Inflation

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Introduction

• We continue to assume an economy with a single asset $M$

• Assume that the government can manage the supply of $M$ over time; i.e., $M_t = \mu_t M_{t-1}$, where $\mu_t > 0$ is the gross rate of money creation
  
  – $\mu_t > 1$ implies expanding money supply
  
  – $\mu_t < 1$ implies contracting money supply

• New money created at date $t$
  
  $$\Delta M_t = (\mu_t - 1)M_{t-1} = (1 - 1/\mu_t)M_t$$
How is money injected/withdrawn?

• The answer to this question generally matters (although, it is often neglected)

• In reality, the Fed (and most central banks) are restricted to use $\Delta M_t$ to purchase assets (primarily government bonds, but in some cases, also private securities)

• We cannot investigate asset swaps yet because we have only one asset

• For now, assume that the fiscal authority manages the economy’s money supply (fiscal authority has the power to tax and purchase goods and services)
The government budget constraint

- Let $G_t$ denote government purchases and let $T_t$ denote net government tax revenue.

- I allow the government to pay interest on money; let $R_t$ denote the gross nominal interest rate.

\[ G_t + (R_t - 1)M_{t-1} = T_t + [M_t - M_{t-1}] \]

- Note that money is like a perpetual bond (has no maturity date).
• Rearrange the expression above...

\[ G_t + \left[ \frac{R_t}{\mu_t} - 1 \right] M_t = T_t \]

• In what follows, assume that \( G_t = 0 \) and that \( T_t \) is a lump-sum tax administered on the old (if \( T_t < 0 \), then \( T_t \) is a transfer)

• If \( \mu_t = R_t \), then \( \Delta M_t \) is sufficient to finance interest on debt (so \( T_t = 0 \))

• If \( \mu_t \geq R_t \) then \( T_t \leq 0 \)

• Sometimes the restriction \( R_t \geq 1 \) is imposed (zero lower bound)
• Define \( \tau_t = v_t T_t / N_{t-1} \)

• A policy is a sequence \( \{R_t, \mu_t, \tau_t\}_{t=1}^{\infty} \) satisfying

\[
\tau_t = \left[ \frac{R_t}{\mu_t} - 1 \right] n \left( \frac{v_t M_t}{N_t} \right)
\]

• In a stationary equilibrium, \( v_t M_t / N_t = k \) (constant)

• In this case, a policy can be represented by a triplet \( (R, \mu, \tau) \) satisfying

\[
\tau = \left[ \frac{R}{\mu} - 1 \right] k
\]
Individual decisions

- A young person’s budget constraints:

\[ c_t^y + v_t m_t = y \]
\[ c_{t+1}^o = v_{t+1} R_{t+1} m_t - v_{t+1} T_{t+1}/N_t \]

or

\[ c_t^y + q_t = y \]
\[ c_{t+1}^o = \frac{R_{t+1}}{\Pi_{t+1}} q_t - \tau_{t+1} \]

combining...

\[ c_t^y + \frac{\Pi_{t+1}}{R_{t+1}} c_{t+1}^o = y - \frac{\Pi_{t+1}}{R_{t+1}} \tau_{t+1} \]
Equilibrium conditions

- Money demand function $q_t$ is characterized by usual “tangency” condition:

$$MRS\left(y-q_t, \frac{R_{t+1}}{\Pi_{t+1}}q_t - \tau_{t+1}\right) = \frac{R_{t+1}}{\Pi_{t+1}} \text{ for all } t \geq 1 \quad (1)$$

- Market-clearing condition:

$$v_t M_t = N_t q_t \text{ for all } t \geq 1 \quad (2)$$

- Government budget constraint:

$$\tau_t = \left[\frac{R_t}{\mu_t} - 1\right] n \left(\frac{v_t M_t}{N_t}\right) \text{ for all } t \geq 1 \quad (3)$$
• Combine (2) and (3) to form

\[ \tau_t = \left[ \frac{R_t}{\mu_t} - 1 \right] nq_t \]

and now combine with (1)...

\[ MRS \left( y - q_t, \frac{R_{t+1}}{\Pi_{t+1}} q_t - \left[ \frac{R_{t+1}}{\mu_{t+1}} - 1 \right] nq_{t+1} \right) = \frac{R_{t+1}}{\Pi_{t+1}} \]

• Market-clearing condition implies...

\[ \Pi_{t+1} = \frac{\mu_{t+1}}{n} \frac{q_t}{q_{t+1}} \]

Combining...

\[ MRS \left( y - q_t, \frac{R_{t+1}}{\mu_{t+1}} nq_{t+1} - \left[ \frac{R_{t+1}}{\mu_{t+1}} - 1 \right] nq_{t+1} \right) = q_t^{-1} \frac{R_{t+1}}{\mu_{t+1}} nq_{t+1} \]
• Which reduces to...

\[
MRS(y - q_t, nq_{t+1}) = q_t^{-1} \frac{R_{t+1}}{\mu_{t+1}} nq_{t+1}
\]  \hspace{1cm} (4)

• Condition (4) is a first-order difference equation in \( \{q_t\}_{t=1}^{\infty} \)

• We restrict attention to policies such that \((R_t, \mu_t) = (R, \mu)\) and steady-states \(q_t = q\); condition (4) then reduces to

\[
MRS(y - q, nq) = nR/\mu
\]  \hspace{1cm} (5)

• Observation: Any policy (in this class of policies) that sets \(R = \mu\) will implement the Golden rule allocation
• Note that the equilibrium (steady-state) inflation rate in this economy is given by

\[ \Pi = \mu/n \]

• Definition: money is said to be superneutral if permanent changes in \( \mu \) have no effect on real allocations

  – contrast this with the definition of money neutrality

• Note that money is superneutral here if new money is injected by way of interest
The inefficiency of inflation

- Set $R = 1$ and consider some $\mu > 1$

- Because this is an endowment economy, inflation will have no effect on the real GDP

- It will, however, distort the allocation $(c^y, c^o)$ relative to the GR allocation

- But what are the welfare effects of this policy (lump-sum transfer to the old, financed by printing money)?
• Let \((c^y, c^o)\) denote the GR allocation; it satisfies

\[
MRS(c^y, c^o) = n
\]
\[
c^y + (1/n)c^o = y
\]

• The competitive monetary equilibrium satisfies

\[
MRS(\hat{c}^y, \hat{c}^o) = n/\mu
\]
\[
\hat{c}^y + (\mu/n)\hat{c}^o = y - (\mu/n)\hat{r}
\]

where

\[
\hat{r} = [1/\mu - 1] n\hat{q}
\]
\[
= [1/\mu - 1] n(y - \hat{c}^y)
\]

• Substitute into budget constraint...

\[
\hat{c}^y + (\mu/n)\hat{c}^o = y - (\mu/n) [1/\mu - 1] n(y - \hat{c}^y)
\]
and rearrange to derive

\[ \hat{c}^y + \left( \frac{1}{n} \right) \hat{c}^o = y \]

- That is, the competitive equilibrium lies on the resource constraint (and on the budget constraint)

- Now, since \( \mu > 1 \), it follows that

\[ MRS(c^y, c^o) = n > MRS(\hat{c}^y, \hat{c}^o) = n/\mu \]

- Conclusion: \( \hat{c}^y > c^y \) and \( \hat{c}^o < c^o \)

  - and lower welfare (display on diagram)
Permanent vs temporary money injections

- Recall that a one-time $\Delta M$ to the initial old is neutral in this model
  - the money supply is *expected* to increase once and forever, so price-level jumps up, but no permanent inflation

- On the other hand, an increase in $\mu$ implies $\{\Delta M, \Delta M, \ldots\}$
  - the money supply is *expected* to increase in every period forever, leading to inflation (and a jump up in the price-level—*why*?)

- I emphasize *expected* to make clear that population has to believe that the policy will be carried through
Does money cause inflation, or vice-versa?

- I have taken the traditional approach of assuming an exogenous money supply path

- But it may make more sense to think of government adopting a policy rule which dictates how the future money supply is to evolve in response to past, current, and/or expected economic variables

- Suppose, for example, it is policy to fully accommodate inflation (inflation expectations)

- Then pick an arbitrary $\Pi$ and it will “cause” $\{\Delta M, \Delta M, \ldots\}$ to adjust accordingly
The limits to seigniorage

- **Seigniorage** refers to the purchasing power a government is able to create by printing money
  - also referred to as the *inflation tax*

- In most developed economies, seigniorage is small potatoes
  - relatively more important for lesser developed economies

- Limits to seigniorage similar to the limits of any distortionary tax (people substitute out of taxed activity)
Financing government purchases

- Recall government budget constraint

\[ G_t + \left[ \frac{R_t}{\mu_t} - 1 \right] M_t = T_t \]

- Now set \( R_t = 1 \) and \( T_t = 0 \)

- Define \( g_t = v_t G_t / N_t \) and rewrite GBC as

\[ g_t = \left[ 1 - \frac{1}{\mu_t} \right] \frac{v_t M_t}{N_t} \]

- Assume that government simply consumes \( g_t \)
• Individual decision-making is unchanged, so $q_t$ determined by

$$MRS(y - q_t, \Pi_{t+1}^{-1}q_t) = \Pi_{t+1}^{-1}$$

• Market-clearing and stationarity gets the usual $\Pi = \mu/n$; define $\hat{q}(\mu)$ by

$$MRS(y - \hat{q}, (n/\mu)\hat{q}) = (n/\mu)$$

• The properties of $\hat{q}(\mu)$ depend on the nature of preferences, but if substitution effect is strong, then $\hat{q}$ is a decreasing function of $\mu$ (inflation)

• It follows that $[1 - 1/\mu] \hat{q}(\mu)$ is first increasing in $\mu$, then decreasing in $\mu$ (Laffer curve)
• Let $\mu^*$ maximize $[1 - 1/\mu] \hat{q}(\mu)$

• Note that $1 < \mu^* < \infty$

• Maximum seigniorage revenue is given by $[1 - 1/\mu^*] \hat{q}(\mu^*)$ and feasibility requires $g \leq [1 - 1/\mu^*] \hat{q}(\mu^*)$

• And for any such $g$, there generally exist two money growth (inflation) rates $1 < \mu_L < \mu_H < \infty$ satisfying

$$g = [1 - 1/\mu_L] \hat{q}(\mu_L) = [1 - 1/\mu_H] \hat{q}(\mu_H)$$

• Allocation associated with high inflation rate is Pareto inferior
Currency substitution

- The limits to how much a government can extract by way of a distortionary tax depends on how easily individuals can “escape” the tax by substituting into other activities.

- In the model above, individuals substitute out of the “cash good” \( (c^o) \) into the “non-cash good” \( (c^y) \)
  
  - this affects the size of the tax base \( q = y - c^y \)

- In a model with multiple currencies (e.g., USD and Argentine peso), people may have an opportunity to substitute into competing currencies (unless prohibited by law, which is frequently the case).
Inflation and output

<table>
<thead>
<tr>
<th>Inflation Episodes</th>
<th>Symbol</th>
<th>Mean Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 March 1952 to September 1965</td>
<td>Diamond</td>
<td>1.3</td>
</tr>
<tr>
<td>2 December 1965 to June 1972</td>
<td>Triangle</td>
<td>4.1</td>
</tr>
<tr>
<td>3 September 1972 to September 1982</td>
<td>Cross</td>
<td>8.3</td>
</tr>
<tr>
<td>4 December 1982 to June 1991</td>
<td>Square</td>
<td>3.8</td>
</tr>
<tr>
<td>5 September 1991 to September 2004</td>
<td>Dash</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Russell and Banerjee, JM 2008
• Output and inflation appear to be positively correlated in the short-run, and negatively correlated in the long-run.

• To explore the impact of inflation on output, we need a theory of output determination.

• Assume that the young are endowed with time and that time can be used to produce output.

• $l_t$ units of time produces $zl_t$ units of output at disutility cost $-h(l_t)$
  
  - $z$ is a productivity parameter, $h$ is an increasing, strictly convex function.
• Let $h'(l_t)$ denote the marginal disutility of labor

• The marginal product of labor is given by the parameter, $z$

• Assume, for simplicity, that the young do not care about current consumption; preferences are

$$U_t = -h(l_t) + \beta u(c_{t+1})$$

• $\beta$ is a preference parameter and $u$ is increasing, concave, marginal utility of future consumption is $\beta u'(c_{t+1})$
• Budget constraints...

\[ vt m_t = zl_t \]
\[ c_{t+1} = v_{t+1} m_t - v_{t+1} T_{t+1}/N_t \]

or,

\[ q_t = zl_t \]
\[ c_{t+1} = \frac{R_{t+1}}{\Pi_{t+1}} q_t - \frac{1}{\Pi_{t+1}} \tau_{t+1} \]

• Plug into objective...

\[ \max_{l_t} \left\{ -h(l_t) + \beta u \left( \frac{R_{t+1}}{\Pi_{t+1}} zl_t - \frac{1}{\Pi_{t+1}} \tau_{t+1} \right) \right\} \]
• Marginal cost/benefit calculation (first-order condition):

\[ h'(l_t) = \frac{R_{t+1}}{\Pi_{t+1}} z \beta u'(c_{t+1}) \]

• We can simplify this considerably (with only some loss of generality) by assuming linear \( u(c) = c \), so that \( u'(c) = 1 \)

• In this case, employment is determined by

\[ h'(l_t) = \frac{R_{t+1}}{\Pi_{t+1}} z \beta \]

(6)

• Employment is increasing in \( R, z, \beta \) and decreasing in \( \Pi \)
• With $l_t$ determined by (6), demand for real money balances is $q_t = zl_t$

• Real GDP is given by $N_t zl_t$

• Imposing market-clearing and stationarity, we have

$$h'(l) = zR\beta$$

• An increase in the (long-run) money growth rate (inflation) leads to a lower level of employment and GDP