Wealth constraints and option contracts in models with sequential investments

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The paper investigates a model where two parties A and B invest sequentially in a joint project (an asset). Investments and the asset value are nonverifiable, and A is wealth constrained so that an initial outlay must be financed by either agent B (insider financing) or an external investor, a bank C (outsider financing). We show that an option contract in combination with a loan arrangement facilitates first best investments and any arbitrary distribution of surplus if renegotiation is infeasible. Moreover, the optimal strike price of the option is shown to differ across financing modes. If renegotiation is admitted, we identify conditions under which the first best can still be attained. Then, either B-financing or C-financing may be strictly preferable, and a combination of insider and outsider financing may be strictly optimal.

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1. Introduction

Parties in an economic relationship often invest sequentially to enhance the value of a joint project. In a research joint venture between a large pharmaceutical company and a specialized biotechnology firm, the research unit is responsible for the early-stage development of a new product, while the subsequent manufacturing and sales tasks are assigned to the downstream company. Similarly, in high tech start-ups, the creativity and devotion of a founder is decisive for the firm’s success in its early stages, while the skills of an experienced management team (often brought in by a venture capitalist) become crucial in later periods.\(^1\)

Typically, the investments of both partners are complex and hard to describe or contract upon. Therefore, the question arises of how to design a governance structure which assigns optimal investments incentives in an indirect manner. The literature has shown that - in contrast to straight or non-conditional ownership structures - option contracts may be best suited to attain this goal. Demski and Sappington (1991) were among the first to consider an agency model with sequential investments.\(^2\) They show that if agent B (who invests subsequently) holds an option to sell the asset to agent A (who invests first) after both parties invested, both parties can be led to exert efficient effort and the first best is attained. Nöldeke and Schmidt (1998) extend this result to a setting where the agents can renegotiate their initial arrangement. They find that an option-to-own contract which grants B the right to buy the asset after both parties invested implements the first best. Moreover, renegotiation does not arise on the equilibrium path, and the first best often remains feasible in a stochastic environment where the asset value is subject to uncertainty.\(^3\)

The present paper reconsiders this double moral hazard framework with sequential investments for a situation where agent A is wealth constrained. Then, two intertwined issues emerge that are well known from the corporate finance literature: First, who should finance an initial outlay that may be necessary to launch the relationship? Second, how can the surplus from the relationship be distributed among the parties involved? Both questions seem relevant in a variety of economic situations where sequential investments play a role. In biotechnology joint ventures, the small research firm often does not have the monetary endowment to finance the high-tech equipment necessary to conduct its research. Likewise, start-up firms often enter a relationship with a venture capital fund in order to obtain a seed financing. In these and other situations, it is important to ask whether wealth constraints interfere with incentive considerations, and which mode of financing generates optimal incentives. To address these questions, our analysis explores two basic financing modes. Under ‘insider financing’, partner B provides the necessary monetary resources. Alternatively, un-

\(^1\)For a recent survey on research alliances, see Hagedoorn et al. (2000). Sahlman (1990) provides a thorough assessment of venture capital financing.

\(^2\)See also Banerjee and Beggs (1989) who, however, focus on a specific production technology.

\(^3\)Edlin and Hermelin (2000) consider a framework where party A is risk-averse, and focus on option-to-buy contracts with an exercise date after the first agent - but before the second agent - invested. For the special case of risk-neutral parties, their results imply that efficiency can then be attained if and only if the parties’ investments are substitutive on the margin.
der ‘outsider financing’, A and B bring in a third party (a bank, say) to finance the initial outlay.

A general finding of the corporate finance literature is that wealth constraints may have an adverse impact on the outcome of economic relationships (below, we provide a brief overview). In the present setting, however, we find that financing constraints are in fact irrelevant if renegotiation can be prevented. Insider as well as outsider financing implements the first best, and the optimal initial contract is a two-part arrangement: it consists of (a) a loan given by either agent B or an outside party C, and (b) an option-to-buy contract between A and B. Specifically, the optimal arrangement under insider financing resembles a convertible debt contract under which B can choose whether to insist on a repayment after both parties invested after both parties invested, or to exercise her option instead.4 We also show that, under insider as well as outsider financing, any arbitrary division of the total surplus is compatible with the efficiency goal. Hence, each point along the Pareto frontier can be reached and distributional issues impose no constraint on the optimal solution.

Despite this congruence in results, we also find that the optimal strike price of B’s option crucially depends on the mode of financing. In case of an outside investor, the efficient option price makes agent B in equilibrium just indifferent between investing and subsequently exercising her option, or not to invest and not to exercise.5 Conversely, under insider financing, the optimal option price is ceteris paribus higher than under outsider financing. This result is counterintuitive at first glance because at a higher strike price B strictly prefers a debt repayment over investing and exercising her conversion option. This puzzle is resolved by observing that when agent A is wealth constrained, B can often not insist on a repayment without expending own effort, because the asset value would then be smaller than her repayment claim. Rather, B must undertake some positive threshold effort to enforce a debt repayment. This effort helps B to secure her claims but at the same time reduces her net payoff from reclaiming her debt relative to a situation where A has sufficient wealth to serve his debt obligations. At the strike price optimal under outsider financing, A would thus exercise even after A underinvested, and A would indeed defect. As a remedy, the strike price must be raised to a level where B is in equilibrium just indifferent between a repayment minus the accompanied default effort on the one hand, and efficient investments and debt conversion on the other.

4In our model, agent B acquires full ownership of the asset as is frequently observed in research joint ventures. According to Arora and Gambardella (1990), for example, many indicators suggest that biotechnology firms are often founded with the intent of later on being sold to a large corporation. In an empirical study on joint ventures, Bleeke and Ernst (1995) find that one partner buys out the other in almost 80% of their sample.

5The logic behind this result is the same as in the standard model without wealth constraints: the option price is chosen in a way that B invests and exercises only after observing efficient investments by agent A. When A invests efficiently, agent B’s net utility thus coincides with her reservation payoff whereas agent A appropriates the total net surplus (minus, in our model, the debt repayment he must make to agent C). Any deviation from optimal investments does not pay off for A because B then refuses to invest, and A is still obliged to serve the debt claim of agent C. For details see Section 3 below.
We then consider a scenario where renegotiation is admitted. Now, the financing constraint may have allocative consequences and preclude an efficient outcome of the relationship. In particular, $A$ may have an incentive to underinvest for strategic reasons in order to extract a larger portion of the surplus. Notably, this problem also appears when $A$ has all the bargaining power ex ante so that the contractual arrangement allows him to appropriate the entire surplus from the relationship. Intuitively, if $A$ underinvests, renegotiation becomes necessary in order to induce $B$ to expend the conditionally efficient effort level. If the initial outlay is positive and/or if the initial contract promises agent $B$ (or agent $C$) a large fraction of social surplus, a defection allows $A$ to default on these claims. Then, $A$ finds a defection indeed optimal if he is in a sufficiently strong bargaining position, and appropriates a large share of the bargaining surplus.

The efficiency of insider and outsider financing differ in a setting with renegotiation. In general, insider financing dominates when $A$'s bargaining power in renegotiations with $B$ and $C$ (which arises after a deviation under outsider financing) is not significantly smaller than in bilateral bargaining with only agent $B$ (which arises under insider financing). Otherwise, however, bank financing renders it easier to mitigate the underinvestment problem so that there may be an efficiency-improving role for third parties. Finally, we argue that a combination of multiple lenders ($B$ and $C$) may be optimal if the first best cannot be attained in a arrangement with a single financier. In particular, we find that multiple lenders combine the advantages of insider and outsider financing: while the lender’s default payoff under insider financing is larger (because $B$ will exert some effort in default) which reduces the renegotiation surplus for party $A$, outsider financing may reduce $A$’s strength in renegotiations because he as the shirking agent now faces two opponents.6

Our results contribute to earlier findings on the optimal governance and financial structure of a wealth-constrained firm. Among others, Bolton and Scharfstein (1990) and Hart and Moore (1995) consider models where the asset (the firm) exists for two periods, in both of which the firm owner can undertake noncontractible productive investments. At the end of each period, a non-verifiable cash flow is realized. If renegotiation is infeasible, the optimal debt contract gives the investor the right to liquidate (part of) the asset after the firm’s default on repayments after the first period. While this liquidation is inefficient, it reduces the firm’s incentives to default strategically. Berglöf and von Thadden (1994), Dewatripont and Tirole (1994) and Bolton and Scharfstein (1996) show that, if renegotiation is feasible, the optimal capital structure calls for a combination of long-term and short term investors with claims of different priority. In line with our results, the presence of multiple investors may reduce the firm’s anticipated renegotiation gain after a default, which renders it less attractive to defect on a repayment obligation on short-term debt. Overall, the present paper draws on these previous contributions, but it also differs from them in several respects. Most importantly, we assume that two parties rather than one have to expend non-contractible investments.

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6In an extension which was suggested by a referee, we also show that outside financing under an option contract with an early exercise date (immediately after $A$ invested) generally achieves efficiency as long as collusion between $B$ and $C$ can be prevented. See Section 4 below.
which makes it efficient to change ownership titles during the course of the relationship. Also, we consider a situation where the asset generates a cash flow only at the end of the game, implying that a debt repayment can happen only then and repayment decisions cannot impose a shutdown threat on the firm owner. Despite these differences, we also find that the optimal contractual arrangement may closely resemble financing schemes that are observed in reality.

Our paper is most closely related to the recent contributions by Schmidt (2003) and Lerner and Malmendier (2004). Schmidt (2003) explores a similar setting where a wealth constrained start-up entrepreneur and a venture capitalist undertake sequential investments. This paper deals exclusively with insider financing, and admits a verifiable asset value which renders shared-equity arrangements feasible. Abstracting from the possibility of strategic default, it is found that an option arrangement which allows the venture capitalist to convert debt into some prespecified equity fraction achieves the first best in a variety of situations. In Lerner and Malmendier (2004), a wealth-constrained biotech company provides the basic research for a pharmaceutical company who also finances the partnership. Their theoretical model focuses on the multi-tasking problem that stems from the disaligned incentives of both partners: in contrast to the pharmaceutical company, the biotech firm may want to generate publishable scientific output, or cross-subsidize different projects. The optimal contractual response to this incentive misalignment is an assignment of termination rights to the financing firm, in combination with the conditional allocation of ownership rights over forthcoming patents. The paper then reports empirical findings from a large sample of biotech alliances which seem well in line with the theoretical predictions.

The remainder of the paper is organized as follows. Section 2 introduces the framework. In Section 3, we analyze the model for the case where renegotiation is infeasible, while Section 4 considers renegotiation. Section 5 contains some brief concluding remarks.

2. The model

We consider a model with two risk-neutral agents $A$ and $B$ who start a relationship and sequentially invest into an asset. At date 0, the partners sign an initial contract, and a monetary seed investment $K \geq 0$ has to be incurred. At date 1, party $A$ can expend an idiosyncratic investment (which will be referred to as effort) $a \in \mathbb{R}_0^+$. At date 2, the initial contract may be renegotiated, before agent $B$ undertakes her own effort $b \in \mathbb{R}_0^+$ at date 3. Both investments are in physical capital so that the asset value neither depends on its final owner nor on the further engagement of either party. The asset value $v(a, b)$ materializes at date 4, and the game ends at date 5 where repayments are made, options may be exercised, and final payoffs are realized.

Figure 1 below illustrates the sequence of events.

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7In further contrast to our setting, Schmidt (2004) models the effort of the venture capitalist (agent $B$ in our model) as a binary rather than a continuous variable, and incorporates uncertainty. See also Lüllesmann (2001) who derives optimal contracts in models with sequential investments and a contractible asset value (but without financing considerations).
We make the following informational and contracting assumptions. Both parties have complete information throughout the game. Moreover, the monetary seed investment $K$ is contractible, while the idiosyncratic effort levels $a$ and $b$ are not. Likewise, the asset value $v(a, b)$ is non-verifiable to outsiders and thus cannot be contracted upon. In what follows, we also impose

**Assumption 1.** The function $v(\cdot)$ is twice continuously differentiable, strictly increasing in both arguments, strictly quasiconcave, and satisfies (subscripts denote derivatives)

\[ a) \quad v(a, b) > 0 \text{ for all } (a, b) > 0 \text{ and } v(0, 0) = 0. \]

\[ b) \quad v_{ii}(\cdot) < 0, \lim_{i \to 0} v_{i}(\cdot) = \infty \text{ and } \lim_{i \to \infty} v_{i}(\cdot) = 0 \text{ for } i \in \{a, b\}. \]

\[ c) \quad v_{ab}(\cdot) \geq 0 \text{ for all } (a, b). \]

According to part a), the joint project has a non-negative gross value for any feasible combination $(a, b)$. If neither party expends any effort, this value is normalized to zero. Part b) ensures that some positive but finite investment levels are optimal provided the project should be started. Finally, part c) states that investments are (weak) complements at the margin. We thus focus on the natural case where the return on, e.g., basic research is small if not combined with complementary skills such as production experience and marketing know-how, and vice versa.\(^8\)

For subsequent reference, we compute the first-best investments $(a^{FB}, b^{FB})$ which maximize the ex-ante surplus,

\[ S(a, b) = v(a, b) - a - b - K. \] (1)

Throughout, we suppose that the relationship should be started, i.e., $v(a^{FB}, b^{FB})$ is sufficiently larger than the monetary seed investment $K$. Then, $(a^{FB}, b^{FB})$ are strictly positive and uniquely defined by the first-order conditions

\[ v_a(a^{FB}, b^{FB}) = 1 \quad \text{and} \quad v_b(a^{FB}, b^{FB}) = 1. \] (2)

Let $\hat{S}(a, b) = v(a, b) - b$ be the joint continuation surplus after $A$ has invested. Also, define $b^*(a) = \arg \max_b \hat{S}(a, b)$ as the ‘conditionally’ efficient investment level of party $B$ for given $a$, so that $b^*(a^{FB}) = b^{FB}$. Notice that $B$ will expend $b^*(a)$ when she expects to be asset owner and therefore residual claimant for the return from her own effort at date 5.

\(^8\)As mentioned in the Introduction, this assumption implies that option contracts with an exercise date before $B$ invested (i.e., at date 2) do not implement the first best under insider financing even if wealth constraints are disregarded (see Edlin and Hermalin, 2000). Most of our subsequent results extend to the case of substitutive investments as well. See also the discussion in footnote 17 below.
After $A$ invested, the parties may find it useful to rescind their initial contractual arrangement and write a new one. For convenience and in line with the literature, we suppose that the outcome of these renegotiations is described by the generalized Nash-bargaining solution. When renegotiations occur at date 2, the agents therefore share the efficiency gain above their respective default payoffs according to a linear sharing rule.$^9$ We parameterize $A$’s bargaining strength in bilateral renegotiations with $B$ by $\gamma \in [0,1]$, while $B$ has a relative bargaining power $(1 - \gamma)$. In trilateral renegotiations with $B$ and an outside investor $C$ (see Section 4 below), we will denote $A$’s Nash bargaining parameter as $\gamma_A \in [0,1]$.

As a useful starting point of analysis, let us first consider a situation where $A$ is not wealth-constrained and finances the initial outlay $K$ out of own funds. Suppose that $A$ is asset owner at date 0 and $A$ and $B$ do not sign an initial contract.$^{10}$ After $A$ invested $a$ at date 1, $B$ will at date 3 undertake the conditionally efficient investment $b^*(a)$ only if she anticipates to become residual claimant. Hence, the parties will (if feasible) renegotiate the initial governance structure at date 2 where $B$ buys the asset. Since $B$ will clearly not invest when $A$ retains ownership, $A$’s default payoff at the renegotiation date 2 is given as $v(a,0)$. Accordingly, and presuming it is efficient to start the project, his maximization program at date 1 reads

$$\max U^A(a) = v(a,0) + \gamma v(a,b^*(a)) - b^*(a) - v(a,0) - K - a,$$

and the unique equilibrium effort $\hat{a}$ is determined by the first-order condition

$$(1 - \gamma)v_a(\hat{a},0) + \gamma v_a(\hat{a},b^*(\hat{a})) = 1.$$

This condition immediately reveals that $\hat{a} < a^{FB}$ when investments are complementary, because $v_a(a,0) < v_a(a,b^*(a))$ in this case.$^{11}$ Only if investments are marginally independent, $A$ invests efficiently and non-conditional ownership attains the first best provided renegotiation is feasible. In the next sections, we analyze option contracts and ask whether this contingent governance structure can overcome the inefficient outcome under non-contingent ownership. Thereby, we disregard the possibility of renegotiation in Section 3, while renegotiation is taken into account in the subsequent Section 4.

### 3. Equilibrium analysis

In related papers, Demski and Sappington (1991) and Nöldeke and Schmidt (1998) have shown that option contracts generically facilitate an efficient outcome. The settings in these papers presume that both parties possess a sufficiently large monetary endowment and, consequently, wealth constraints are no matter of concern. Also, no monetary outlay is required at the start of the relationship. As we will see below, an efficient outcome remains feasible

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$^9$This efficiency gain is the difference between the maximal joint continuation surplus $\hat{S}(a,b^*(a))$, and the sum of the default payoffs.

$^{10}$Notice that $A$ will not expend any effort when $B$ initially owns the asset.

$^{11}$Similarly, if investments are substitutive, we have $\hat{a} > a^{FB}$. 

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if a seed investment is needed but A is not wealth constrained. After analyzing this benchmark, we incorporate a wealth constraint on A’s side who then cannot finance K, and also cannot assure B’s participation in the venture by providing an upfront payment. Thereafter, we consider a situation where B finances the asset start up. The final subsection examines outsider financing by a bank C that has no further productive role.

**Wealthy agents**

Suppose A and B sign the following arrangement \((L, R, p)\) at date 0. Under the terms of contract, A initially owns the asset and B provides some (possibly negative) monetary loan \(L\). At date 5, B can then either insist on (or provide) a repayment \(R\), or she can alternatively exercise an option-to-buy. If B exercises, she disburses a strike price \(p\) and acquires asset ownership. Notice that B will find it profitable to invest and to exercise her option if and only if \(v(a, b^*(a)) - b^*(a) - p \geq R\). We now demonstrate that this conditional ownership arrangement in combination with a debt-like financial structure induces A to invest efficiently if A is not wealth constrained. Consider a strike price

\[
p^* = v(a^{FB}, b^{FB}) - b^{FB} - R
\]

for some arbitrary combination \((L, R)\) that is compatible with each agent’s participation constraint. Imagine A undertakes an effort \(a \geq a^{FB}\) at date 1. Then, B invests \(b^*(a)\) and exercises her option-to-buy at date 5 because her associated continuation payoff \(v(a, b^*(a)) - b^*(a) - p^*\) at least weakly exceeds \(R\). For obvious reasons, A will never invest more than \(a^{FB}\) because B reaps the return on any excess effort. We must also show that A will not invest less than \(a^{FB}\). Suppose \(a < a^{FB}\). Then, B refrains from any investment and insists on the repayment \(R\) at date 5 because \(R > v(a, b^*(a)) - b^*(a) - p^*\). Hence, A’s payoff is \(U^A(a < a^{FB}) = S(a, 0) + L - R\) which is strictly smaller than \(U^A(a = a^{FB}) = S(a^{FB}, b^{FB}) + L - R\). As a consequence, \(a < a^{FB}\) cannot be A’s preferred choice for strike price \(p^*\). Notice that this result applies for any feasible loan and repayment levels. Thus, if A is not wealth-constrained, \(L\) as well as \(R\) can be selected to divide the ex-ante surplus \(S(a^{FB}, b^{FB})\) in any desired way, which allows us to remain agnostic about the initial bargaining positions of both parties.

We can state the following proposition which is a slight extension of results found in Demski and Sappington (1991) and Nöldeke and Schmidt (1998).\(^{12}\)

**Proposition 1.** Suppose that A is not wealth-constrained. For any \(K \geq 0\), an option-to-buy contract \((L, R, p)\) with strike price \(p^* = v(a^{FB}, b^{FB}) - b^{FB} - R\) implements efficient investments and distributes the joint surplus in any desired way by appropriate choice of \((R, L)\).

\(^{12}\)As mentioned in the Introduction, Demski and Sappington analyze option-to-sell rather than option-to-buy contracts. One can show that both contracting types have identical economic consequences whether or not wealth constraints are admitted. Therefore, we can without loss of generality focus on option-to-buy contracts to shorten the exposition.
Proposition 1 shows that the start-up cost $K$ per se does not hamper an efficient outcome of the relationship.\footnote{I thank a referee for suggesting a perhaps more intuitive scheme under which the same result prevails: $A$ finances $K$ directly, and the option strike price is equal to $p^* + K$. This scheme, of course, is feasible only if $A$ is not wealth constrained.} If $A$ is not wealth-constrained, it is in fact irrelevant which party bears the monetary seed investment. For example, if one switches from $L = 0$ to $L = K$ so that $B$ finances the asset start up, a reduction of the option price by an amount $R = K$ leaves the equilibrium payoffs of both parties unaffected, and does not distort their incentives to exert value-enhancing effort.

**Wealth constraints**

We now explore whether the implementation result of Proposition 1 carries over to a setting where $A$ is wealth constrained, which seems relevant in many real-life situations (including venture capital financing and research joint ventures). Recall that Proposition 1 applies for any conceivable level of the initial monetary payment $L$ that flows from $B$ to $A$. Therefore, even if $A$ is wealth-constrained, $B$’s participation and the venture’s start up can be ensured under an option contract with strike price $p^*$ if both parties continue to invest efficiently. This reasoning seems to suggest that $A$’s lack of monetary resources does neither affect the optimal contract nor the implementable outcome. Perhaps surprisingly, though, we show that an option contract with price $p^*$ may lead party $A$ to underinvest. Fortunately, a modified strike price is found to restore a first-best outcome under insider financing.

While a detailed formal analysis is relegated to the Appendix, it is useful to give an intuitive explanation for these results. Recall that if $A$ is not wealth constrained, he will not under-invest under an option-to-buy contract with strike price $p^*$ because $B$ then refuses to invest any $b > 0$ and insists on a repayment $R$. Since $A$ appropriates the maximum overall surplus $S(a^{FB}, b^{FB})$ minus a constant $(R - L)$ when he invests efficiently and $B$’s continuation payoff cannot fall below $R$, any deviation from $a^{FB}$ can only hurt $A$ and is thus self defeating.

If $A$ is wealth-constrained, however, $B$’s default strategy $b = 0$ after observing $a < a^{FB}$ may no longer be optimal. To see this, consider $R \geq L = K$ and assume $A$ expends an effort so small that $v(a, 0) < R$. Notice that effort levels with this property exist for any $K > 0$ by Assumption 1.\footnote{They also exist if $B$ has to be promised a positive share of total surplus, i.e., if $R > 0$ even if $K = L = 0$.} Then, $A$ has to default on repayment $R$ when $B$ exerts no effort, and $B$ has a legal claim on the asset and becomes owner at date 5. However, if $A$ indeed goes bankrupt and $B$ seizes the collateral, $b = 0$ cannot be her date-3 best response on $A$’s defective action. It is now useful to state the following definition.

**Definition 1.**

Let

$$
\tilde{b}(a, R) = \begin{cases} 
\max\{ b \mid v(a, b) \leq R \} & \text{if } v(a, 0) \leq R \\
0 & \text{otherwise.}
\end{cases}
$$
The threshold investment $\hat{b}(a, R)$ represents the minimal effort of $B$ for given $a$ to enforce the repayment $R$.\footnote{Throughout the main text, we will without loss of generality concentrate on loan levels $L = K$. For $L > K$ where the initial loan exceeds the seed investment, $A$ retains a monetary endowment $L - K$. Hence, he can repay his debt for given $(a, b)$ whenever $(L - K) + v(a, b) \geq R$. In order to leave $B$’s equilibrium surplus unaffected, an increase in $L$ must be accompanied by an identical increase in $R$. Accordingly, the threshold investment $\hat{b}(a, L, R(L))$ does not vary in $L$ for any given distribution of total surplus. All proofs in the Appendix allow for loan levels $L > K$, so that $\hat{b}(\cdot)$ is there defined as $\max\{b | v(a, b) \leq R - (L - K)\}$ if $v(a, 0) \leq R - (L - K)$, and $\hat{b}(\cdot) = 0$ otherwise.} If $\hat{b}(a, R)$ is positive, $B$’s best reply is one of two actions: either, she undertakes the threshold investment $\hat{b}(\cdot)$ where $A$ is just able to repay. If this effort level is the optimal response, $B$ insists on a repayment and her continuation payoff becomes $R - \hat{b}(a, R)$. The corresponding payoff of $A$ is then strictly smaller than $S(a^{FB}, b^{FB}) - (R - L)$ so that a deviation from $a^{FB}$ cannot be profitable. Alternatively, the best continuation action for $B$ may be to expend $b^*(a)$ and to appropriate the asset. If $b^*(a) < \hat{b}(a, R)$, this strategy is always dominant because $b^*(a)$ by definition maximizes the continuation value of the asset, and $B$ acquires it costlessly at date 5 where $A$ is still unable to repay. Again, a deviation cannot be worthwhile for $A$ who is left with a non-positive payoff. Finally, though, consider a situation where

\[ \hat{b}(a, R) < b^*(a) \] (C1)

and notice that this condition holds (at least) for deviations $a$ close to $a^{FB}$,\footnote{Since $R < v(a^{FB}, b^{FB})$ is necessary to satisfy $A$’s participation constraint, we must have $\hat{b}(a \rightarrow a^{FB}, R) < b^*(a \rightarrow a^{FB}) = b^{FB}$.} Clearly, $B$ can then again exert $\hat{b}(a, R)$ and claim $R$ (which, of course, remains optimal if $\hat{b}(\cdot) = 0$). Alternatively, however, she may undertake an investment $b^*(a)$, subsequently exercise her option-to-buy at strike price $p^*$, and obtain a continuation payoff $v(a, b^*(a)) - b^*(a) - p^*$. In this latter case, $A$ reaps a total payoff $U^A(a) = p^* - a$ which is clearly larger than his utility when investing efficiently. One can easily check that, if (C1) is satisfied, $b^*(a)$ is agent $B$’s optimal response on $A$’s defection if $v(a, b^*(a)) - b^*(a) - p^* > R - \hat{b}(a, R)$. Inserting $p^*$ and $R$, this latter condition translates into

\[ [v(a^{FB}, b^*(a^{FB})) - b^{FB}] - [v(a, b^*(a)) - b^*(a)] < \hat{b}(a, R). \] (C2)

We can now state the following preliminary result.

**Lemma 1.** Suppose $A$ is wealth-constrained and the parties sign an option contract with strike price $p^*$. Then, this contract fails to implement the first best whenever there exists some investment level $a \geq 0$ which satisfies conditions (C1) and (C2). In particular, efficient investments cannot be implemented if $v(a^{FB}, 0) < R$.

**Proof.** See Appendix A.
if $R = K$. If he is wealth constrained, $A$ can ‘force’ $B$ to exert a positive investment even after a defection. $B$ may then even find it beneficial to expend $b^*(a)$ and to buy the asset, although this reduces her overall surplus below $(R - L)$. In a word, $A$ may sacrifice efficiency and exploit his wealth constraint strategically in order to extract a larger share of the social surplus.

It is easy to see that defection always arises if $v(a^{FB}, 0) < R$, i.e., if $K > 0$ and/or $B$ has some bargaining power ex ante, and if $B$’s investment is sufficiently important. In these cases, $\hat{b}(a, R)$ is strictly positive for any deviation $a < a^{FB}$. At least for a slight underinvestment of party $A$, agent $B$ then strictly prefers to invest $b^*(a) > \hat{b}(a, R)$, and buys the asset at price $p^*$ because condition (C2) is satisfied. To illustrate this, consider the generalized Cobb-Douglas production function $v(a, b) = a^{y}b^{x}$, $x + y < 1$. For this functional form, $v(a, 0) < K$ and $\hat{b}(a, R) = R^{1/\gamma}a$ for any positive $a, K$. Suppose $A$ invests slightly less than $a^{FB}$ and observe that $\hat{b}(a, R) < b^*(a)$ for this deviation. For $a \to a^{FB}$, the left-hand side of (C2) converges to zero while $\hat{b}(a, R)$ converges to a strictly positive value for any $R \geq K$. Accordingly, an option contract with strike price $p^*$ does not implement efficient investments.

We now show that a modified option price may nevertheless facilitate an efficient outcome of the relationship. Specifically, consider the strike price

$$p^{**} = v(a^{FB}, b^{FB}) - b^{FB} - R + \hat{b}(a^{FB}, R).$$

We can now state

**Proposition 1.** Consider insider financing. Then, an option-to-buy contract with strike price $p^{**}$ generically implements efficient investments, and distributes by proper choice of $R$ the joint surplus $S(a^{FB}, b^{FB})$ in any desired way. The optimal strike price is strictly larger than $p^*$ unless $\hat{b}(a^{FB}, R) = 0$.

**Proof.** see Appendix A.

At first glance, one may wonder how $B$ can be induced to invest and to exercise her option at a strike price $p^{**} > p^*$. Even if $A$ invests efficiently, $B$ at this larger strike price strictly prefers a repayment $R$ over her option-to-buy and the accompanied investments. Using our previous arguments, though, we can easily solve this puzzle. If $A$ is wealth-constrained and $v(a^{FB}, 0) < R$, $B$ cannot recover $R$ without expending own investments $\hat{b}(a, R)$, which reduces her continuation payoff below $R$. At an option price $p^*$ and for $a = a^{FB}$, agent $B$ therefore has a strict rather than a weak preference to choose $b^{FB}$ and to acquire the asset. Moreover, $A$ will exploit this fact because he can underinvest (at least in some range) without running the risk that $B$ does not exercise subsequently.

The steeper option price $p^{**} = p^* + \hat{b}(a^{FB}, R)$ resolves this problem. The argument proceeds in two steps. First, provided $A$ invests efficiently, $B$ is at price $p^{**}$ just indifferent between investing efficiently and exercising her option, or to undertake the threshold investment $\hat{b}(a^{FB}, R)$ and to claim $R$. To see this, recall that $B$ prefers the former strategy at
strike price $p$ if and only if

$$v(a, b^*(a)) - b^*(a) - p \geq R - \hat{b}(a, R).$$

(5)

By construction of $p^{**}$, this condition is satisfied with equality for $a = a^{FB}$. Second, we must show that $B$ will not exercise her option for any $a < a^{FB}$, but instead choose the threshold investment $\hat{b}(a, R)$ and insist on a debt repayment. From (5), this behavior is indeed optimal if $v(a, b^*(a)) - b^*(a) + \hat{b}(a, R)$ is strictly increasing in $a$. In the Appendix, we demonstrate that this monotonicity condition is always satisfied if investments are independent or complementary at the margin, i.e., under Assumption 1c). Accordingly, since $B$ will invest efficiently and disburse $p^{**}$ only if $a \geq a^{FB}$, agent $A$ will not deviate from $a^{FB}$ and an efficient outcome is attained. We should emphasize that the above reasoning applies for any feasible repayment level $R$. Hence, one can again choose $R$ arbitrarily in order to divide the social surplus $S(a^{FB}, b^{FB})$ between both parties.

Under insider financing, wealth constraints thus do not endanger efficient sequential investments if renegotiation is infeasible.\(^{17}\) In contrast to the framework where $A$ has a sufficient monetary endowment, though, the optimal option price may be one which does not make party $B$ indifferent between exercising her option on the one hand, and to claim a repayment on the other. Rather, the option price may be so large that she has strict preferences for a repayment but anticipates that an enforcement of this claim will require costly effort.

**Outsider financing**

We now suppose that $A$ does not rely on $B$ to finance the initial outlay, i.e., $L = 0$. Instead, he signs a debt contract with an external investor $C$ (e.g., a bank) who at date 0 provides a loan $L_C \geq K$ and holds a repayment claim $R_C$ to be due at date 5. Specifically, consider the following arrangement: $A$ signs an option-to-buy contract $(p^*, L = R = 0)$ with $B$ and a debt contract $(L_C, R_C)$ with the outside investor $C$. Suppose $A$ invests efficiently. Then, $B$ will undertake $b^{FB}$ and exercise her option at date 5, and $A$ repays his debt $R_C$ (which is smaller than $p^*$) to agent $C$. It remains to show that $A$ cannot gain by investing less than $a^{FB}$. If he does, $B$ will expend no effort as this would yield a negative continuation payoff whereas she can guarantee herself a reservation payoff of zero by not investing. Accordingly, $A$’s payoff in case of a deviation is either zero [if $v(a, 0) < R_C$ in which case $C$ seizes the asset], or it is $S(a, 0) - (R_C - L_C)$ [if $v(a, 0) \geq R_C$ so that $A$ can repay]. Either payoff is smaller than $S(a^{FB}, b^{FB}) - (R_C - L_C)$ so that a defection cannot be profitable. Hence,

**Proposition 3.** Suppose $A$ is wealth-constrained and an outside investor $C$ loans the initial

\(^{17}\)To show this, we relied on the assumption that investments are weakly complementary, $v_{ab}(\cdot) \geq 0$ [Ass. 1]. While an inefficient outcome may possibly arise for substitutive investments, I was unable to construct an example where this is actually the case. Note that - with the exceptions of Proposition 2 and Proposition 4(b) below - all results in the present paper immediately carry over to settings where investments are strict substitutes.
outlay. Then, an option-to-buy contract \( p^*_0 \equiv (p^*, R = L = 0) \) implements efficient investments and a first-best outcome prevails. Also, any division of surplus among all three parties can be ensured via a fixed payment from \( C \) to \( B \).

**Proof.** See Appendix A.

The logic behind this efficiency result is simple. If \( B \) does not finance the initial outlay, she is not locked into the relationship before expending own investments. Therefore, \( A \) cannot exploit his limited liability to hold up \( B \) and to force her to invest in order to protect her repayment claims after \( A \) defected. Relying on an external investor prevents hold up exactly because this outside investor has no productive role. In addition, the arrangement allows to assign any arbitrary share of the total surplus to parties \( B \) and \( C \). To see this, consider a contract extension under which \( B \) receives from \( C \) an unconditional fixed payment, say \( T \). Clearly, the size of this lump sum transfer has no effect on efficiency and the parties’ equilibrium rents are \( U^B = T \), \( U^C = R^C - L^C - T \), and \( U^A = S(a^{FB}, b^{FB}) - (R^C - L^C) \).

A simple contractual arrangement among \( A \), \( B \) and an external investor \( C \) thus implements the first best if renegotiation is infeasible. While this outcome replicates the implementation result under insider financing, the construction of the optimal strike price of \( B \)’s option evidently differs across financing regimes. In particular, and in contrast to insider financing, the optimal strike price now exactly coincides with the one in a model where wealth constraints and financing issues do not arise. Consequently, when external investors are admitted and renegotiation can be prevented, there is no loss of generality in confining attention to an option-to-buy contract with strike price \( p^*_0 = v(a^{FB}, b^{FB}) - b^{FB} \).

### 4. Renegotiation

According to our discussion of Section 3, the relationship between \( A \) and \( B \) yields an efficient outcome if renegotiation is infeasible after \( A \) invested. We now ask under which conditions this finding extends to a scenario where renegotiation is allowed for. Clearly, private parties are in principle free to rescind their bilateral or multilateral arrangements, and to set up a new contracting scheme whenever they find this valuable. In a setting which concentrates on insider financing, Nöldeke and Schmidt (1998) have found that the possibility to renegotiate imposes no binding constraint on the attainable outcome, and the first best can still generically be attained. The present paper incorporates wealth constraints and considers insider financing, outsider financing and hybrid financing in turn.

**Insider financing with renegotiation**

It is convenient to start with a summary of results.

**Proposition 4.** Consider insider financing with renegotiation.

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\(^{18}\)Notice that outsider financing generically implements the first best even if investments are strict substitutes. Hence, this financing mode may strictly dominate insider financing for \( v_{ab}(\cdot) < 0 \). See also footnote 17.
(a) Suppose \( A \) is not wealth constrained. Then, an option-to-buy contract with strike price \( p^* \) attains efficient investments and the initial contract is not renegotiated. Hence, the results in Proposition 1 fully apply.

(b) Suppose \( A \) is wealth constrained and consider an option-to-buy contract with strike price \( p^{**} \) and repayment \( R \geq 0 \). This contract implements a first-best outcome unless there exists some \( a \geq 0 \) for which \( \tilde{b}(a, R) < b^*(a) \) [condition \((C1)\)], and

\[
p^{**} - a^{FB} < \gamma[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))] - a \tag{C3}
\]

jointly apply. Conversely, if \((C1)\) and \((C3)\) are satisfied for some \( a \), \( A \) underinvests and no option contract with repayment level \( R \) can implement a first-best outcome. This inefficiency can arise for any feasible \( R > 0 \), but only if \( A \)'s bargaining parameter \( \gamma \) is sufficiently large.

**Proof.** see Appendix A.

Proposition 4 affirms that in a setting where \( A \) is not wealth constrained, our previous findings continue to apply if renegotiation is admitted. In fact, Proposition 4(a) is a straightforward extension of results in Nöideke and Schmidt (1998). Even if renegotiation is feasible, \( B \) can always insist on a repayment so that her overall equilibrium utility cannot fall below \((R - L)\). Likewise, she cannot insist on renegotiation if \( A \) invests efficiently (see the proof of Lemma 1). Since \( A \) reaps the entire social surplus from the relationship minus the constant \((R - L)\) when he expends efficient effort, any deviation cannot raise his surplus and efficiency prevails.

The second part of the proposition, however, demonstrates that the opportunity to renegotiate can hamper an efficient outcome of the relationship if \( A \) is wealth constrained. According to Proposition 4(b), \( A \) may now defect and choose an investment level smaller than optimal. Using our previous arguments, this result has an intuitive explanation. Under an option contract with strike price \( p^{**} \) and repayment level \( R \), \( B \) will on a deviation never respond with a default investment \( b > \tilde{b}(a, R) \). If \( b^*(a) \leq \tilde{b}(\cdot) \), her preferred default effort is \( b^*(a) \) and \( B \) simply seizes the asset. For obvious reasons, \( A \) will not shirk in this way in the first place. Conversely, for investments \( a \) such that \( b^*(a) > \tilde{b}(\cdot) \), \( B \)'s default effort \( \tilde{b}(\cdot) \) falls short of the conditionally efficient level. Therefore, the parties renegotiate at date 2 and \( A \) reaps a fraction \( \gamma \) of the bargaining surplus \([v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))]\). If this payoff net of the corresponding investment \( a \) exceeds \( p^{**} - a^{FB} \), \( A \) finds a defection profitable and an efficient outcome becomes infeasible. Intuitively, \( A \) may underinvest under insider financing because this forces \( B \) into renegotiation, and diminishes her overall utility significantly below \((R - \tilde{b}(a^{FB}, R) - L)\) if \( A \)'s own bargaining position is sufficiently strong.\(^{19}\)

We should emphasize that for expositional reasons, Proposition 4(b) treats \( R \) as parametric even though it is actually a contractual variable. This is important because even in situations

\(^{19}\)Notice that no option price other than \( p^{**} \) can possibly achieve efficiency under option contracts with an exercise date \( t = 5 \). Also, recall that option contracts with an earlier exercise date (i.e., \( t = 2 \)) fail to implement first best under insider financing even if \( A \) is not wealth constrained (Edlin and Hermalin, 2000).
where not every feasible $R$ can ensure efficient investments, there may exist a subset of feasible repayment levels for which no investment $a < a^{FB}$ satisfies conditions (C1) and (C3). We do not analyze this set for two reasons. First, it will be difficult to identify this set as any typical element $R^*$ will in general not coincide with the minimum or maximum feasible repayment levels: condition (C1) is easier and condition (C3) harder to satisfy the larger $R$. Second, the repayment levels which are optimal from an investment point of view will likely be in tension with distributive considerations. Recall that $B$’s equilibrium utility is equal to $R$ minus any lump-sum payment she may make to agent $A$. Any repayment level $R^*$ may then be too small to achieve $B$’s desired surplus share, and a corresponding second-best outcome may require the parties to sacrifice investment efficiency in favor of a more desired distribution of total payoff.

Before concluding, we show that a defective outcome is a real possibility under insider financing (and for an arbitrary $R > 0$). To see this, consider once again a situation where $\tilde{b}(a^{FB}, R) > 0$ for any $R > 0$, e.g., the Cobb-Douglas value function. For any $\gamma$, the maximizer of the right-hand side of (C3) with respect to $a$ will then be strictly smaller than $a^{FB}$ because $\tilde{b}(\cdot)$ decreases in $a$ in the relevant range.\(^{20}\) Since at the same time (C3) holds with equality for $a = a^{FB}$ and $\gamma = 1$, (C3) will be satisfied for any $a$ being sufficiently close to $a^{FB}$ and $\gamma$ being sufficiently close to one. Since an investment $a$ sufficiently close to $a^{FB}$ also meets the additional requirement $b^*(a) > \tilde{b}(a, R)$ [condition (C1)], we can conclude that for any $R > 0$, agent $A$ will defect under the optimal option contract if his bargaining position is sufficiently strong. This suggests that renegotiation imposes a serious constraint on the effectiveness of option contracting with wealth-constrained parties and renegotiation.

**Outsider Financing and Renegotiation**

Again, it is useful to state the result first.

**Proposition 5.** Consider outsider financing and allow the parties to renegotiate. Then, an option-to-buy contract with strike price $p_0^*$ between $A$ and $B$ and a debt contract with repayment level $R^C$ between $A$ and $C$ guarantees a first-best outcome if there does not exist some $a \geq 0$ with $\tilde{b}(a, R^C) > 0$ such that

$$p_0^* - R^C - a^{FB} < \gamma_A[v(a, b^*(a)) - b^*(a) - v(a, 0)] - a.$$  \hspace{1cm} (C4)

Otherwise, $A$ underinvests under any date-5 option contract with outsider financing.

**Proof.** see Appendix A.

Outsider financing may also be subject to a defective outcome when renegotiation is taken into account (see, however, the Excursion below). The argument here is quite different from the corresponding result in Proposition 4. If $A$ underinvests, he cannot hold up $B$

\(^{20}\)This maximizer is increasing in $\gamma$, and converges to $a^{FB}$ iff $\gamma = 1$ and $d\tilde{b}(a^{FB}, R)/da = 0$. 

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who will simply not invest and obtain her reservation utility. For any investment such that 
\( b(a, R_C) > 0 \) which implies \( v(a, 0) < R_C \), though, \( A \) causes an externality on \( C \) who cannot 
recover his loan at date 5 and in default seizes the asset as a collateral. In ensuing date-2 
renegotiations among all three involved parties, \( A \) then again captures a fraction \( \gamma_A \) of the 
negotiation gain \( [v(a, b^*(a)) - b^*(a) - v(a, 0)] \).

If (C4) does not apply, \( A \) does not deviate and a first-best outcome remains feasible. Conversely, if (C4) applies for some \( a \) for which \( A \) is unable to repay his initial loan, a defection cannot be avoided and the possibility of 
renegotiation imposes a binding constraint on the feasible outcome.

To further assess these findings, it is interesting to note that (C4) is never satisfied when \( A \)'s bargaining strength in trilateral renegotiations with \( B \) and \( C \) is small. Hence, if \( \gamma_A \) is 
sufficiently smaller than \( \gamma \) because, for instance, the outside investor has a strong bargaining 
position, outsider financing may facilitate the first best while insider financing does not.

On the other hand, suppose \( \gamma_A = \gamma \) so that \( A \)'s bargaining strength in negotiations with \( B \) 
and \( C \) is no smaller than under insider financing. Then, insider financing dominates outsider 
financing when renegotiation is admitted: since \( B \) expends no default investment after a 
development under outside financing, but exerts a positive default effort under inside financing, 
the available bargaining surplus is strictly smaller in the latter case.

In addition, the non-deviation payoff in (C4) is larger than the one in (C3). Taken both effects together, insider 
financing renders it easier to implement efficient investments if \( \gamma_A = \gamma \) and may thus be the 
unambiguously preferred financing mode in this case.

**Excursion: Date-2 Option Contracts and Outside Financing**

Let us extend the model and consider option contracts with an exercise date immediately 
after \( A \) invested, say, at a date 2. We can show that option contracts of this type generically 
achieve the first best if collusion can be ruled out. This positive outcome is quite surprising 
because, in a setting without the outside agent \( C \), options with an early exercise date have 
been shown to be strictly dominated if renegotiation is allowed for (see Edlin and Hermalin, 
2000).

While technical details have been relegated to Appendix B, it is useful to briefly explain

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21 While we presume Nash bargaining for simplicity, a qualitatively identical result is obtained for more 
sophisticated bargaining rules such as, for instance, the Shapley value.

22 The outcome of Proposition 4 does not change if we allow \( B \) and \( C \) to engage in collusive behavior. To 
see this, note first that collusion makes potentially sense only if \( B \) does not exercise her option at date 5, and 
if \( A \) cannot repay \( R_C \) to \( C \) in which the bank seizes the asset. However, the available joint (ex post) surplus 
of \( B \) and \( C \) is then smaller than \( R_C \) whereas their joint surplus would be \( R_C \) if \( B \) had invested properly and 
exercised her option. Collusive behavior thus cannot pay off.

23 Similar observations on the benefits of outsider financing have been made in the literature. In a 
simultaneous-investments setting, Aghion and Tirole (1994) show that outsider financing can improve the 
bargaining position of a client vis-a-vis the researcher (the agent) in a research project, and facilitate an 
efficient allocation of property rights even if the agent is wealth constrained.

24 In technical terms, we have \( v(a, 0) < R - b(a, R) = v(a, b(\cdot)) - b(\cdot) \) because \( v(a, b) - b \) is increasing in \( b \) 
for any \( b < b^*(a) \), while \( b(\cdot) < b^*(a) \) is a necessary condition for a deviation.

25 I am very grateful to an anonymous referee for suggesting this extension and the following scheme to me.
the basic idea behind this finding. Under the optimal scheme, all parties initially agree on the following contract. The bank $C$ lends an amount $K$ to $A$ with a repayment date $T$. $A$ is asset owner initially but $B$ holds an option to buy it at date 2 at a strike price $p^*_0 = v(a^{FB}, b^{FB}) - b^{FB}$. If $B$ does not exercise her option, she has to pay a certain ‘fine’ $F$ to $C$, who in addition becomes asset owner without having to provide any compensation to $A$.

This scheme indeed implements the first best by the following chain of arguments. If $B$ does not exercise, $C$ cashes in $F$ and appropriates the asset. $B$ and $C$ then subsequently renegotiate the ownership status, $B$ buys the asset from $C$ and exerts the conditionally efficient investment. Accounting for this continuation of the game, there exists a unique fine level $F$ for which $B$ just breaks even if she does not exercise and $A$ expended $a^{FB}$. If $A$ invests efficiently, $B$ is then just indifferent between exercising or not (at strike price $p^*_0$, she receives zero anyway). Conversely, one can show that for any $a \leq a^{FB}$ $B$ strictly prefers not to exercise. Since the asset is taken out of $A$’s hands and he does not realize any positive surplus when $B$ does not exercise, it is now clear that $A$ will not deviate and the above scheme implements the first best for any feasible $R^C$. Again, the three parties can achieve any arbitrary division of total surplus by setting up an additional lump-sum payment from $C$ to $B$.²⁶

The above scheme nicely illustrates a more general insight. As has been emphasized in the agency literature, a third-party involvement is often helpful because outside parties help to overcome a budget-balancedness constraint (see, e.g., Holmström, 1982; Aghion and Tirole, 1994). When an agent $C$ is available, a defection of $A$ can be punished by shifting surplus and ownership rights to the outside party. $A$ is then no longer needed to realize the bargaining gain after his defection, which prevents him from defecting in the first place.

Unfortunately, the idea that detrimental renegotiation in partnerships can be prevented through third parties also has a serious drawback: in particular, the above scheme is extremely vulnerable with respect to the possibility of collusion which is easily seen. Suppose $A$ indeed invested $a^{FB}$. Then, $B$ and $C$ can revise their initial contractual arrangement according to which $B$ pays a fine to $C$ if she does not exercise. Subsequently, $B$ does not exercise her option so that $C$ becomes owner, and $B$ and $C$ renegotiate to share the entire surplus when $B$ does not exercise, it is now clear that $A$ will not deviate and the above scheme implements the first best for any feasible $R^C$. Again, the three parties can achieve any arbitrary division of total surplus by setting up an additional lump-sum payment from $C$ to $B$.

²⁶Conversely, a similarly designed option contract with exercise date 5 will not generically achieve an efficient outcome. To see this, let $C$ again become asset owner if $B$ does not exercise her option at date 5, and consider some strike price at which $B$ invests efficiently and exercises subsequently if and only if $a \geq a^{FB}$. If renegotiation is ruled out, this contract will again guarantee a first best outcome. However, this is not true if renegotiation cannot be prevented. First, suppose all three parties are needed to renegotiate at date 2. If $A$ defects, he will then again reap a fraction of the bargaining gain at date 2, and efficiency does not generally prevail. Second, imagine $B$ and $C$ can renegotiate at date 3 without $A$’s involvement (see also the discussion of collusion proofness below). Notice that whatever investment level $A$ has chosen, $B$ and $C$ now have a joint interest not to exercise the option at date 5, in which case $C$ obtains asset ownership. Their optimal date-2 side contract then specifies that (a) $B$ does not exercise, and (b) an asset sale from $C$ to $B$ at a fixed price $t$ immediately after $C$ became owner at date 5. Choosing $t \leq v(a, b^*(a)) - b^*(a)$ then ensures that irrespective of $a$, $B$ exerts $b^*(a)$ whereas $A$ loses the asset. Anticipating this outcome, $A$ will not invest and efficiency fails.
surplus in some fashion. Anticipating this hold-up, A will not undertake any effort at all and an extreme form of underinvestment prevails.

**Mixed Financing can be Optimal**

Before concluding, we now want to argue that a combination of insider and outsider financing may be strictly preferable if renegotiation is feasible and a single investor triggers a suboptimal outcome.\(^\text{27}\) A full exploration of this issue must remain beyond the scope of the paper, but some partial analysis will be useful. Imagine that B provides a loan of size \(\beta K, \beta \in [0,1]\), and C a loan of size \((1-\beta)K\) to cover the start-up cost. Indicate the lenders’ repayment claims as \(R^B\) and \(R^C\), respectively, and notice that \(R^B \geq \beta K\) and \(R^C \geq (1-\beta)K\) so that \(R = R^B + R^C \geq K\).

Suppose first that B and C hold debt claims of the same priority. In addition, the initial contract assigns to B the right to buy the asset at some strike price \(\hat{p}\) at date 5. If she exercises this option, B cannot reclaim her debt. Consider now an investment \(a\) of A with the property \(\tilde{b}(a, R) < b^* (a).\(^\text{28}\) Given this investment, B’s default continuation payoff is as follows: if she invests some \(b\) from the interval \([0, \tilde{b}(a, R)]\), A is unable to serve his aggregate debt obligation at date 5. B and C then seize the asset and share its value \(v(a, b)\) proportionally to the amounts of their initial loans, so that B reaps \(\beta v(a, b) - b\). Supposing that \(\beta\) is sufficiently large, the default investment which maximizes B’s payoff in the considered range is the boundary \(\tilde{b}(\cdot)\). Alternatively, B may invest \(b^*(a)\) and exercise her option subsequently in which case her final payoff is \(v(a, b^*(a)) - b^*(a) - \hat{p}\). In analogy to our discussion in the previous sections, B will prefer the first alternative for any \(a < a^{FB}\) and she will prefer the second alternative for any \(a \geq a^{FB}\) whenever \(\hat{p} = \hat{p}^* \equiv v(a^{FB}, b^{FB}) - b^{FB} - \tilde{b}(a^{FB}, R) - R^B\).

Consider now an initial loan arrangement with \(\beta\) being sufficiently large that B’s locally optimal default investments is indeed \(\tilde{b}(a, R)\) for given \(a < a^{FB}\) and for an option contract with strike price \(\hat{p}^*\). A will then deviate from \(a^{FB}\) if and only if there exists some investment \(a\) which satisfies \(\tilde{b}(a, R) < b^*(a)\), and

\[
\hat{p}^* - R^C - a^{FB} < \gamma_A [v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))] - a. \tag{C5}
\]

One can immediately verify that condition (C5) is stronger than (C4). This is because the right-hand side of (C5) is strictly smaller than the right-hand side of (C4), whereas the left-hand side of (C5) is larger than its counterpart, \(\hat{p}^* - R^C = p_0^* - R + \tilde{b}(a, R) > p_0^* - R\). Moreover, for \(\gamma_A < \gamma\), (C5) is also more demanding than condition (C3) whenever \(\gamma_A < \gamma\) which is easily seen because both conditions are identical except for the size of A’s bargaining

\(^{27}\) Mixed financing as described here is empirically relevant. To give just one example, the internet broker priceline.com recently announced its plans to expand into Europe (see Priceline Press release, June 28, 2000). Priceline.com Europe is a new company in which priceline.com and the venture capital fund General Atlantic are investors and jointly fund the company. Under the terms of contract, priceline.com purchases a convertible note allowing the company to take up to a 50% equity stake in priceline.com Europe under certain conditions. Until that note is converted, priceline.com will not hold an equity stake in the new venture.

\(^{28}\) Recall that any profitable deviation of A must satisfy this condition.
parameter. Accordingly, $A$ will find it less attractive to deviate if $B$ and $C$ jointly finance the initial outlay.\(^{29}\)

Before concluding, we show that financing structures where one loan has priority over the other can never be optimal in the present setting. To see why, consider first the case where $C$’s loan has priority. $B$’s default payoff from investments in the relevant interval $[0, \tilde{b}(a, R)]$ is then $\max\{0, v(a, b) - R_C\} - b$ and her default investment will be one of the boundary solutions $b = 0$ or $b = \tilde{b}(a, R)$, respectively.\(^{30}\) In the former case, the total bargaining gain from which $A$ accrues a fraction $\gamma_A$ remains the same as under straight bank financing. In the latter case, the bargaining gain shrinks to the same level as under straight insider financing. At the same time, though, $C$’s debt claim will be fully repaid in default because $R_C < v(a, \tilde{b}(a, R))$ by construction. Reasonably, renegotiation will then only take place between $A$ and $B$ so that again $\gamma_A = \gamma$ and no improvement over pure insider financing is attained. Finally, suppose $B$’s loan has priority. Then, her default investment in the relevant range will be implicitly given by $v(a, \hat{b}) = R_B$ with $\hat{b}(\cdot) < \tilde{b}(\cdot)$ whenever $R_B < R$, which raises the bargaining gain after a default relative to a situation where the claims of both investors have equal priority.

5. Conclusion

In this paper, two parties $A$ and $B$ sequentially invest into an asset whose setup requires an initial outlay. The model is suited to represent a variety of interesting economic scenarios, e.g., the relationship between a start-up firm and a venture capitalist, or between a biotech firm and a pharmaceutical company in a research joint venture. In a setting where the effort levels of both parties as well as the final asset value are non-verifiable, previous work implies that an option-to-buy contract generically implements the first best if there are no financing constraints. As a first main result of the present paper, we establish that this outcome extends to a setting with wealth constrained agent if renegotiation can be prohibited. In this case, a debt contract between $A$ and either $B$ or an outside investor in combination with an option-to-buy contract facilitates efficient investments. The optimal strike price of $B$’s option depends on which of those financing modes is chosen. We then go on to show that, under certain conditions, a first best can also be attained if renegotiation is feasible. In general, however, renegotiation impedes on efficiency and the performance of insider and outsider financing differs substantially. We characterize conditions under which one financing form dominates the other, and show that a combination of insider and external investors may be valuable and even restore the first best. In an extension, we also show that an option contract with an early exercise date generically facilitates efficiency if collusion can be prevented. Overall, our findings shed some light on the relative benefits of insider and outsider financing in relationships with sequential investments. They also reconfirm

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\(^{29}\)Other papers have identified a variety of circumstances where multiple lenders can be beneficial. Dewatripont and Tirole (1994) show that multiple outside investors can alleviate the problem that a long-term project is stopped prematurely after first-period profits turned out low. Berglöf and von Thadden (1994) and Bolton and Scharfstein (1996) show that multiple (long-term and short-term) creditors reduce the firm’s payoff in renegotiations and accordingly reduce its incentives to default strategically.

\(^{30}\)Recall that $\tilde{b}(\cdot)$ maximizes $v(a, b) - b$ on the interval $[0, \tilde{b}(\cdot)]$ whenever $\tilde{b}(a, R) < b^*(a)$. 

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that option-to-buy arrangements can be a valuable tool to govern the incomplete-contracting relationship between parties who invest sequentially, and show how these arrangements should be augmented to account for financing constraints. Nevertheless, our results also point to some limitations of option contracting in situations where agents are wealth constrained and where the possibility to renegotiate is prevalent. Whether other contractual instruments restore efficiency is a challenging issue for future research.

Appendix A

Proofs of Lemma 1, Propositions 2, 4 and 5 follow.

Proof of Lemma 1. Consider the following arrangement \((L, R, p^*)\). At date 0, \(B\) contributes a payment \(L \geq K\). At date 5, \(B\) can then demand a repayment \(R \geq K\), or can alternatively exercise her option and acquire the asset at a price \(p^* = v(a^{FB}, b^{FB}) - b^{FB} - R\).

Suppose \(A\) invested \(a^{FB}\) at date 1. We must distinguish between two subcases. If \(B\) expends \(b = b^{FB}\) at date 3 (note that a larger investment is never profitable), she will subsequently exercise her option-to-buy at date 5 because \(v(a^{FB}, b^{FB}) - p^*\) strictly exceeds \(R\). Accordingly, \(B\)'s continuation payoff after date 0 is \(v(a^{FB}, b^{FB}) - b^{FB} - p^* = R\) and she attains an overall payoff \(U^B = R - L\). Second, she may undertake an investment \(b < b^{FB}\) and insist on a repayment at date 5. If \(v(a^{FB}, b) + [R - L] \geq R\), \(A\) can meet his repayment obligation and \(B\)'s overall payoff is weakly smaller than \(R\) which cannot be optimal. Conversely, if \(v(a^{FB}, b) + [R - L] < R\), \(A\) cannot meet his repayment obligation. In this case, \(B\) seizes the asset as well as \(A\)'s remaining cash endowment \([R - L]\) as a collateral. Again, her continuation payoff is then weakly smaller than \(R\) so that \(B\) will undertake \(b^{FB}\) at date 3 provided that \(A\) invests efficiently at date 1.

We now examine whether \(A\) gains by deviating from \(a^{FB}\). Observe that his overall payoff for efficient investments is \(U^A = S(a^{FB}, b^{FB}) - [R - L]\), and his participation constraint requires \(R < S(a^{FB}, b^{FB}) + L\). To start with, notice that a deviation \(a > a^{FB}\) cannot be optimal because \(B\) still exercises her option-to-buy so that \(A\) recovers no return on any excess investment. Consider now an arbitrary deviation \(a < a^{FB}\). Let \(\tilde{b}(a, R)\) be the threshold investment level as defined in Definition 1 for \(L = R\) and as defined in footnote 15 for \(L > K\). \(A\) is able to repay his debt if \(B\) chooses an effort level \(b \geq \tilde{b}(\cdot)\). Notice that \(A\) will never choose an investment \(a\) for which \(\tilde{b}(a, R) = 0\) because \(B\) can then insist on a repayment \(R\) without expending own effort. Hence, we can concentrate on deviations \(a\) where \(\tilde{b}(a, R)\) is positive.

Consider first deviations from \(a^{FB}\) where \(\tilde{b}(a, R) > b^*(a)\). If \(B\) responds by some \(b < \tilde{b}(a, R)\), \(A\) cannot repay and \(B\) seizes the asset as well as \(A\)'s monetary endowment \((L - K)\) at date 5. Moreover, \(b = b^*(a)\) is \(B\)'s optimal response because she appropriates the entire continuation surplus \(v(a, b^*(a)) - b^*(a)\) as well as \(A\)'s remaining cash endowment when exerting \(b^*(a)\). Since \(A\) is left with a non-positive surplus, she will never deviate from \(a^{FB}\) in a way that \(\tilde{b}(a, R) \geq b^*(a)\).

Consider now deviations such that \(\tilde{b}(a, R) < b^*(a)\). If \(B\) responds by some \(b < \tilde{b}(a, R)\), she
again seize the asset at date 5. Her locally best reply is an effort level close to \( \tilde{b}(a, R) \), which gives \( B \) a continuation payoff slightly smaller than \( v(a, \tilde{b}(\cdot)) - \tilde{b}(\cdot) + (L - K) \). Alternatively, \( B \) can undertake an investment from the complementary interval \( b \geq \tilde{b}(a, R) \). If she does, she can insist on a repayment in which case \( b = \tilde{b}(a, R) \) is her best reply, and her continuation payoff becomes \( R - \tilde{b}(a, R) = v(a, \tilde{b}(\cdot)) - \tilde{b}(\cdot) + (L - K) \). Accordingly, a response \( b < \tilde{b}(a, R) \) is dominated, and \( A \) will not deviate whenever \( B \) chooses \( \tilde{b}(a, R) \) because his overall payoff is then non-positive. Finally, though, \( B \) may anticipate to exercise her option-to-buy at date 5 in which case she expends \( b^*(a) \) if \( \tilde{b}(a, R) \) at date 3. In this latter case, her continuation payoff becomes \( v(a, b^*(a)) - b^*(a) - p^* \). For \( \tilde{b}(a, R) < b^*(a) \), this last strategy is \( B \)'s best response if \( v(a, b^*(a)) - b^*(a) - p^* > R - \tilde{b}(a, R) \), which translates into

\[
[v(a^{FB}, b^{FB}) - b^{FB}] - [v(a, b^*(a)) - b^*(a)] < \tilde{b}(a, R). \tag{C2}
\]

If \( \tilde{b}(a, R) < b^*(a) \) and (C2) is satisfied (which implies \( \tilde{b}(a, R) > 0 \)), \( b^*(a) \) is \( B \)'s optimal reaction on a deviation \( a < a^{FB} \) and she exercises her option at price \( p^* \). Then, \( A \) accrues an overall payoff \( p^* + (L - K) - a \) which strictly exceeds \( S(a^{FB}, b^{FB}) - (R - L) \), so that \( A \) deviates from \( a^{FB} \) if and only if conditions (C1) and (C2) are violated for some \( a < a^{FB} \). Finally, we show that defection generically occurs if \( v(a^{FB}, 0) < R \) so that \( \tilde{b}(a, R) > 0 \) for any \( a \leq a^{FB} \). Consider \( a \rightarrow a^{FB} \). Then, \( \tilde{b}(a, R) < b^*(a) \) because \( A \)'s participation constraint implies \( R < v(a^{FB}, b^{FB}) \). In addition, the left-hand side of (C2) converges to zero while the right-hand side is strictly positive. Accordingly, \( A \) deviates and underinvestment generically occurs at strike price \( p^* \) if \( v(a^{FB}, 0) < R \). \( Q.E.D. \)

**Proof of Proposition 2.** Consider an option price

\[
p^{**} = v(a^{FB}, b^{FB}) - b^{FB} - R + \tilde{b}(a^{FB}, R)
\]

and notice that \( \tilde{b}(a^{FB}, R) > 0 \) iff \( v(a^{FB}, 0) < R + (L - K) \). Suppose first that \( A \) exerts an effort that satisfies \( b(a, R) \geq b^*(a) \), and verify that any such effort level is smaller than \( a^{FB} \). By the arguments given in the proof of Lemma 1, such a deviation cannot be profitable because \( B \) will then expend \( b^*(a) \) and seize the asset as a collateral. Next, consider investment levels with the property \( \tilde{b}(a, R) < b^*(a) \). \( B \) will then either invest \( b^*(a) \) and exercise her option (in which case \( A \) deviates), or undertake the threshold investment \( \tilde{b}(a, R) \) and insist on \( R \) in which case \( A \)'s surplus is smaller than \( S(a^{FB}, b^{FB}) - (R - L) \) and he will not deviate. At strike price \( p^{**} \), \( B \) pursues the former strategy iff

\[
v(a, b^*(a)) - b^*(a) - [v(a^{FB}, b^{FB}) - b^{FB} - \tilde{b}(a^{FB}, R)] + \tilde{b}(a, R) \geq 0.
\]

This condition holds with equality if \( a = a^{FB} \) so that \( B \) will invest efficiently and exercise her option if \( A \) indeed expires \( a^{FB} \). Since \( A \) will never invest more than \( a^{FB} \), he will not deviate if the above condition is violated for any \( a < a^{FB} \). To see that this holds true, consider the derivative of the left-hand side of the above inequality with respect to \( a \), which yields (by the implicit function theorem)

\[
v_a(a, b^*(a)) - \frac{db(a, R)}{da} = v_a(a, b^*(a)) - \frac{v_a(a, \tilde{b}(a, R))}{v_b(a, \tilde{b}(a, R))}.
\]

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Since \( b(a, R) < b^*(a) \), we have \( v_a(a, b^*(a)) \geq v_a(a, \tilde{b}(a, R)) \) and \( v_b(a, \tilde{b}(a, R)) > v_b(a, b^*(a)) = 1 \) if investments are weak complements, \( v_ab(\cdot) \geq 0 \). Under Assumption 1, the left-hand side of the inequality is thus strictly increasing in \( a \). Since \( B \) will not exercise her option when \( a < a^{FB} \) and \( A \)’s payoff is decreasing in \( a \) for any \( a \geq a^{FB} \), a contractual arrangement \((L, R, p^*)\) implements efficient investments.

Finally, we show that any arbitrary distribution of the social surplus \( S(a^{FB}, b^{FB}) \) can be achieved by proper choice of \( R \) and \( L \geq K \). To see this, notice that \( B \)'s equilibrium payoff \( U^B = R - L - \tilde{b}(a^{FB}, R) \) strictly increases in \( R \) as long as \( \tilde{b}(a^{FB}, R) < b^*(a^{FB}) = b^{FB} \), which guarantees \( dU^B/dR = 1 - 1/v_b(a^{FB}, \tilde{b}(\cdot)) > 0 \). Hence, we must show that - for any arbitrary split of total surplus - the accompanied repayment level \( R \) satisfies \( \tilde{b}(a^{FB}, R) < b^{FB} \).

To see that this property indeed holds, notice that \( \tilde{b}(a^{FB}, R) < b^* \) for any \( R \leq \tilde{R} \equiv v(a^{FB}, b^{FB}) + (L - K) \) by the definition of \( \tilde{b}(\cdot) \). Inserting \( \tilde{R} \) in \( U^B \), we obtain \( U^B(R = \tilde{R}) = v(a^{FB}, b^{FB}) - b^{FB} - K > S(a^{FB}, b^{FB}) \). For \( R = \tilde{R} \), agent \( B \) appropriates more than the entire social surplus so that \( R \geq \tilde{R} \) violates the participation constraint of agent \( A \). Conversely, for any \( R < \tilde{R} \), \( \tilde{b}(a, R) < b^{FB} \) is satisfied which completes the proof. \( Q.E.D. \)

**Proof of Proposition 4.** Proposition 4(a) follows immediately from Proposition 1 and results in Nöldeke and Schmidt (1998). If \( A \) expends \( a \geq a^{FB} \), \( B \) has no credible threat to invest less than \( b^{FB} \) at date 3 so that renegotiation does not arise. Also, \( A \) will never invest less than \( a^{FB} \) because \( B \) can assure herself a continuation payoff \( R \) by not investing and insisting on the repayment at date 5, and because date-2 renegotiations will only raise this payoff. Since \( A \) reaps \( S(a^{FB}, b^{FB}) - (R - L) \) when investing efficiently, any deviation reduces his payoff and hence cannot be part of an equilibrium strategy.

To prove Proposition 4(b), observe first that \( B \) cannot credibly insist on renegotiation if \( A \) chooses an investment \( a \geq a^{FB} \) (see Lemma 1). If \( A \) invests \( a < a^{FB} \), \( B \)'s default response under an option contract with strike price \( p^{**} \) is (a) \( b^*(a) \) if \( b^*(a) \leq \tilde{b}(a, R) \), and (b) \( \tilde{b}(a, R) \) if \( b^*(a) > \tilde{b}(a, R) \) (see Proposition 2). In the former case, no renegotiation occurs and \( B \) seizes the asset as a collateral at date 5 because \( A \) cannot repay. Hence, \( A \) reaps a non-positive payoff and a deviation cannot be optimal. In the latter case, \( B \)'s default investment \( \tilde{b}(a, R) \) is strictly smaller than the conditionally efficient level. Accordingly, the parties renegotiate at date 2, and \( B \) acquires the asset in order to ensure the conditionally efficient investment \( b^*(a) \). In these negotiations, \( A \) appropriates a payoff \( \gamma[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))] \) under Nash bargaining. He therefore deviates from \( a^{FB} \) if and only if there exists some \( a \) with \( \tilde{b}(a, R) \in [0, b^*(a)] \) for which

\[
p^{**} - a^{FB} = v(a^{FB}, b^{FB}) - b^{FB} - R + \tilde{b}(a^{FB}, R) < \gamma[v(a, b^*(a)) - b^*(a) - (R - \tilde{b}(a, R))] - a\]

is satisfied (condition (C3)). In the main text, we show that a deviation might indeed arise. Finally, no strike price other than \( p^{**} \) can induce an efficient outcome either: for any strike price \( p < p^{**} \), \( A \) will underinvest because \( B \) then exercises her option for some effort levels \( a < a^{FB} \). Conversely, for \( p > p^{**} \), \( B \) never exercises at \( a = a^{FB} \) by construction of \( p^{**} \). \( Q.E.D. \)
Proof of Proposition 5. To prove Proposition 5, notice again that \( A \) will never undertake \( a > a^{FB} \). Hence, consider a deviation \( a < a^{FB} \). For any such deviation, \( B \) will expend zero effort if renegotiation fails. Also, notice that \( A \) will never deviate in a way that \( b(a, R^C) = 0 \), i.e., \( v(a,0) > R^C \), because \( C \) can then insist on a repayment \( R^C \) at date 5 and will never agree on a smaller payoff in any renegotiations. Hence, consider deviations which satisfy \( b(a, R^C) > 0 \), i.e., \( v(a,0) < R^C \), and suppose date-2 renegotiations are unsuccessful. Then, \( A \) cannot repay \( R^C \) at date 5, and \( C \) seizes the asset and obtains a continuation payoff \( v(a,0) \). In equilibrium, renegotiation is successful and allows to realize a bargaining gain \( [v(a,b^*(a)) - b^*(a) - v(a,0) - a] \). Assuming that the outcome of renegotiation is described by the 3-persons Nash-bargaining rule, agent \( A \) appropriates a fraction \( \gamma_A \) of this bargaining surplus. Therefore, he will deviate from \( a = a^{FB} \) if and only if there exists some \( a \) which satisfies \( b(a, R^C) > 0 \) and

\[
p^* - a^{FB} = v(a^{FB}, b^{FB}) - a^{FB} - b^{FB} - R^C < \gamma_A [v(a,b^*(a)) - b^*(a) - v(a,0) - a] \quad (C4).
\]

To give an example, suppose the asset value is \( v(a,b) = 2a^{1/2} + 2b^{1/2} \). For this functional form, one obtains \( a^{FB} = b^*(a) = b^{FB} = 1 \) and \( S(a^{FB}, b^{FB}) = 2 - K \). Note that \( A \) will choose \( a = 0 \) when he deviates so that \( v(a,0) = 0 < R^C \) which implies \( b(a, R^C) > 0 \). Accordingly, condition (C4) reads \( 2 - R^C < \gamma_A [2(b^{FB})^{1/2} - b^{FB}] = \gamma_A \) which is satisfied at least if \( K > 2 - \gamma_A \). Hence, outsider financing cannot implement an efficient outcome if \( K \in (2 - \gamma_A, 2) \). Q.E.D.

Appendix B

Efficiency proof for the excursion in Section 4.

Suppose \( A, B, \) and \( C \) sign the following contract \( (L = K, R^C, p, F) \). \( C \) lends \( A \) an amount \( K \) with a repayment of \( R^C \) at date 5. \( A \) is asset owner initially but \( B \) holds an option to buy it at date 2 at a strike price \( p \). If \( B \) does not exercise her option, she has to make a payment \( F \) to \( C \), who in addition becomes asset owner without compensating \( A \). Also, \( C \) is then allowed to sell the asset to \( B \). Let \( p = p_0 = v(a^{FB}, b^{FB}) - b^{FB} \) and \( F = \tilde{g}_B[v(a^{FB}, b^{FB}) - b^{FB} - v(a^{FB}, 0)] \) where \( \tilde{g}_B \in [0, 1] \) indicates \( B \)'s bargaining strength in bilateral negotiations with \( C \).

Suppose first that \( B \) does not exercise at date 2. Then, \( C \) becomes asset owner and in addition receives a monetary payment \( F \) from \( B \). Subsequently (but prior to date 3), \( B \) and \( C \) renegotiate on an ownership transfer and \( B \) acquires the asset. The bargaining gain in these negotiations is \( [v(a,b^*(a)) - b^*(a) - v(a,0)] \) from which \( B \) reaps a fraction \( \tilde{g}_B \). Her overall payoff from not exercising her option is thus

\[
\tilde{g}_B[v(a,b^*(a)) - b^*(a) - v(a,0)] - F.
\]

Conversely, if \( B \) exercises, no renegotiation arises and her overall payoff is

\[ v(a,b^*(a)) - b^*(a) - p. \]

Accordingly, \( B \) will exercise at date 2 for a given \( a \) if and only if

\[
(1 - \tilde{g}_B)[v(a,b^*(a)) - b^*(a)] + \tilde{g}_B v(a,0) \geq (1 - \tilde{g}_B)[v(a^{FB}, b^{FB}) - b^{FB}] + \tilde{g}_B v(a^{FB}, 0).
\]
Inspection reveals that $B$ exercises if and only if $a \geq a^{FB}$. Since $A$’s overall payoff is non-positive for any $a < a^{FB}$ and positive for $a = a^{FB}$, he will not deviate and invest efficiently for any feasible level of $R^C$. Consequently, $A$ exercises her option and invests $b^{FB}$ so that a first best outcome is attained. Q.E.D.

References


contract, loan, outlay $K$

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