Econ 387: Spring 2006
Assignment 4: Term Structure of Interest Rates

1. Consider a closed populated by a representative agent with preferences defined over time-dated consumption (c_1, c_2, c_3). These preferences are given by:

$$U(c_1, c_2, c_3) = \ln c_1 + \beta \ln c_2 + \beta^2 \ln c_3;$$

so that:

$$\begin{aligned} MRS(c_1, c_2) &= \frac{1}{\beta}\frac{c_2}{c_1}; \\ MRS(c_2, c_3) &= \frac{1}{\beta}\frac{c_3}{c_2}; \\ MRS(c_1, c_3) &= \frac{1}{\beta^2}\frac{c_3}{c_1}; \end{aligned}$$

where $0 < \beta < 1$.The representative agent has an exogenous endowment (y_1, y_2, y_3). Let $\gamma_{j+1} = y_{j+1}/y_j$ for $j = 1, 2$ (the gross growth rate in real per capita GDP). Let R_{12} denote the (gross) short-term real rate of interest (i.e., the interest rate prevailing on a risk-free debt instrment issued in period 1 and maturing in period 2). Let R_{23} denote the *future* short-term interest rate (i.e., the interest rate prevailing on a risk-free debt instrument issued in period 2 and maturing in period 3). Let R_{13} denote the long-term interest rate (i.e., the interest rate prevailing on a risk-free debt instrument issued in period 1 and maturing in period 3).

   (a) Derive the general equilibrium interest rates $(R^*_{12}, R^*_{23}, R^*_{13})$.

   (b) Show that $R^*_{13} = R^*_{12}R^*_{23}$. Provide an interpretation of this condition.

   (c) Define the interest rate $R^*_L = \sqrt{(R^*_{12}R^*_{23})}$. Explain how this long-term interest rate is related to the average (real) growth rate of the economy from period 1 to period 3.

   (d) The slope of the real term structure is defined as $R^*_L - R^*_{12}$. Business economists claim that when the slope of the term structure is positive, that this signals higher future growth. Conversely, when the slope of the term-structure is negative (a so-called inverted yield curve), that this signal lower future growth. According to this model, is there any merit in this view? Explain.

2. Let's assume that the price-level is determined by a simple QTM condition; i.e., $p^*_j = M_j/y_j$, for $j = 1, 2, 3$, where M_j denotes the money supply at date j. Assume that $M_{j+1}/M_j = \mu_{j+1} > 1$ for $j = 1, 2$ (i.e., the monetary authority is expanding the money supply at gross rate μ_{j+1}). Let I_{ij} denote the (gross) nominal interest rate between periods i and j.

   (a) Assuming that the nominal interest rate is determined by the Fisher equation, derive an expression for the slope of the nominal term structure of interest rates.

   (b) Most business economists do not make a distinction between real and nominal interest rates. Is this a mistake? In particular, explain how it is possible to observe a positively sloped nominal yield curve and a negatively sloped real yield curve.