

Econ 387  
March 9, 2006

- [1] [10 Marks]. What is the definition of an indifference curve? Why are indifference curves an important component of macroeconomic analysis? (Provide two reasons).

*An indifference curve is defined as the set of all commodity bundles that an individual ranks identically. Indifference curves (preferences) are important in macro because: [1] they describe an individual's willingness to substitute across commodities (thereby helping us make predictions); and [2] the relative position (or level) of indifference curves allows us to evaluate the welfare consequences of any change in the economic environment.*

- [2] [10 Marks]. According to the *Intertemporal Substitution of Leisure Hypothesis*, a transitory wage change is likely to induce a much stronger labor supply response relative to a permanent wage change of identical size. Explain the logic.

*A transitory increase in the wage increases the price of leisure today relative to tomorrow. Hence, it makes sense to substitute leisure today (when leisure is expensive) for leisure tomorrow (where leisure is cheap). In other words, increase labor today (where it is fruitful to work) and take a break tomorrow (when the return to work is lower). A permanent increase in the wage does not change the relative price of leisure across time; therefore, this intertemporal substitution effect is absent.*

- [3] [30 Marks]. Consider a closed economy that is populated by a representative agent with preferences for  $(c_1, c_2)$  such that  $MRS = c_2/(\beta c_1)$  and a nonstorable endowment  $(y_1, y_2)$ , where  $y_2 = \gamma y_1$  and  $\gamma > 0$ . Assume that there is a government that desires to consume some fraction  $0 \leq \theta_j < 1$  of GDP in each period  $j = 1, 2$ ; i.e., so that  $g_j = \theta_j y_j$ . The government finances its expenditures with a set of lump-sum taxes  $(\tau_1, \tau_2)$ . Individuals (and the government) are free to borrow and lend in a risk-free debt market; let  $R$  denote the (gross) real rate of interest.

- [a] Write down a mathematical statement describing the choice problem facing individuals, given some interest rate  $R$  and given some sequence of taxes  $(\tau_1, \tau_2)$ .

*Choose  $(c_1, c_2)$  to maximize  $u(c_1, c_2)$  subject to  $c_1 + c_2/R \leq y_1 - \tau_1 + (y_2 - \tau_2)/R$ .*

- [b] Write down a mathematical statement describing the government's budget constraint.

*$g_1 + g_2/R \leq \tau_1 + \tau_2/R$  or  $\theta_1 y_1 + \theta_2 y_2/R \leq \tau_1 + \tau_2/R$  (either is correct).*

[c] Solve for the equilibrium interest rate  $R^*$  as a function of  $\gamma, \beta, \theta_1$  and  $\theta_2$ .

We know that the following must be true:

$$\frac{c_2}{\beta c_1} = R; \quad (1)$$

$$c_1 + \frac{c_2}{R} = y_1 - \tau_1 + \frac{y_2 - \tau_2}{R}; \quad (2)$$

$$g_1 + \frac{g_2}{R} = \tau_1 + \frac{\tau_2}{R}; \quad (3)$$

$$c_1 + g_1 = y_1; \quad (4)$$

$$c_2 + g_2 = y_2. \quad (5)$$

Since this is a representative agent economy, we know that there will be no trade in equilibrium. Consequently, we do not have to solve for the consumer demand functions to answer this question. That is, we can just combine (1) with (4) and (5) to derive:<sup>1</sup>

$$R^* = \frac{1}{\beta} \frac{(y_2 - g_2)}{(y_1 - g_1)} = \frac{1}{\beta} \frac{(y_2 - \theta_2 y_2)}{(y_1 - \theta_1 y_1)} = \frac{\gamma (1 - \theta_2)}{\beta (1 - \theta_1)}.$$

[d] Explain how  $R^*$  depends on the parameters  $\gamma$  and  $\beta$  (provide economic intuition and an interpretation of these parameters).

The parameter  $\gamma = y_2/y_1$  represents the growth rate in real GDP (or income). An increase in  $\gamma$  leads to an increase in  $R^*$ . An example here is if individuals receive ‘news’ that their future income will be higher than originally anticipated. Consumption-smoothing motives imply that individuals would like to increase their consumption at all dates. The only way to increase current consumption is by borrowing. But since everyone wants to borrow, there is an excess demand for loans. Therefore, the interest rate must rise to clear the financial market.

The parameter  $\beta$  is a measure of ‘patience.’ An increase in  $\beta$  leads to a reduction in  $R^*$ . That is, if everyone suddenly becomes more patient, then they want to move consumption from the present to the future. The only way to do this is by saving more. If everyone saves more, there will be an excess supply of loanable funds. Therefore, the interest rate must fall to clear the financial market.

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<sup>1</sup>Alternatively, one could solve for the consumer demand function:

$$c_1^D = \left( \frac{1}{1 + \beta} \right) \left[ y_1 + \frac{y_2}{R} - \tau_1 - \frac{\tau_2}{R} \right].$$

Substituting in the GBC and imposing the market-clearing condition  $c_1^D + g_2 = y_1$  would yield:

$$\left( \frac{1}{1 + \beta} \right) \left[ y_1 + \frac{y_2}{R^*} - g_1 - \frac{g_2}{R^*} \right] + g_1 = y_1.$$

One can then solve this equation for  $R^*$ .

- [e] Imagine that initially, the Ministry of Finance sets taxes to balance the budget in each period; i.e.,  $\tau_1 = \theta_1 y_1$  and  $\tau_2 = \theta_2 y_2$ . Imagine further that there is an exogenous shock (e.g., a war) that requires a transitory increase in government spending (i.e.,  $\Delta\theta_1 > 0$ ). What effect does this shock have on the interest rate and why?

According to  $R^* = \left(\frac{\gamma}{\beta}\right) \left(\frac{1-\theta_2}{1-\theta_1}\right)$ , an increase in  $\theta_1$  will lead to an increase in  $R^*$ . This can be explained as follows. First, the increase in  $g_1$  must imply lower household wealth, leading to a decrease in both  $c_1^D$  and  $c_2^D$ . Note that since the impact on consumption is spread throughout time (consumption smoothing again), we have  $|\Delta c_1^D| < |\Delta g_1|$  (the decrease in consumer demand is less than the increase in government demand for current output). Hence, the aggregate demand for current period consumption rises. But now we have an excess demand for current consumption; for the market to clear, the price of current consumption ( $R$ ) must rise.<sup>2</sup>

- [f] How does your answer to the previous question change if the government keeps current taxes fixed at their initial level (so that  $\Delta\tau_1 = 0$ )?

The answer to the previous question would not change at all. In this model, the conditions of the Ricardian Equivalence Theorem are satisfied. Therefore, financing the  $\Delta g_1$  via a deficit serves only to postpone taxes, leaving household wealth unchanged. In other words, the increase in the interest rate will be exactly the same, whether the expansionary fiscal policy is financed by higher (current) taxes or a deficit (higher future taxes). The timing of taxes does not matter here.

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<sup>2</sup> Alternatively, you could have answered in the following way. From the definition of desired domestic saving:

$$s^D = y_1 - c_1^D - g_1.$$

In general equilibrium (in this model) we know that  $R$  will adjust such that  $s^D = 0$ . Suppose now that  $g_1$  rises by one unit. We know (from consumption smoothing motive) that  $c_1^D$  will fall by less than one unit. Therefore,  $\Delta s^D < 0$  at the original interest rate. In order to restore  $s^D = 0$ , the interest rate must rise.