the good. Because of this externality, the equilibrium investments will coincide with the first-best level \( e^{FB} \) when they are purely selfish, while the seller will underinvest whenever her investment is cooperative to some degree. Conversely, when investments are primarily cooperative, equilibrium effort is at its maximum when the parties do not sign any initial agreement, and simply rely on bargaining and spot contracting after effort has been expended. The seller then obtains a fraction \( \gamma \) of the overall return from the relationship, which leads her to invest some positive amount even if investments are purely cooperative.\(^{15}\)

For the purpose of this paper, we are interested in the case where investments are primarily selfish, \( \gamma \leq \tilde{\gamma}. \)\(^{16}\) For \( \bar{e} = \hat{e} = 0 \), we found that a fixed-price contract \( t \) is optimal and implements the effort level \( e^{*} \) which equates the marginal reduction in production costs, and the marginal costs of effort. Throughout, we assume that \( e^{*} \) is large enough to satisfy \( S^i(e^{*}) > 0, i \in \{uc, c\} \). Hence, the relationship has value and should be started from an ex-ante point of view if the parties succeed in implementing \( e^{*}. \)\(^{17}\)

---

\(^{15}\)In an intermediate interval \( \gamma \in (\bar{\gamma}, \hat{\gamma}) \), more general contracts may be second-best optimal. See Che and Hausch (1999).

\(^{16}\)When investments are primarily cooperative, the best initial contract is no contract and as will become clear below, strategic defection has no value for the seller.

\(^{17}\)We will also consider situations where \( S^i(e^{**}) > 0, i \in \{uc, c\} \), i.e., where the relationship has
3 Investment Threshold and Strategic Shirking

Imagine that the state $s$ materializes only after the buyer acquired the good, so that trade is completed under uncertainty ($UC$). As a consequence, the parties can base their contract renegotiation and trade decisions only on the observed investment level $e$. In addition, suppose that $\bar{e} > 0$, i.e., the parties should not trade when the seller’s investments are smaller than some positive threshold. We will find that the analysis of this scenario yields two interesting insights: first, a positive $\bar{e}$ may make it impossible to implement $e^*$ under fixed-price contracting, regardless of the initially contracted trade price. Second, if a fixed-price contract indeed fails to induce $e^*$, no general contract can implement an investment large enough to trigger subsequent trade, and the parties would do better not to start their relationship in the first place.

To begin with, consider again a deterministic fixed-price contract $t$. At first glance, one may think that the maximand of the seller’s optimization program remains $e^*$, because for any $e < \bar{e}( < e^*)$ the joint continuation surplus $\phi^{uc}(e)$ is zero. As we will find out now, however, the seller may rather have an interest to defect, that is, she may shirk and expend an effort level so small that subsequent trade becomes inefficient ex post. To see why this can happen for any arbitrary trade price, observe that the seller’s utility is now given by the piecewise defined function

$$U_S(e) = \begin{cases} t - c(e) - \psi(e) & \text{for } e \geq \bar{e} \\ t - c(e) + \gamma[0 - (V(e) - c(e))] - \psi(e) & \text{for } e < \bar{e}. \end{cases}$$

If $S$ undertakes $e < \bar{e}$ at stage 1, the parties will at date 3 renegotiate the initial contract to ensure a now efficient no-trade outcome. In this bargaining process, the value and trade is efficient even if only the effort level $e^{**}$ (refer Proposition 1) can be implemented.  

\textsuperscript{18}Note that the analysis in this scenario is the same as for a deterministic setting, where valuation and costs only depend on the seller’s investment.
seller receives a fraction $\gamma$ of the renegotiation gain, which is the difference between the (zero) joint surplus when no trade occurs, and the (negative) joint surplus when the initial contract is executed.

Notice that $e^*$ is the seller’s optimal choice from the subset of effort levels $e \geq \bar{e}$ which trigger trade. For the complementary set $e < \bar{e}$, the local maximizer is some defection effort $e^D \in [0, \bar{e})$. The second-best effort level $e^*$ then continues to be selected under a fixed-price contract if and only if

$$t - c(e^*) - \psi(e^*) \geq t - (1 - \gamma)c(e^D) - \gamma V(e^D) - \psi(e^D).$$

Whether (8) applies is unaffected by the size of the initially agreed trade price. As a consequence, potential shirking cannot be avoided by simply adjusting the terms of trade between both partners. An analysis of the above condition produces the following result.

**Proposition 2.** Consider a fixed-price contract, and suppose that $\bar{e} > 0$. If investments are purely selfish, the first-best effort level $e^{FB}$ is implemented. Suppose now that investments are hybrid but primarily selfish, and define

$$\gamma_{uc} = \frac{c(e^D) - c(e^*) - \psi(e^*) - \psi(e^D)}{c(e^D) - V(e^D)} (> 0).$$

Then, the second-best effort $e^*$ cannot be implemented if $\gamma_{uc} < \tilde{\gamma}$, and if the seller’s bargaining parameter is some $\gamma \in (\gamma_{uc}, \tilde{\gamma})$.

**Proof:** We first show that the first-best investment $e^{FB} = e^*$ are implementable if investments are purely selfish, i.e., $V(e) \equiv V \forall e$. To see this, notice that (8) holds iff

$$c(e^*) + \psi(e^*) < (1 - \gamma)c(e) + \gamma V(e) + \psi(e) \quad \forall e < \bar{e}.$$
Since $S^{uc}(e^*) > 0$ by assumption, we have $c(e^*) + \psi(e^*) < V < c(e)$ for all $e < \bar{e}$ and the result follows. Next, if investments are hybrid, (8) is violated if $\gamma > \gamma_{uc}[> 0].^{19}$ Hence, defective shirking under a fixed-price contract arises when the seller’s bargaining parameter $\gamma$ is (a) larger than $\gamma_{uc}$, and (b) smaller than $\tilde{\gamma}$, the requirement for primarily selfish investments. In the Appendix (and the end of the present Section), we provide an example which satisfies these conditions, so that a fixed-price contract cannot implement $e^*$ for a non-empty range of bargaining parameters $(\gamma_{uc}, \tilde{\gamma})$. $\square$

One might ask whether some contract which goes beyond a simple fixed-price contract is able to implement $e^*$, or at least an effort level large enough to support subsequent trade.$^{20}$ A negative answer is given in

**Proposition 3.** Suppose that investments are primarily selfish, i.e. $\gamma < \tilde{\gamma}$. If a fixed-price contract does not implement $e^*$, there exists no general mechanism which renders trade feasible.

**Proof:** By the revelation principle, we can restrict attention to a direct revelation mechanism which prescribes the (pre-renegotiation) contract terms as a function of both parties’ announcements on the investment level $e$. Define these announcements as $e_B$ and $e_S$, respectively. Then, the mechanism $\{\delta(e_B, e_S), t(e_B, e_S)\}$ specifies the probability of trade $\delta \in [0, 1]$ and the monetary transfer $t$ to the seller in dependence of the announcement profile. Let $e_1$ and $e_2$ be two different investment levels with $e_1 > \bar{e} > e_2$, and define the parties’ post-renegotiation utilities (gross of the seller’s investment costs) as $u^B(e)$ and $u^S(e)$, respectively. Due to

---

$^{19}$The threshold parameter $\gamma_{uc}$ is strictly positive because $c(e^*) + \psi(e^*) < c(e^D) + \psi(e^D)$ by definition of $e^*$, and $c(e) - V(e) > 0$ for all $e < \bar{e}$.

$^{20}$Clearly, implementable investment levels in presence of the additional non-shirking constraint can never exceed $e^*$. 

14
the efficiency of renegotiation and since trade is realized only if $e \geq \tilde{e}$, we have $u^S(e_1) + u^B(e_1) = V(e_1) - c(e_1)$ and $u^S(e_2) + u^B(e_2) = 0$. Suppose first that $S$ expended $e_2$. Then, incentive-compatibility of the direct mechanism requires that

$$u^S(e_2) \geq t(e_2, e_1) - \delta(e_2, e_1)[c(e_2) + \gamma(V(e_2) - c(e_2))].$$

(11)

Next, assume that $S$ expended $e_1$. Then, $B$ announces truthfully iff

$$u^B(e_1) \geq -t(e_2, e_1) + \delta(e_2, e_1) V(e_1) + (1 - \delta(e_2, e_1))(1 - \gamma)[V(e_1) - c(e_1)].$$

(12)

Since the possibility of renegotiation ensures that $u^S(e_1) + u^B(e_1) = V(e_1) - c(e_1)$, condition (12) translates into

$$u^S(e_1) \leq t(e_2, e_1) + \gamma(1 - \delta(e_2, e_1)) V(e_1) - [\delta(e_2, e_1) + (1 - \delta(e_2, e_1)\gamma)c(e_1)].$$

(13)

Combining (11) and (12) yields

$$u^S(e_1) - u^S(e_2) \leq \gamma(1 - \delta(e_2, e_1)) V(e_1) - [\delta(e_2, e_1) + (1 - \delta(e_2, e_1)\gamma)c(e_1)]$$

$$+ \delta(e_2, e_1) [(1 - \gamma)c(e_2) + \gamma(V(e_2))] \equiv \xi(e_1, e_2, \delta(\cdot)).$$

(14)

Fix $(e_1, e_2)$ and consider the derivative of $\xi(\cdot)$ with respect to $\delta$,

$$\frac{d\xi(\cdot)}{d\delta(\cdot)} = (1 - \gamma)[c(e_2) - c(e_1)] - \gamma[V(e_1) - V(e_2)].$$

(15)

This derivative is always positive if investments are primarily selfish, i.e., $\gamma < \tilde{\gamma}$. Accordingly, $\delta(\cdot) = 1$ implements the seller’s maximum payoff difference $\Delta u^S(e_1, e_2, \delta) \equiv \xi(e_1, e_2, \delta) - [\psi(e_1) - \psi(e_2)]$ between any two investment levels characterized by $e_1 > \tilde{e} > e_2$. Notice that $e^*$ can be implemented only if $\Delta(e^*, e_2, 1) \geq 0$ for all $e_2 < \tilde{e}$, and consider $e_2 = e^D < \tilde{e}$. Verify that the seller’s payoff difference $\Delta(e^*, e^D, 1)$ between $e^*$ and $e^D$ is then equivalent to the corresponding payoff difference under a fixed-price contract. Consequently, a revelation mechanism can
implement $e^*$ if and only if a fixed-price arrangement does. Finally, consider a situation where a fixed-price contract leads to defection, i.e., $\Delta(e^*, e^D, 1) < 0$. Since $e_1 = e^*$ maximizes $\Delta(e_1, e_2, 1) = [c_2 - c_1] + \gamma[V(e_2) - c(e_2)] - [\psi(e_1) - \psi(e_2)]$ for arbitrary $e_2$, we also have $0 > \Delta(e^*, e^D, 1) \geq \Delta(e_1, e^D, \delta)$ for all $e_1 \geq \bar{e}, \delta \in [0, 1]$, which completes the proof. □

If a fixed price contract fails to implement the second-best investment level due to strategic shirking, there exists no contract which ensures final trade to arise in equilibrium. A natural alternative candidate to consider would be the contract with a prescribed quantity level $q = 0$, or equivalently a relationship with no initial contract at all. An obvious advantage of this ‘contract’ is the absence of any strategic shirking incentive on the seller’s side, because she has nothing to gain from blocking subsequent trade. To make things interesting, suppose that the no-contract effort $e^{**}$ in the standard holdup framework (refer Proposition 1) satisfies $e^{**} > \bar{e}$.\textsuperscript{21} Even then, a trade outcome will remain infeasible because the seller invests $e^{**}$ only if this effort guarantees him a non-negative final payoff, $\gamma S^{nc}(e^{**}) - \psi(e^{**}) \geq 0$. As is easily seen from the proof of Proposition 3, this participation condition must be violated when a fixed-price contract fails to implement a trade outcome.\textsuperscript{22} Accordingly, the interplay between participation and shirking constraints is the key to understanding the above impossibility result.

To conclude, we illustrate the significance of strategic shirking in a numerical example (which is fully discussed in the Appendix). Consider functional forms

\textsuperscript{21}If not, the $q = 0$ contract will trivially not induce trade in the present setting.

\textsuperscript{22}To see this, consider a revelation mechanism that specifies $\delta(e_1, e_2) = 0$ for $(e_1 = e^{**}, e_2 = 0)$. The corresponding payoff difference $\Delta(e_1, e_2, 0) = \gamma[V(e^{**}) - c(e^{**})] - \psi(e^{**})$ is then identical to the seller’s payoff from choosing $e^{**}$ in absence of an initial contract. According to the proof of Proposition 3, this payoff difference must be strictly negative whenever $\Delta(e^*, e^D, 1) < 0$. Notice that participation constraints are irrelevant for implementation in the standard setting where $\bar{e} = 0$. \hfill 16
\[ V(e) = s + me, \ c(e) = f - ke \text{ and } \psi(e) = e^2 / 2. \] For \( k = 6, m = 4, \) investments are primarily selfish if \( \gamma < 6/10 = \hat{\gamma}. \) Also, suppose \( s = 60 \) and restrict attention to fixed costs \( f \in (60, 102]. \)\(^{23}\) We then obtain \( e^{FB} = 10, \ e^* = 6, \ e^D = 6 - 10\gamma, \ S^{FB} = 110 - f, \) and \( S^{uc}(e^*) = 102 - f. \)\(^{24}\) The second-best effort level \( e^* \) can not be implemented and trade fails whenever \( \gamma \in (\gamma_{uc}(f), \hat{\gamma}], \) where \( \gamma_{uc} = 12/5 - f/50. \) This defection range is indeed non-empty. If both parties have equal bargaining power, \( \gamma = 1/2, \) strategic shirking prevents trade for the fixed costs interval \( f \in [95, 102]. \) For \( \gamma \) sufficiently close to \( 6/10, \) trade already fails for any \( f \in (90, 102), \) that is, for about \( 1/4 \) of all permissible cost parameters. In contrast, strategic shirking does not arise if the seller’s bargaining strength is sufficiently small, in our example, for \( \gamma < 9/25. \) Intuitively, this is true because the seller’s bargaining gain when renegotiating to a no-trade outcome shrinks as \( \gamma \) decreases.\(^{25}\)

Overall, these figures highlight strategic shirking as a potentially serious hazard to long-term relationships in which some minimum effort is needed for bilateral trade.

## 4 Trade under Certainty

We will now show that strategic shirking may become an even more serious threat when the buyer’s valuation is already known at the date of trade, i.e., when the state \( s \) is realized prior to date 4. To analyze this scenario, define \( \hat{s}(e) = \sup \{ s \mid v(e, s) - c(e) \leq 0 \} \) as a threshold state so that trade becomes ex-post efficient for all \( s \geq \hat{s}(e). \)

\(^{23}\)These specifications ensure that \( c(e^{FB}) > 0, \) and \( S^{uc}(e^*) \geq 0. \)

\(^{24}\)Note that we require \( e^D < \bar{e} = (f - 60)/10, \) which is satisfied if \( f > 120 - 100\gamma. \) This constraint is never binding.

\(^{25}\)Suppose that \( S \) undertakes the no-contract investment \( e = e^{**} = 10\gamma \) that prevails when \( \bar{e} = 0. \) This effort level triggers trade and yields \( S^{uc}(e^{**}) = 75\gamma + 60 - f \) which is positive as long as \( f < 75\gamma + 60. \) However, the seller is interested in his own utility rather than joint surplus, and selects \( e^{**} \) only if \( \gamma [V(e^{**}) - c(e^{**})] - \psi(e^{**}) \geq 0. \) From our previous findings, this condition is violated for any \( \gamma-f \)-combinations which trigger strategic defection under a fixed-price contract.
Also, indicate $q(e) = 1 - F(\hat{s}(e))$ as the (equilibrium) trade probability for given investment level. Since our focus is on a situation where the agents trade with less than full (and possibly zero) ex-ante probability if the seller does not invest, let $\hat{e} > 0$ with the consequence $q(0) < 1$. Finally, to make the setting comparable to the analysis in Section 3 above, suppose that $S^*(e^*) > S^e(e)$ for all $e < e^*$, and that trade is always efficient if the second-best effort level $e^*$ is implemented. Hence, $e^*$ generates a larger joint surplus than any smaller investment, and $q(e^*) = 1$ so that trade is efficient in every state $s$ if $e^*$ can be implemented.\footnote{For $q(e^*) < 1$, the second-best effort level would be smaller than $e^*$ and implicitly determined by $-q(e)c_e(e) = \psi(e)$.}

In absence of a strategic defection motive, a fixed-price contract $t$ would again implements the second-best efficient effort $e^*$ if investments are primarily selfish. For subsequent reference, define $V^+(e) = \int_{s \geq \hat{s}(e)} v(e, s) dF(s)/q(e)$ as the buyer’s expected valuation for the commodity for given ex-post optimal trade decision and given $e$, and let $V^-(e) = \int_{s < \hat{s}(e)} v(e, s) dF(s)/(1 - q(e))$ be his expected valuation over the complementary set of states. Under a fixed-price contract, $S$ then chooses $e$ to maximize

$$U_S(e) = t - c(e) + [1 - q(e)]\gamma[c(e) - V^-(e)] - \psi(e),$$

where the expression in brackets again represents the gain from renegotiation if trade is ex-post inefficient, and $[1 - q(e)]$ denotes the likelihood of this event for given $e$. Since $q(0) < 1$, this payoff function is not necessarily concave.\footnote{Observe that $e^*$ is the local maximum in the range $e \geq \hat{e}$ and also constitutes the global optimum if $U_S(e)$ is strictly concave.}

\begin{equation}
U_S(e^*) = t - c(e^*) - \psi(e^*) \geq t - [1 - q(e)]\gamma[V^-(e) - c(e)] - \psi(e) \quad \forall \ e < \hat{e}.
\end{equation}
We will now show that condition (17) can be violated. In addition, we establish that the seller shirks and induces a breakdown of trade even in situations where she would not do so in scenario $UC$. To validate these claims, one can confine attention to a subset $e < \bar{e}$ of possible deviations: since $\bar{e} \geq \bar{e}$, the range of possible defections is (weakly) larger in $C$ than in $UC$. The seller’s expected utility for $e = e^*$ coincides in scenarios $UC$ and $C$ for given $t$. Now consider a deviation $e < \bar{e}$. Then, the seller’s payoff from shirking in scenario $C$ is larger than in scenario $UC$ if, by (7) and (16),

$$c(e) - V(e) < [1 - q(e)][c(e) - V^-(e)].$$

Recalling that $V(e) = (1 - q(e))V^-(e) + q(e)V^+(e)$, we can rewrite (18) as $q(e)V^+(e) > q(e)c(e)$, which holds for any $e < \bar{e}$ and $q(e) > 0$ by the definition of $V^+(\cdot)$. In terms of bargaining parameters and recalling that $e^D$ indicates $S$’s optimal shirking effort in $UC$, the seller will shirk in scenario $C$ at least if

$$\gamma > \gamma_c = \frac{[c(e^D) - c(e^*)] - [\psi(e^D) - \psi(e^*)]}{c(e^D) - V(e^D) + q(e^D)[V^+(e^D) - c(e^D)]}.$$  

Note that $\gamma_c \leq \gamma_{uc}$, and that the strict inequality $\gamma_c < \gamma_{uc}$ applies whenever $q(e^D) > 0$. Along the lines of the proof of Proposition 3, one can also show that no other more general mechanism can improve upon the outcome of fixed-price contracting. Accordingly, we can state

**Proposition 4.** Suppose that investments are primarily selfish, the agents trade under certainty, and trade is efficient in any state for the second-best investment $e^*$. Then, it is impossible to implement $e^*$ if the seller’s bargaining parameter is some $\gamma \in (\gamma_c, \tilde{\gamma})$ with $\gamma_c \leq \gamma_{uc}$ (and strict inequality at least if $q(e^D) > 0$). Hence, under certainty, strategic shirking occurs over a wider range of parameters.
There is an intuitive explanation behind these results. Suppose the seller shirks in scenario $UC$ and both parties renegotiate in a position of uncertainty about the true state $s$. A no-trade outcome then prevails in every state, including those where retaining the initial trade contract would have been beneficial for both agents. The seller thus incurs some (locally) negative bargaining surplus in renegotiations, which diminishes her shirking incentives. Clearly, no similar disincentive arises when trade and renegotiation takes place under certainty. As a consequence, shirking is more attractive in $C$, because the additional information in this regime encourages strategic defection.

The Proposition thus suggests that agents in a trade relationship may benefit from finalizing their transaction in a state of uncertainty. In fact, examples with this counterintuitive outcome can easily be constructed. However, one should be aware of a counterbalancing effect in favor of informed trading: shirking in scenario $C$ will usually not cause a total failure of the relationship, since trade is still carried out in good states.\footnote{Relatedly, we would expect the seller’s shirking effort in scenario $C$ to be larger, in anticipation of subsequent trade in some states.} Shirking, if it is to occur, will then be less detrimental when both partners trade in full knowledge of the buyer’s valuation.

It is instructive to reconsider the example set up in the previous Section. Let $q = \text{prob}\{s = \bar{s}\} = 1/2$ and for consistency reasons, $\bar{s} = 40$ so that for an average value $s = 60$, $\bar{s} = 80$. Given this specification, trade in the good state $\bar{s}$ is optimal for any $f \leq 100$ even if the agents exerts no effort. Conversely, a trade outcome in the low state is realized only if $e \geq \hat{e} = (f - 40)/10$. If $S$ decides to shirk, she chooses $e^D = \arg\max \{t - c(e^D) - q\gamma[v(e^D, \bar{s}) - c(e^D)] - \psi(e^D)\} = 6 - 5\gamma$ under the (nonbinding) constraint $e^D < \hat{e}$. Note that $\hat{e} < e^* = 6$ is satisfied for $f < 100$. The permissible range of fixed costs is then $f \in (60, 100)$, and the effort level $e^*$ followed
by trade in both states is indeed second-best optimal. We can now easily establish that the seller will shirk and choose $e^D$ whenever $f > 100 - 25\gamma$, which translates into $\gamma > \gamma_c = 4 - f/25$. For $\gamma = 1/2$, shirking arises if $f > 87.5$, encompassing about a third of the feasible cost range. Figure 2 below illustrates the shirking intervals for both alternative scenarios.

<table>
<thead>
<tr>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
</tr>
<tr>
<td>90-</td>
</tr>
<tr>
<td>85</td>
</tr>
<tr>
<td>80-</td>
</tr>
<tr>
<td>70-</td>
</tr>
<tr>
<td>60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 2: Example

5 Conclusion

This paper investigates a standard long-term bilateral trade relationship where the seller can undertake investments with a cost-decreasing and a value-enhancing effect. Our focus is on a natural scenario where subsequent trade becomes inefficient (or inefficient in certain states) when the seller’s investments are too low. In this setting, the concavity assumptions usually imposed in the literature do not apply and we find that ‘strategic shirking’ may become detrimental for the relationship. Specifically, the seller may - in anticipation of renegotiation - deliberately exert a very low effort.
to render subsequent trade inefficient. We show that this motive may make it impossible to implement the benchmark second-best effort level with no threshold investment. Moreover, if strategic shirking arises, it shuts down trade completely if the trade transaction takes place under uncertainty, or if it is deterministic. The relationship of the two trade partners then has no value and starting it wastes a positive amount of investment resources. While strategic defection may have less adverse consequences when trade arises in knowledge of the buyer’s valuation, shirking occurs more frequently in this latter scenario. Perhaps counterintuitively, trading under uncertainty may thus be preferable for the relationship. It is worth emphasizing that while our setting was one of bilateral trade, our arguments are likely to apply to other economic relationships as well. To give just one example, our findings may contribute towards an understanding of ‘golden handshakes’ in managerial compensation. Identifying and analyzing economic environments with the potential for strategic shirking must await future research.
6 Appendix

Proof of Proposition 1

By the revelation principle, we can restrict attention to a direct revelation mechanism which prescribes the (pre-renegotiation) contract terms as a function of both parties’ announcements on the state \( \theta \) where \( \theta = (e) \) in scenario \( UC \), and \( \theta = (e, s) \) in scenario \( C \). Define these announcements as \( \theta_B \) and \( \theta_S \), respectively. Then, the mechanism \( \{\delta(\theta_B, \theta_S), t(\theta_B, \theta_S)\} \) specifies the probability of trade \( \delta(\cdot) \in [0,1] \) and the monetary transfer \( t(\cdot) \) to the seller in dependence of the announcement profile. Define the parties’ post-renegotiation utilities (gross of the seller’s investment costs) as \( u_B(\theta) \) and \( u_S(\theta) \), respectively. Then, by the constant-sum nature of the game, we have \( u_S(\theta) + u_B(\theta) = B(\theta) - c(\theta) \). Finally, define \( \phi(\theta) = B(\theta) - c(e) \), where \( B(\theta) = V(\theta) \) in scenario \( UC \), and \( B(\theta) = v(\theta) \) in scenario \( C \). Consider now two states \( \theta, \theta' \), and suppose first that the true state is \( \theta' \). Then, incentive-compatibility of the direct mechanism requires that

\[
\begin{align*}
  u_S(\theta') &\geq t(\theta', \theta) - \delta(\theta', \theta)c(\theta') + (1 - \delta(\theta', \theta))\gamma\phi(\theta'). 
\end{align*}
\]

(20)

Next, suppose the true state is \( \theta \). Then, \( B \) announces truthfully iff

\[
\begin{align*}
  u_B(\theta) &\geq -t(\theta', \theta) + \delta(\theta', \theta)B(\theta) + (1 - \delta(\theta', \theta))(1 - \gamma)\phi(\theta).
\end{align*}
\]

(21)

Since the possibility of renegotiation ensures that \( u_S(\theta) = B(\theta) - c(\theta) - u_B(\theta) \), conditions (20) and (21) translate into

\[
\begin{align*}
  u_S(\theta) - u_S(\theta') &\leq \gamma(1 - \delta(\cdot))[\phi(\theta) - \phi(\theta')] - \delta(\cdot)[c(\theta) - c(\theta')]. 
\end{align*}
\]

(22)
Fixing $\theta' = (e', s)$ in $C$ (and $\theta' = e'$ in UC), this inequality implies that

$$\frac{\partial u^s(\theta)}{\partial e} = \limsup_{e' \to e} \frac{du^s(\theta) - u^s(\theta')}{e - e'} \leq \gamma \phi_e(\theta') - \liminf_{e' \to e} \delta(\cdot)[\gamma v_e(\theta) + (1 - \gamma)c_e(\theta)] \begin{cases} \leq \gamma \phi_e(\theta) & \text{if } \gamma \geq \hat{\gamma} \\ < \phi_e(\theta) & \text{if } \gamma < \hat{\gamma} \\ \leq -c_e(e) & \text{if } \gamma \leq \tilde{\gamma}. \end{cases}$$

(23)

Noting that $U^s_e(\theta) \leq u^s_e(\theta) - \psi_e(e)$ in UC and $U^s_e(\theta) \leq \int_s u^s_e(\theta)dF(s) - \psi_e(e)$ in scenario $C$, first-best investments are unfeasible unless investments are purely selfish or $\gamma = 1$. For $\gamma \geq \hat{\gamma}$, the expression in brackets in (23) is non-negative so that $\delta(\cdot) = 0$ implements the maximal marginal utility difference between two neighboring states. Accordingly, it is second-best optimal to sign no initial contract. Finally, for $\gamma < \hat{\gamma}$, the expression in brackets in (23) is non-positive. Thus, $\delta(\cdot) = 1$ implements maximal investments which coincide with those under a fixed-price contract. □

Example

Suppose the buyer’s valuation function is $v(e, s) = s + me$, the seller’s production costs are $c(e) = f - ke$, and her investment costs are $\psi(e) = e^2/2$. Let $m, f, k$ be non-negative parameters, and $s \in \{\bar{s}, \tilde{s}\}$ be a state of the world. State $\bar{s}$ is realized with probability $q \in (0, 1)$, and $\bar{s} > f > \tilde{s}$ implying that $\dot{e} = [f - \bar{s}]/(m + k) > 0$. Let $s =: q\bar{s} + (1 - q)\tilde{s} < f$ so that $\bar{e} = [f - s]/(m + k) > 0$, and notice that $\bar{e} > \bar{e}$ and $V(e) = s + me$. In this framework, $\tilde{\gamma} = \hat{\gamma} = k/(m + k)$. To simplify the analysis, we suppose that $\gamma = 1/2$ and, in order to focus on primarily selfish investments, assume $k > m$. If first-best and second-best investments are positive, these levels are $e^{FB} = m + k$ and $e^* = -c_e(e^*) = k$, respectively. To guarantee positive second-best investments, the following condition must hold:

$$S^{uc}(e^*) > 0 \iff f - s < \frac{1}{2}k^2 + mk. \quad (C1)$$

24
In UC, $S$ defects under a fixed-price contract $t$ if condition (8) is violated for some $e \in [0, \bar{e}]$. In case of defection, he chooses $e^D = k - [m + k]/2$ (for an interior solution; otherwise, defection cannot be optimal). Inserting $e^D$ and rearranging terms, defection thus arises if

$$f - s > k^2 - [e^D]^2 = \frac{3}{4}k^2 + \frac{1}{2}mk - \frac{1}{4}m^2. \quad (C2)$$

Combining conditions (C1) and (C2), we find that defection cannot be avoided and (by Proposition 3) the bilateral relationship has no value whenever

$$\frac{3}{4}k^2 + \frac{1}{2}mk - \frac{1}{4}m^2 < f - s < \frac{1}{2}k^2 + mk. \quad (24)$$

These conditions hold for a generic set of parameter values. In particular, as long as $k < m(1 + \sqrt{2})$, there exists a nonempty set of parameter differences $(f - s)$ for which (24) is satisfied. (Note that (24) cannot hold if investments are purely selfish, i.e., $m = 0$.)

Finally, we check that defection in UC (and thus in C as well) can arise if $e^*$ is the second-best optimal effort in scenario $C$. If $V(e^*) - c(e^*) - \psi(e^*) > q[v(\hat{e}, \bar{s}) - c(\hat{e})] - \psi(\hat{e})$ for $\hat{e}$ implicitly defined by $-qc(\hat{e}) = \psi(\hat{e})$ with $\hat{e} < \hat{e}$, $e^*$ is indeed second-best optimal in $C$. In our example, $\hat{e} = qk$ and we can rewrite the above condition as

$$f - s < \left[\frac{1}{2}k^2 + mk\right](1 - q^2) - q[\bar{s} - f]. \quad (C1')$$

As expected, (C1’) is more restrictive than condition (C1) for any $q \in (0, 1]$. For $q \to 0$, (C1’) is identical to (C1), while (C1’) cannot hold for $q \to 1$ because $\bar{s} < f$ when $\bar{e} > 0$. Since the right-hand side of (C1’) is monotonically decreasing in $q$, defection in UC is compatible with (C1’) for $q$ being sufficiently small.
References


