Econ 807: Macroeconomic Theory and Policy
Assignment 5: Computing Optimal Growth Trajectories

1. Consider the optimal growth model with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

with $0 < \beta < 1$ and $c_t = Ak_t^{\alpha} + (1-\delta)k_t - k_{t+1}$. Let $k_0 > 0$ be given.Write down the conditions that characterize the optimal capital stock trajectory.

2. On the course webpage, there is a GAUSS code that solves for the optimal trajectory for the parameter values $\beta = 0.96$, $\alpha = 0.35$, $A = 0.7287$, $\delta = 0.10$, and $k_0 = 0.25k^*$. Note that the parameter A was chosen so as to normalize the steady-state level of output to unity. Study this code.

   (a) How far below (in percentage terms) is the initial level of GDP relative to its steady-state value? How many years (periods) does it take for the GDP to grow within one percent of its long-run value? Plot the GDP trajectory on a graph.

   (b) Assume now that $\delta = 1$. In this case, we know that the optimal capital stock trajectory must satisfy $k_{t+1} = \alpha\beta Ak_t^{\alpha}$, with $k_0 > 0$ given. Solve for the optimal trajectory using the solution algorithm in the GAUSS code and compare your solution to the true solution.

3. Instead of solving for the optimal sequence $\{k_{t+1}^*\}$, one could try to solve for the optimal policy function $k' = g(k)$ using the Coleman algorithm (note: k' denotes the 'next' period's capital stock and k denotes the current period's capital stock). Write a GAUSS code that solves for the function g on a finite grid and use a linear interpolation procedure to approximate the true solution function. Using your solution, simulate the optimal capital stock trajectory beginning with $k_0 = 0.25k^*$ and compare the results to what you calculated in question 2.