Econ 809 Spring 2006
Assignment

1. Time is discrete and the horizon is infinite; $t = 0, 1, ..., \infty$. Each period is divided into two subperiods; stage 1 and stage 2 (day and night). There is a unit mass of infinitely-lived *ex ante* identical agents $i \in [0, 1]$, with preferences defined over stochastic sequences:
$$\{e_t(i), c_t(i), n_t(i) : t \geq 0\},$$
where $e_t(i)$ denotes effort during stage 1; $c_t(i)$ denotes consumption during stage 2; and $n_t(i)$ denotes effort during stage 2. These preferences are represented by an expected utility function:
$$E_0 \sum_{t=0}^{\infty} \beta^t [-\alpha e_t(i) + u(c_t(i)) - \theta_t(i) g(n_t(i))],$$
with $0 < \beta < 1$, $\alpha > 0$, and where $\theta_t(i) \in \{\omega_L, \omega_H\}$, $0 < \omega_L < \omega_H < \infty$. Let $\Pr[\theta_t(i) = \omega_L] = 1/2$ for all i and for all t. Assume that $u'' < 0 < u'$ and $0 < g', g''$ with $g(0) = 0$ and $u(0) = u_0 \geq -\infty$. The available production technologies are as follows. Assume that $e_t(i) \in \mathcal{R}$, so that first-stage effort is unbounded from above and below. Second stage output is produced according to $y_t(i) = n_t(i)$. Assume that $n_t(i) \geq 0$ and that output is nonstorable.

   (a) Characterize the autarkic allocation and welfare level.

   (b) Characterize the solution to a planner's problem that maximizes *ex ante* welfare. Is this in any sense a 'credit' economy? Describe the pattern of exchanges recommended by the planner. Could such an allocation be decentralized in any way? Explain.

   (c) Assume now that $\theta_t(i)$ is private information. Characterize the constrained-efficient allocation (assuming that sequential IR is not a problem). Provide economic intuition.

   (d) Assume now that the planner can observe $\theta_t(i)$, but that agents cannot commit. Assume that the planner can banish individuals to a state of autarky. Characterize the constrained-efficient allocation. Provide economic intuition.