Banking Crises in Monetary Economies*

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Abstract

This paper analyzes the effect of inflation on banking crises in a model in which money and banks play essential roles. The model’s equilibrium replicates some key features of actual banking crises; namely, the partial suspension of payments, and the desire to hold cash even in the absence of pressing liquidity needs. When banks have access to a stable foreign currency, inflation has a threshold effect on banking crises: higher inflation reduces the likelihood of crises when inflation is below the threshold; the reverse happens when inflation exceeds the threshold. This result appears to be broadly consistent with available evidence.

Keywords: Inflation; Banking Crisis; Monetary Policy
JEL Categories: E40, E50

1 Introduction

This paper is concerned with explaining what appears to be a U-shaped relationship between banking crises and inflation; that is, the fact that banking crises appear much

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more likely to occur in either very ‘low’ inflation environments or in very ‘high’ inflation environments. Low inflation (and deflation) environments like the depression-era U.S., or Japan throughout its ‘lost decade’ of the 1990s, for example, were infamous for their widespread banking-sector troubles. Similar banking-sector problems appear to be present in several economies that feature very high rates of inflation; see Table 1, and the formal econometric investigation in Demirguc-Kunt and Detragiache (1998, 2005). Why this should be the case is not known. The purpose of this paper is to develop a possible rationale for this phenomenon.

<table>
<thead>
<tr>
<th>Banking crises</th>
<th>Net annual inflation (%) at start of crises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina 1980-1982</td>
<td>100.76</td>
</tr>
<tr>
<td>Argentina 1989-1990</td>
<td>3079.81</td>
</tr>
<tr>
<td>Bolivia 1986-1988</td>
<td>276.34</td>
</tr>
<tr>
<td>Brazil 1990</td>
<td>2947.73</td>
</tr>
<tr>
<td>Brazil 1994-1999</td>
<td>2075.89</td>
</tr>
<tr>
<td>Israel 1983-1984</td>
<td>145.64</td>
</tr>
<tr>
<td>Lebanon 1988-1990</td>
<td>127.84</td>
</tr>
<tr>
<td>Peru 1983-1990</td>
<td>111.15</td>
</tr>
<tr>
<td>Sierra Leone 1990-1993</td>
<td>110.95</td>
</tr>
<tr>
<td>Turkey 1994</td>
<td>106.26</td>
</tr>
</tbody>
</table>

SOURCE: IMF/IFS.
NOTES: The crisis episodes are identified in Demirguc-Kunt and Detragiache (2005).

To address the issue at hand, it seems clear that any passable theory will have to include, at the very least, the following three elements: [1] a role for banks; [2] a role for money; and [3] a set of shocks that can potentially trigger ‘crisis’ events. There are several potential modelling choices that one may take here concerning the role/nature of each one of these elements. There is also a question concerning the actual definition of what constitutes a banking ‘crisis.’ Let me first briefly review some of the relevant literature and then justify the general approach that I take.

First, it is almost conventional wisdom to suppose that banks are somehow ‘different’ from other private agencies (including other intermediaries, like insurance com-

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1Jonker and van Zanden (1995) study banking crises in the inter-war period and point out that ‘... As a rule it would seem as if crises occurred in countries which, following the collapse of the post-war boom, implemented deflationary policies in the run-up towards restoration of the gold standards. ... In Denmark, Sweden, and the Netherlands deflationary policies were introduced which combined with the depression to produce banking crises. Between 1920 and 1923 the price level in these countries fell by 21%, 23% and 27% respectively (Maddison, 1991:app.E).’
panies or pension funds). This difference stems from the peculiar liability structure of those agencies we label ‘banks;’ i.e., the demandable nature of their debt instruments. While this liability structure obviously has an economic purpose (with bank liabilities serving as an important payment instrument, and their demandable nature serving as a low-cost form of insurance against idiosyncratic liquidity needs), it allegedly opens the door to a form of inherent ‘fragility’ or ‘instability.’ That is, if everyone (for some unexplained reason) chooses to exercise their redemption option simultaneously (i.e., make a ‘run’ on the bank), then the bank will be forced to liquidate even positive net-present-value investments (at fire-sale prices) to make good on its obligations. With a sequential service constraint in place, depositors who are slow to act may be left with nothing. It is this fear that justifies ‘running’ (on the expectation that others will do) and which renders a bank-run a self-fulfilling prophecy.

Diamond and Dybvig (1983) were the first to attempt formalizing the concept of a bank-run as an equilibrium phenomenon (see also: Waldo, 1985, Cooper and Ross, 1998, Loewy, 1991, and Peck and Shell, 2003). I choose not to go this route for a number of reasons. First, most of these papers (with the exception of Loewy, 1991) feature models that are static in nature with no role for money. Second, the existence of multiple equilibria in these environments appears to be an artifact of exogenously imposed sub-optimal bank contracts (see Green and Lin, 1996, 2003). Third, it is by no means clear that actual bank crises are the product of expectations-driven shocks or other events related to a change in underlying fundamentals; see, for example, Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), Allen and Gale (1998), Morris and Shin (2000), and Goldstein and Pauzner (2005).

Thus, the approach I take here is that ‘crisis’ events are triggered by fundamentals; or, to be more precise, shocks to information relating to fundamentals (along the lines of Allen and Gale, 1998). But I also need a dynamic model and, in particular, a model that features money and banks. The basic framework I adopt here is based on Champ, Smith and Williamson (1996) and Smith (2003), who introduce money via an overlapping generations structure where banks exist to insure agents against random needs for liquidity. As with these authors, I do not focus on modelling a ‘crisis’ as a bank-run per se; rather, a ‘crisis’ is defined by particular behavior that maps into real-world phenomena that are commonly associated with crisis events (as in the widespread demand for liquidity by agents that would not normally desire it, along with what - on the surface at least - looks like a partial suspension of payments). Unlike these authors (see also Loewy, 2003), however, I choose to view the shocks precipitating crisis events as ‘technology’ shocks that alter the real value of a bank’s assets, rather than an exogenous shock to the aggregate demand for liquidity. My approach is also related in many ways to that of Loewy (1998), whose monetary Diamond-Dybvig model features ‘information-based’ shocks along the lines of Jacklin and Bhattacharya (1988); but whose analysis is focussed primarily on understanding a specific episode in U.S. banking history (1929-33).

The interpretative setup I choose is therefore based on a combination of Smith
As in Smith (2003), there is a friction that allows money to coexist with capital, even in the absence of aggregate uncertainty, and despite money being dominated in rate of return. Banks exist in the model to provide agents with insurance against idiosyncratic shocks to their liquidity needs. An optimal banking arrangement requires that banks take deposits, invest them in a portfolio that consists of capital investment and reserves of cash. The liabilities that they issue are made demandable for cash. Following Allen and Gale (1998) (and in contrast to Smith, 2003), I introduce an aggregate technology shock (in the form of ‘news’ that reveals the future return on the bank’s capital investment). The bank’s portfolio decision (cash versus capital) must be made before the arrival of this information. The presence of aggregate uncertainty generates an additional demand for money in the form of ‘precautionary balances,’ since the rate of return on money (the inverse of the inflation rate) is stable relative to the risky return on capital. The inflation rate is dictated solely by monetary policy.

In this environment, *ex ante* behavior (prior to the arrival of information about the productivity shock) all looks the same (a by-product of the fact that technology shocks are *i.i.d.* and that agents live for two periods only): the bank always chooses the same currency/deposit ratio. But *ex post* behavior falls into one of two classes, depending on whether the realization of the shock falls above or below some critical value (determined endogenously by the rate of return on money and the bank’s currency-to-deposit ratio). In ‘normal’ times (associated with news that the return on a bank’s capital is above the critical value), only depositors who are subject to a liquidity shock hold cash and in doing so, forgo the higher yield on capital (their interest-bearing deposits at the bank). Those who do not require liquidity enjoy the high return on their bank deposits and do not wish to hold any cash. Occasionally, however, people receive bad news that the return on the bank’s investment on capital will be abnormally low (below the critical value). In this event, even depositors without pressing liquidity needs request some cash to offset the low return on capital. In this event too, all depositors experience a lower-than-normal return on their deposits—an event I associate with a partial suspension of payments. Admittedly, this does not capture all of the features that one would normally associate with a banking crisis, it does appear to generate behavior that along some dimensions, at least, resembles observed behavior in many crisis episodes. As in Allen and Gale (1998), given the existence of liquidity and productivity shocks, banking crises are just part of an optimal risk sharing scheme: it allows depositors to share liquidity risk in the face of an unfavorable productivity shock.

I then ask, within the context of this model, how the probability of a crisis (so defined) is related to the conduct of monetary policy (inflation). The key thing to recognize here is that the frequency with which the economy experiences ‘good’ or ‘bad’ news depends on the frequency with which the *ex post* return to capital falls above or below the above-mentioned critical value. This critical value serves as a sort of ‘hurdle’ that the return to capital must exceed if the economy is to avoid an *ex post* allocation that I have associated with a ‘crisis’ event. Holding fixed all other
parameters (in particular, those governing the realization of technology shocks), I find that higher inflation reduces the probability of a crisis; in particular, it decreases the above-mentioned critical value. There are two effects associated with a higher inflation or lower return on cash: a direct and an indirect effect. The direct effect is that a decrease in the rate of return on money makes it more likely that the return to capital will exceed it. The indirect effect is that a lower rate of return on money induces a substitution in the bank’s portfolio away from cash and into capital, making it more likely now that the residual claimants (people that hold their bank deposits for longer periods of time) will receive a higher payoff. As a result, depositors without liquidity needs are more likely to prefer holding on to their bank deposits rather than cashing out, which in turn, means a lower probability of crises. This captures the downward part of the U-shaped relationship between banking crises and inflation.

To account for the whole U-shaped relationship between banking crises and inflation, I follow Antinolfi, Landeo and Nikitin (2006) to introduce a third asset - a stable foreign currency called ‘dollars’ - that can also provide insurance against liquidity and productivity shocks. With legal restriction on the domestic currency/capital ratio, the model is able to generate the threshold effect of inflation on banking crises. In particular, when domestic inflation is below a threshold, higher domestic money growth rate and inflation reduce the likelihood of banking crises, with the reverse happening when domestic inflation is above the threshold. The threshold is determined by the real rate of return of the foreign currency. When domestic inflation is low, domestic currency dominates foreign currency in rate of return. In this case, only domestic currency is used to insure against liquidity and productivity shocks, generating behavior that was described earlier. When domestic inflation is high, dollars dominate domestic currency in rate of return and are used to insure against liquidity and productivity shocks along with the domestic currency. The legal restriction on domestic currency/capital ratio binds, and dollars compete with domestic assets. In this case, higher domestic inflation induces the bank to substitute away from domestic capital and into dollars making it more likely now that the residual claimants will receive a lower payoff. As a result, depositors without liquidity needs are less likely to prefer holding on to their bank deposits rather than cashing out, which in turn, means a higher probability of crises.

The paper is organized as follows. Section 2 presents the basic model in which only domestic assets are available and investigates the effect of inflation on banking crises. Section 3 extends the basic model to include a stable foreign currency and re-investigates the relationship between inflation and banking crises. Section 4 concludes.

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2Antinolfi, Landeo and Nikitin (2006) use a similar framework to explain the threshold effect of inflation on capital investment and output.
2 The Basic Model

In this section, I study banking crises in a model with only domestic assets. I first describe the physical environment and talk about the role of money and banking in such an environment. I then characterize banking crises and examine the effect of inflation on banking crises.

2.1 Physical Environment

The economy consists of overlapping generations people who live for two periods and the initial old generation that lives for one period only. People inhabit on two locations, which I refer to as 'islands'. At each date \( t = 1, 2, 3, \ldots \infty \), a new generation is born on each location consisting of continuum of \( \textit{ex ante} \) identical young agents with unit mass. For simplicity, assume that people care only for consumption when old (this renders the saving decision trivial and allows me to focus on portfolio allocation). Each person is subject to an idiosyncratic relocation shock that is realized at the end of the first period of life. Let \( 0 < \pi < 1 \) denote the probability of being relocated (applying the law of large numbers, this also represents the fraction of young agents who transit from one location to the other - note that these flows are symmetric across locations). The expected utility of a representative young agent is given by:

\[
U = E \left[ \pi u(c_m) + (1 - \pi)u(c_n) \right];
\]

where \( c_m \) and \( c_n \) denote the consumption of movers and non-movers respectively, and \( u(c) = \ln c \).

Each young agent is endowed with \( y > 0 \) units of output. In addition, there is a storage technology where \( k \) units of investment (which must be made at the beginning of the agent’s first period of life) yields \( xk \) units of future output. Assume that \( x \) (an aggregate shock determining the realized return on capital investment) follows an exogenous stochastic process with cumulative distribution function \( F(a) \equiv \Pr[x \leq a] \). The distribution of \( x \) is \( i.i.d. \) over time. Assume further that \( x \) is realized when agents are young - at the same time as the realization of their idiosyncratic relocation shock. Hence, \( x \) takes the form of 'news' concerning the future return to contemporaneous capital expenditure. Finally, assume that capital depreciates fully after it is used in production and that goods are not transportable across locations.

Following Smith (2003), I also assume that private liabilities issued in one location cannot be used in the other location. As in Arouba, Waller, and Wright (2006),

\[ u(c) = c^{1-\sigma}/(1 - \sigma), \]

\( 0 < \sigma < 1 \).

Refer to Beaudry and Portier (2003) for a detailed discussion of ‘news’ shocks. For simplicity, I assume that the ‘news’ contains accurate information about the future return on capital.
one can suppose, for example, that private liabilities can be costlessly counterfeited outside the location in which they were issued.

If young agents (who are \textit{ex ante} identical) cannot communicate with each other across locations (assume that this is so), then the resulting allocation is essentially autarkic. That is, young agents can do no better than invest their entire endowment and then hope for the best (in particular, hope that they do not experience a relocation shock, and hope for a high return on their capital investment). It is, however, possible for young agents to attain a superior allocation if they have access to a public record-keeping technology or, absent this, fiat money (with fiat money serving as an imperfect substitute for an absent record-keeping technology; see Kocherlakota, 1998). I explain how in the following subsection.

\subsection*{2.2 Money and Banking}

Let the initial old (in each location) be endowed with $M_0$ units of fiat money and assume that the government expands the supply of money at an exogenously specified (gross) rate $z$, so that $M_t = zM_{t-1}$. New money is used to finance government purchases of output.\footnote{Alternatively, one could assume that new money is distributed to agents as a lump-sum transfer.} Fiat money cannot be counterfeited and so (unlike private money) can be used on both islands. In this environment then, fiat money will be valued for two reasons. First, as in Smith (2003), fiat money can provide insurance against idiosyncratic relocation shocks (usefully interpreted now as ‘liquidity’ shocks). Second, fiat money (offering more stable rate of return) can function as ‘precautionary balances’ to insures agents against low realizations of return to capital. These insurance properties of fiat money ensure that it can be valued even if it is dominated in expected rate of return by capital investment (which is assumed to be the case throughout the paper).

As in Smith (2003), I view a ‘bank’ as a local coalition of young agents. The bank (in each location) takes as a deposit $y$ units of output from the young. Prior to the realization of the ‘news’ shock, the bank must make a portfolio decision that allocates deposits across ‘reserves’ of fiat money (which it purchases from the old) and capital expenditure. In return for their deposit, each young agent is issued a bank liability (private money) that is made redeemable for fiat on demand. This redemption option is necessarily exercised by those agents that experience the relocation shock. These agents take their fiat money to the other location where they can use it in the next period to purchase output (from the new generation bank that demands cash reserves). Those agents that do not move take their bank money into the next period where it is then redeemed for the output that is produced with the maturing capital project. Depending on the realization of the productivity shock, however, even non-movers may find it optimal to redeem a part of their bank money for cash.
With money and banking, the sequence of events involving generation $t$ agents are described by Figure 1:

**Figure 1: Timeline**

- A. Young generation $t$ agents are born;
- B. Generation $t$ bank is formed;
- C. Bank makes portfolio decision $(q,k)$;
- D. Relocation shock is realized; ‘news’ about $x$ is received;
- E. Movers withdraw cash;
- F. Movers move;
- G. Bank’s capital project matures;
- H. Non-movers share return from the bank’s asset;
- I. Old generation $t$ agents exchange money for output with generation $t+1$ bank;
- J. Old generation $t$ agents consume.

Formally, the choice problem facing a local bank can be written as follows:

$$\max_{q,k,c_m,c_n} \int [\pi u(c_m) + (1 - \pi)u(c_n)]dF(x);$$

subject to:

$$q + k = y;$$
$$\pi c_m(x) \leq R(x)q;$$
$$\pi c_m(x) + (1 - \pi)c_n(x) = R(x)q + xk.$$

In formulating the problem above, note that the portfolio choice cannot depend on $x$ (as this decision must be made prior to the arrival of news). Consumption allocations, on the other hand, can be made conditional on the realized return to capital. $R(x)$ denotes the (gross) real return on fiat money (the inverse of inflation rate $\gamma(x)$, which is potentially a function of the productivity shock). The second constraint above asserts that the (locally) aggregate level of future consumption for movers must be financed entirely out of bank reserves, the return of which depends on realized inflation. The last constraint simply reflects a (local) resource constraint.

At this stage, it is possible to deduce the equilibrium inflation rate (and the rate of return on money). To do so, I exploit the fact that $x$ follows an i.i.d. process and that I am focusing on a stationary allocation (so that $q$ remains constant over time). In this case, market-clearing at every date $t \geq 1$ can be expressed as:

$$M_t = p_t q;$$

where $p_t$ denotes the period price-level. It follows then that $\gamma^*(x) = z$ and $R^*(x) = 1/z$. In other words, the equilibrium inflation rate (and rate of return on money) depends only on the rate of money expansion (and not on the realized technology shock).

Let me now characterize optimal behavior conditional on knowing that, in equilibrium, it must be the case that $R^*(x) = 1/z \equiv R$. The problem (P1) can be solved
recursively. To do so, let me first take the portfolio decision \((q, k)\) as given and assume that \(x\) is now known. Conditional on this, the problem entails choosing an optimal allocation of consumption across movers and non-movers; i.e.,

\[
\max_{c_m, c_n} \pi u(c_m) + (1 - \pi) u(c_n);
\]

subject to:

\[
\pi c_m(x) \leq Rq; \\
\pi c_m(x) + (1 - \pi) c_n(x) = Rq + x(y - q).
\]

In the problem above, the first constraint is either slack or it is not. If the constraint binds, then \(c_m(x) = Rq/\pi\) and \(c_n(x) = x(y - q)/(1 - \pi)\). That is, consumption for movers is independent of \(x\) and depends only on the predetermined level of cash reserves (and the rate of inflation). Consumption for non-movers, on the other hand, is an increasing function of \(x\) (the realized return on their bank deposits). If, on the other hand, the first constraint is slack, then it is a simple matter to establish that full consumption insurance is desirable; i.e., \(c_n(x) = c_n(x) = c(x)\), with \(c(x) = Rq + x(y - q)\). Whether the first constraint binds or not depends on the configuration of parameters \((x, q, R)\). This result is summarized in the following Lemma.

**Lemma 1** For a given \((q, R)\), there exists a \(w > 0\) such that an optimal consumption allocation satisfies:

\[
c_m(x) = c_n(x) = c(x) = Rq + x(y - q) \text{ if } x < w(q, R); \\
c_n(x) = x(y - q)/(1 - \pi) \text{ and } c_m(x) = c_m = Rq/\pi \text{ if } x \geq w(q, R)
\]

with \(w(q, R) = [(1 - \pi)/\pi]Rq/(y - q)\).

The formal proof of this can be found in the appendix. The intuition though is relatively straightforward. *Ex ante*, depositors prefer to smooth their consumption across states of nature (i.e., whether they experience the relocation shock or not). It is desirable to smooth perfectly if the realized return on investment is low enough (i.e., below the critical value \(w\)). To take an extreme example, imagine that \(x = 0\). In this case, it is optimal for both movers and non-movers to finance their consumption entirely out of cash. Since depositors are *ex ante* identical, full insurance is desirable. On the other hand, imagine that the realized return on capital investment is very high (above the critical value \(w\)). In this case, consumption smoothing is still desirable, but is not feasible (remember that goods cannot be transported across locations). The best the bank can do in this case is to let movers have all of the cash (which earns a relatively low rate of return) and let non-movers enjoy the high return on capital. Figure 2 displays this result in the form of a diagram.
The critical value $w$ depends positively on $q$ and $R$. Higher $q$ (and thus lower $k$) and $R$ imply higher return to depositors who redeem their bank money for cash and lower return to depositors who choose to hold on to their bank deposits, inducing the latter to require a higher return on capital to bridge the gap.

Let me now return to the problem (P1), given what we know about the nature of the (conditional) solution in (P2). Formally, the choice problem may now be written as:

$$\max_q \int_{w(q,R)}^w u(c(x))dF(x) + \int_{w(q,R)}^{w} [\pi u(c_m) + (1 - \pi)u(c_n(x))]dF(x);$$

where $c(x) = Rq + x(y-q)$, $c_m = Rq/\pi$ and $c_n(x) = x(y-q)/(1 - \pi)$. Differentiation with respect to $q$ yields:

$$\int_{w-q}^{w} (R - x)u'(c(x))dF(x) + \int_{w}^{w} [Ru'(c_m) - xu'(c_n(x))]dF(x)$$

$$+ \{u(c(w)) - [\pi u(c_m) + (1 - \pi)u(c_n(w))]\}f(w)\partial w/\partial q.$$  

Since $c(w) = c_m = c_n(w)$ (see Figure 1), the term in the curly brackets equals zero,

\[ \text{Figure 2: Solution to (P2)} \]

\[ c_n(x) \]

\[ c_m(x) \]

\[ w(q,R) = \left[ \frac{1 - \pi}{\pi} \right] Rq/(y-q) \]

\[ Rq + x(y-q) \]

\[ Rq/\pi \]

\[ 0 \]

\[ w(q,R) \]

\[ x \]
and the solution $\hat{q}(R)$ is characterized by:

$$\int \hat{w}(R-x)u'(R\hat{q} + x(y-\hat{q}))dF(x) + \int [Ru'(R\hat{q}/\pi) - xu'(x(y-\hat{q})/(1-\pi))]dF(x) = 0;$$

where $\hat{w}(R) = w(\hat{q}(R), R) = [(1-\pi)/\pi]R\hat{q}/(y-\hat{q})$.

Equation (1) says that the expected marginal benefit of an extra unit of investment in cash for movers must be equal to the expected marginal benefit of an extra unit of investment on capital for non-movers. With $\hat{q}$ determined in this manner, the equilibrium level of capital spending is simply $\hat{k} = y - \hat{q}$, and the equilibrium level of government purchases is given by $\hat{g} = (1 - R)\hat{q}$. The equilibrium consumption allocation may be expressed as follows:

$$\hat{c}_m(x) = [1 - I(x, \hat{w})] [R\hat{q} + x(y-\hat{q})] + I(x, \hat{w})R\hat{q}/\pi;$$
$$\hat{c}_n(x) = [1 - I(x, \hat{w})] [R\hat{q} + x(y-\hat{q})] + I(x, \hat{w})x(y-\hat{q})/(1-\pi);$$

where $I(b, c) = 1$ if $b \geq c$ and $I(b, c) = 0$ otherwise.

What happens in the equilibrium is that young agents deposit their entire endowment with the bank and receive in return the bank’s liability (bank money). The bank makes investment decision to invest $\hat{q}$ on real money holdings, and $\hat{k}$ on capital. After the liquidity shock is realized and the ‘news’ about future productivity arrives, movers go to the bank to redeem their bank money in government cash and relocate, and non-movers hold on to their bank deposits. At the end of each date, movers exchange cash for consumption goods, and non-movers share the return on the bank’s remaining assets. With money and banking, agents are able to insure themselves against both types of risks (though not completely).

### 2.3 Banking Crises

In the equilibrium described above, when $x$ is above $\hat{w}$ movers are paid a fixed amount $R\hat{q}/\pi$ and agents without liquidity needs do not want to hold cash. However, when $x$ is below $\hat{w}$, movers are paid $R\hat{q} + x\hat{k}$ which is only a fraction of the payment when $x$ is above $\hat{w}$; at the same time, non-movers desire to hold cash even in the absence of pressing liquidity needs. I define such situations as banking crises, which happen

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The assumption that $\lim_{c \to 0} u'(c) = \infty$ implies that the demand for cash is positive (remember that goods are non-tranportable and non-movers’ consumption must be financed by cash). The assumption that money is dominated in expected rate of return by capital guarantees a positive demand for capital investment. To see this, let us look at the first-order derivative of bank’s objective function with respect to real cash balances $q$ at $q = y$, which is given by $\int u'(Ry)(R - x)dF(x) = u'(Ry)\int (R - x)dF(x)$ and is negative given the assumptions that $u' > 0$ and money is dominated in expected rate of return by capital. The optimal solution requires $q < y$, or $k > 0$.  

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with probability $F(\hat{w})$.$^8$

In the real world, banking crises are relatively rare events, which means that $F(\hat{w})$ is usually a small number. Most of the time, the bank’s capital earns a return that is above the critical value $\hat{w}$, only depositors with liquidity needs hold cash and in doing so, forgo the higher yield on capital (their interest-bearing bank deposits). Depositors without liquidity needs enjoy the high return of the bank’s capital and do not demand cash. Occasionally, however, depositors receive bad news that the return on the bank’s capital will be abnormally low (below the critical value $\hat{w}$). In this event, even depositors without pressing liquidity need request to hold some cash to offset the low return on capital. At the same time, all depositors experience a lower-than-normal return on their deposits which look like a partial suspension of payments. Admittedly, this does not capture all of the features that one would normally associate with a banking crisis, it does appear to generate behavior that along some dimensions, at least, resembles observed behavior in many crisis episodes. The result is consistent with Rolnick and Weber’s (1982) study on banking problems in the free banking era, which concludes that most of the free banking problems were ‘caused by capital losses that banks suffered when market forces drastically pushed down the prices of state bonds, a significant part of all free bank portfolios’. It is also consistent with the empirical evidence that banking crises often occur during economic slow-downs.$^9$

Note that as in Allen and Gale (1998), banking crises are optimal because they allow depositors to achieve complete risk sharing when the economy is hit by adverse productivity shocks. Depositors know that their payoffs will depend on the realization of economic fundamentals, and getting smaller-than-usual payment in case of an unfavorable productivity shock is just part of the optimal contract. In the model economy, the bank does not explicitly announce suspension of payments to depositors; it simply allocates resources according to the optimal contract which requires lower payment to depositors in case of weak fundamentals.

Note also that the explanation about the cause of banking crises is different from the aggregate liquidity risk models (including Champ, Smith and Williamson, 1996, Smith, 2003, and Loewy, 2003). In my model, banking crises arise when depositors receive negative signals about the return on capital. According to the aggregate liquidity risk models, banking crises occur due to exhaustion of bank reserves in response to high realizations of aggregate liquidity needs. The policy implication is also different. The economy in aggregate liquidity risk models faces a shock to money demand, and a discount window policy that makes money supply contingent on the realization of the aggregate liquidity shock will totally eliminate crises. In my model, since each young generation has the same demand for liquidity (remember that there is no aggregate liquidity risk) and precautionary balances (remember that

$^8$When there is a banking crisis ($x < \hat{w}$), the severity of the crisis can be measured by the fraction of cash requested by non-movers: $[R(\hat{q} + x\hat{k})(1 - \pi) - x\hat{k}] / (R\hat{q}) = (1 - \pi)(1 - x/\hat{w})$.

$^9$For example, see Gorton (1988) and Demirguc-Kunt and Detragiache (1998, 2005).
productivity shocks are $i.i.d.$ across time), the demand for real cash balances stays the same at each date. With a constant demand for real cash balances, a similar discount window policy will only affect the price level, but will not eliminate crises.

### 2.4 Effect of Inflation on Banking Crises

Using the above developed model, I am now ready to investigate the effect of inflation on banking crises.

**Lemma 2** The probability of a crisis, $F(\bar{w})$, is a decreasing function of $z$.

Lemma 2 states that higher inflation reduces the probability of banking crises. Refer to the appendix for the formal proof. The intuition is as follows. Given the distribution of productivity shocks, the probability of banking crises is determined by the probability of the ex post return to capital falling below the critical value ($\bar{w}$). That is, the return to capital must exceed a ‘hurdle’ if the economy is to avoid an ex post allocation that I have associated with a ‘crisis’ event. A higher inflation or a lower rate of return to cash has a direct and indirect effect on banking crises. The direct effect is that a decrease in the rate of return on money makes it more likely that the return to capital will exceed it. The indirect effect is that a lower rate of return on money induces the bank to substitute away from cash and into capital, making it more likely that the residual claimants (people who hold their bank deposits for longer periods of time, here the non-movers) will receive a higher payoff. The two effects reinforce with each other making it more likely that non-movers will choose to hold on to their bank deposits instead of cashing out, which in turn, reduces the probability of crises.

The basic model described here predicts that lower inflation increases the likelihood of banking crises, which captures the downward part of the U-shaped relationship between inflation and banking crises, and is consistent with the observation in the Great Depression and Japan’s lost decade.

However, from table 1, one can see that many banking crises are associated with high or hyper inflation, which contradicts the model’s prediction. The basic model’s ‘failure’ to account for the positive relationship between inflation and probability of banking crises is due to the simplifying assumption that only two assets - domestic fiat money and capital - are available. In section 3, I extend the basic model by introducing a third asset; this extended model features a U-shaped relationship between banking crises and inflation.
3 Threshold Effect of Inflation on Banking Crises

Following Antinolfi, Landeo and Nikitin (2006), I now introduce a third asset into
the basic model. Like domestic fiat money, the third asset is also liquid and yields
a stable real rate of return. In this paper, I interpret it as a stable foreign currency
named ‘dollar’. Dollars are in perfectly elastic supply and can be exchanged for
goods at a fixed (real) rate. It is assumed that only the bank has access to the
foreign exchange market. The (gross) rate of return of dollars is fixed at $R_0$, and
like the domestic currency, dollars are non-counterfeitable and can be used on both
islands. The features of dollar enable it to perform similar roles as domestic currency
by providing insurance against liquidity and productivity shocks. To ensure positive
demand for domestic currency in all situations, I assume that the government imposes
a legal restriction that the bank must hold $\theta k$ ($\theta > 0$) units of real balances of
domestic currency. Let $d$ be the real balances of dollars held by the bank. The bank’s problem
is now described by (P3) as follows:

$$\max_{q, k, d, c_m, c_n} \int [\pi u(c_m) + (1 - \pi)u(c_n)] dF(x);$$

(P3)

subject to:

$$q + k + d = y;$$

$$\pi c_m \leq Rq + R_0 d;$$

$$\pi c_m + (1 - \pi)c_n = Rq + R_0 d + xk;$$

$$\theta k \leq q.$$

When $R \geq R_0$, domestic currency dominates dollars in rate of return so that
$\theta = 0$. (P3) is equivalent to (P1) plus the legal restriction, which binds when $R$
is below a threshold value $R_L$, with $R_L$ solving $\hat{q}(R_L)/\hat{k}(R_L) = \theta$. For simplicity, I
assume that $R_0 > R_L$, i.e., when domestic currency offers a return of $R_0$ or higher,
the legal restriction does not bind, and (P3) has the same solution as (P1).

---

10 Since the bank maximizes the representative depositor’s welfare, even if depositors have access
to the foreign exchange market themselves, they are willing to deposit their total endowment with
the bank and let the bank access the exchange market on their behalf. The assumption that only
the bank has access to the foreign exchange market does not change the optimal allocation that can
be achieved (compared to the case that only depositors have access to the foreign exchange market),
but it does have the advantage of making the identification of banking crises more straightforward.

11 The assumption that reserve requirement is imposed only on domestic capital is motivated by the
practice that higher reserve requirement is imposed on domestic deposits than on foreign currency
deposits.

12 Again, I concentrate on the stationary equilibrium where inflation rate and rate of return on
domestic money are dictated solely by the domestic money growth rate.

13 There are three cases to consider if I make the alternative assumption that $R_0 < R_L$. Case (i):
$R > R_L$. In this case, $d = 0$ and the legal restriction does not bind so that (P3) is equivalent to
(P1) of the basic model. Case (ii): $R_0 < R < R_L$. In this case, $d = 0$ and the legal restriction
When \( R < R_0 \), dollars offer higher rate return than domestic currency, the legal restriction binds, and (P3) can be rewritten as:\(^\text{14}\)

\[
\max_{d_t,k_t,c_m,c_n} \int [\pi u(c_m) + (1 - \pi)u(c_n)]dF(x); \quad (P4)
\]

subject to:

\[
(1 + \theta)k + d = y;
\]
\[
\pi c_m \leq R\theta k + R_0 d;
\]
\[
\pi c_m + (1 - \pi)c_n \leq R\theta k + R_0 d + xk.
\]

One can solve (P4) (which is similar to (P1)) recursively in two steps. The first step takes as given the portfolio decision \( d \) (and \( k = (y-q)/(1+\theta), q = \theta k \)) and solves payment/consumption schedules to maximize the expected utility of a representative young depositor. The second step determines the optimal portfolio taking into consideration the optimal consumption schedules. As in the basic model, the solution to the problem in the first step involves a threshold strategy. When \( x \) is below a critical value \( w(d, R) \), the bank retains some of the cash (domestic and foreign currencies) for the non-movers and all depositors get the same consumption. When \( x \) is higher than \( w(d, R) \), all of the cash is paid to movers, and non-movers divide the output produced from the bank’s capital and enjoy higher consumption than movers. The critical value \( w(d, R) \) is derived by solving the following equation for \( x \):

\[
\frac{R(y-d)\theta/(1+\theta) + R_0 d}{\pi} = \frac{x(y-d)/(1+\theta)}{1-\pi}. \quad (2)
\]

The left-hand-side and right-hand-side of the equation are the consumption by movers and non-movers respectively when the bank gives all the cash to movers and uses only output from capital to pay non-movers. The solution to equation (2) is:

\[
w(d, R) = \frac{1 - \pi}{\pi} \left( \theta R + (1 + \theta)R_0 \frac{d}{y-d} \right).
\]

\(^\text{14}\)To ensure positive demand for domestic assets, it is assumed that dollars are dominated in expected rate of return by domestic assets:

\[
R_0 < \int \left( \frac{\theta}{1+\theta} R + \frac{1}{1+\theta} x \right) dF(x).
\]
In the second step, one solves the following problem:

$$\max_d \int_{w(d,R)} u(c(x))dF(x) + \int_{w(d,R)} [\pi u(c_m) + (1 - \pi)u(c_n(x))] dF(x);$$

where $c(x) = \theta R(y - d)/(1 + \theta) + R_0d + x(y - d)/(1 + \theta)$, $c_m = [\theta R(y - d)/(1 + \theta) + R_0d] / \pi$, $c_n(x) = [x(y - d)/(1 + \theta)] / (1 - \pi)$, and $w = [(1 - \pi) / \pi] [\theta R + (1 + \theta)R_0d/(y - d)]$.

Differentiation with respect to $d$ yields:

$$\int_{w} [-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]u'(c(x))dF(x)$$
$$+ \int_{w} \{[-\theta R/(1 + \theta) + R_0]u'(c_m) - [x/(1 + \theta)]u'(c_n(x))\} dF(w)$$
$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))]f(w)\partial w / \partial d.$$  

Since $c(w) = c_m = c_n(w)$, the last term equals zero, and the solution $\tilde{d}(R)$ is characterized by:

$$\int_{w} [-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]u'(c(x))dF(x)$$
$$+ \int_{w} \{[-\theta R/(1 + \theta) + R_0]u'(c_m) - [x/(1 + \theta)]u'(c_n(x))\} dF(w) = 0.$$  

where $\tilde{w} = w(d(R), R) = [(1 - \pi) / \pi] [\theta R + (1 + \theta)R_0\tilde{d}/(y - \tilde{d})]$.

With $\tilde{d}$ determined, the equilibrium level of capital spending $\tilde{k}$ is simply $(y - \tilde{d})/(1 + \theta)$, and the equilibrium demand for real domestic currency balances $\tilde{q}$ is $(y - \tilde{d})\theta/(1 + \theta)$. The probability of banking crises can be calculated as $F(\tilde{w})$.

The equilibrium government spending is given by $\tilde{g} = (1 - 1/z)\tilde{q}$. The equilibrium consumption can be expressed as follows:

$$\tilde{c}_m(x) = [1 - I(x, \tilde{w})](R\tilde{q} + R_0\tilde{d} + x\tilde{k}) + I(x, \tilde{w})(R\tilde{q} + R_0\tilde{d}) / \pi;$$
$$\tilde{c}_n(x) = [1 - I(x, \tilde{w})](R\tilde{q} + R_0\tilde{d} + x\tilde{k}) + I(x, \tilde{w})x\tilde{k} / (1 - \pi).$$

**Proposition 1** When $z \leq 1/R_0$, higher inflation reduces the probability of banking crises; when $z > 1/R_0$, higher inflation increases the probability of banking crises.

Refer to the appendix for the proof. The intuition is as follows.

When domestic inflation is low ($z \leq 1/R_0$), agents use only domestic currency to insure themselves against the two types of risks since it dominates dollars in rate of

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\(^{(15)}\)As in the basic model, in the case of a crisis, the severity of the crisis can be measured by $(1 - \pi)(1 - x/\tilde{w})$. 

16
return. As discussed in the basic model, higher inflation reduces the probability of banking crises.

When domestic inflation is high \((z > 1/R_0)\), dollars dominate domestic currency in rate of return and are used to insure against liquidity and productivity shocks along with the domestic currency. The legal restriction on domestic currency/capital ratio binds, and dollars compete with domestic assets (domestic currency plus capital are viewed as a bundle when the legal restriction binds). As in the basic model, a banking crisis occurs when depositors receive news that the future return to the bank’s capital will be below the critical value \((\bar{\omega})\), and even those without liquidity needs demand to hold cash. Again, there are direct and indirect effects associated with higher domestic inflation and lower return on domestic currency. The direct effect is that lower return to domestic currency lowers the return to cash (domestic plus foreign currency) increasing the probability that the rate of return on capital exceeds it and thus reducing the probability of crises. The indirect effect is that a lower rate of return on domestic currency (and thus domestic asset) induces a substitution in the bank’s portfolio away from domestic capital into dollars making it more likely now that the residual claimants will receive a lower payoff. As a result, depositors without liquidity needs are more likely to prefer cashing out instead of holding on to their bank deposits, which in turn, means a higher probability of crises. In the model, the second effect dominates so that higher inflation raises the probability of crises.

The extended model in this section thus captures the empirical observation that when inflation in beyond a threshold, higher inflation is associated with higher probability of banking crises.\(^{16}\) I provide a numerical example below.

**Example.** Let \(y = 1, \pi = 0.1, \theta = 1/9, R_0 = 0.97\) and \(x\) be distributed uniformly over the range \([0.965, 1.155]\). Figure 3 graphs the demand for real balances of domestic currency, capital investment, the demand for real dollar balances and the probability of banking crises against the net domestic inflation rate.

Finally, I would like to point out that adding dollars into aggregate liquidity risk models (including Champ, Smith and Williamson, 1996, Smith, 2003, Loewy, 2003) will not generate the U-shaped relationship between the probability of banking crises and inflation that is supported by the data. When inflation is low, domestic currency dominates dollars in rate of return and this corresponds to the basic model with only domestic assets. Lower inflation causes the bank to hold more cash decreasing the probability of banking crises (remember that in aggregate liquidity risk model, banking crises occur because of exhaustion of bank reserves in case of high liquidity needs). When inflation is high, domestic currency is dominated in rate of return by dollars. Legal restriction binds, and domestic currency and capital are bundled together as domestic asset. Higher inflation induces the bank to hold more dollars

\(^{16}\) As a by-product, the model offers an explanation about the ‘twin crises’ phenomenon; i.e., the tendency for banking and currency crises to occur in tandem (refer to Kaminsky and Reinhart, 1999, and Glick and Hutchison, 2001).
reducing the probability of banking crises (higher dollar holding means that the bank can meet higher liquidity needs). The aggregate liquidity risk models will generate an inverted U-shaped relationship between the probability of banking crises and inflation.

Figure 3: Threshold effect of inflation on banking crises

\[ \text{Net domestic inflation} \rightarrow \text{Real domestic currency balances} \rightarrow \text{Capital investment} \rightarrow \text{Real dollar balances} \rightarrow \text{Probability of banking crises} \]

4 Conclusion

In this paper, I analyze the effect of inflation on banking crises in an overlapping generations model in which money is valued for its insurance roles against liquidity and productivity risks, and the banking sector acts as a mechanism through which depositors pool their liquidity risk. The model’s equilibrium is consistent with some key features of actual banking crises; namely, the partial suspension of payments and depositors demanding cash even in the absence of liquidity needs. According to the model, banking crises are due to adverse information about the future return on the bank’s capital, rather than exogenous fluctuations in the demand for liquidity as in Champ, Smith and Williamson (1996), Smith (2003) and Loewy (2003); this result is consistent with the empirical observation that banking crises tend to occur during economic slow-downs.

The model is able to explain the U-shaped relationship between inflation and banking crises. There are three assets: domestic currency, a stable foreign currency
called dollars and domestic capital. Both currencies can be used to insure liquidity and productivity shocks. There is legal restriction on the domestic currency/capital ratio. When domestic inflation is low, domestic currency dominates foreign currency in rate of return and only domestic currency is used to insure against liquidity and productivity shocks. Higher inflation reduces the rate of return to domestic currency and induces the bank to invest more on capital; this makes it more likely that the residual claimants prefer holding on to bank deposits than cashing out and reduces the probability of banking crises. When domestic inflation is high, domestic currency is dominated in rate of return by dollars, the legal restriction binds and domestic currency and capital are viewed as a bundle. Higher inflation reduces the return to domestic asset (currency plus capital) inducing the bank to invest more on dollars and less on capital, which makes it less likely that the residual claimants prefer keeping bank deposits instead of cashing out and increases the likelihood of banking crises.

The model in this paper can be extended in a number of directions. First, I have concentrated on a class of simple monetary policies: fixed money growth to finance government spending. It would be interesting to study more sophisticated monetary policies which make money growth contingent on the realization of productivity shocks and to deduce the welfare implications of such policies. The model is simplistic and the analysis in the paper is qualitative in nature. Investigating the quantitative implications of a suitably developed model should prove a useful endeavor.
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Appendix

Proof of Lemma 1

The Lagrangian is \( \mathcal{L} = (1-\pi)u((Rq + xk - \pi c_m)/(1-\pi)) + \pi u(c_m) + \lambda(Rq - \pi c_m) \).

The first-order condition with respect to \( c_m \) is given by: \( u'(c_m) = u'(c_m) + \lambda \).

There are two cases to be considered.

When \( \lambda > 0 \), the constraint \( \pi c_m \leq Rq \) binds with equality, and I have \( c_m = Rq/\pi \), \( c_n = x(y - q)/(1-\pi) \), \( u'(c_n) > u'(c_m) \Rightarrow c_n < c_m \), and \( Rq/\pi > x(y - q)/(1-\pi) \), or \( x < w \).

When \( \lambda = 0 \), the constraint \( \pi c_m \leq Rq \) is slack, and I have \( \pi c_m < Rq \), \( u'(c_n) = u'(c_m) \Rightarrow c_n = c_m = c = Rq + x(y - q) \), and \( Rq/\pi > x(y - q)/(1-\pi) \) or \( x > w \). \( \blacksquare \)

Proof of Lemma 2

I first show that \( \partial \hat{q}/\partial R > 0 \). It then follows naturally from \( R = 1/z \) and the expression for \( \hat{w} \) that \( \hat{w}'(z) < 0 \) or that higher inflation reduces the probability of banking crises.

Let \( G(q, R) = \int_w^w [(R - x)u'(c(x))]dF(x) + \int_w^w [Ru'(c_m) - xu'(c_n(x))]dF(x) \]
\( + [u(c(w)) - \pi u(c_m) - (1-\pi)u(c_n(w))]f(w)\partial w/\partial q; \)

where \( c(x) = Rq + x(y - q) \), \( c_m = Rq/\pi \), \( c_n(x) = x(y - q)/(1-\pi) \) and \( w = [(1 - \pi)/\pi]Rq/(y - q) \).

Differentiating \( G(q, R) \) with respect to \( q \), one gets:

\[
G_q = \int_w^w u''(c)(R - x)^2dF(x) + \int_w^w [u''(c_m)R^2/\pi - u''(c_n)x^2/(1-\pi)]dF(x)
+ [u'(c)(R - w) - u'(c_m)(-w) + 2u'(c)R]f(w)\partial w/\partial q
+ [u'(c) - \pi u'(c_m) - (1-\pi)u'(c_n(w))]f(w)(\partial w/\partial q)^2
+ [u(c(w)) - \pi u(c_m) - (1-\pi)u(c_n(w))]f'(w)(\partial w/\partial q)^2
+ [u(c(w)) - \pi u(c_m) - (1-\pi)u(c_n(w))]f(w)\partial^2 w/\partial q^2.
\]

Using \( c(w) = c_m(w) = c_m \), I can simplify \( G_q \) as follows: \( G_q = \int_w^w (R - x)^2u''(c)dF(x) + \int_w^w [u''(c_m)R^2/\pi - u''(c_n)x^2/(1-\pi)]dF(x) < 0. \)
Differentiating $G(q, R)$ with respect to $R$, I get:

$$G_R = \int_w [u''(c)(c - xy) + u'(c)] dF(x) + \int_w [u''(c_m)c_m + u'(c_m)] dF(x)$$

$$+ \{u'(c(w))(R - w) - [u'(c_n(w))(-w) + u'(c_n)R]\} f(w)\partial w/\partial R$$

$$+ [u'(c(w)) - \pi u'(c_m) - (1 - \pi)u'(c_n(w))] f(w)(\partial w/\partial q)(\partial w/\partial R)$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f'(w)(\partial^2 w/\partial q \partial d)$$

$$= \int_w [u''(c)(R - x)q + u'(c)] dF(x) + \int_w [u''(c_m)c_m + u'(c_m)] dF(x)$$

$$= \int_w [u''(c)(c - xy) + u'(c)] dF(x) + \int_w [u''(c_m)c_m + u'(c_m)] dF(x).$$

Since $u(c) = \ln c$ has the property that $u''(c)c = -u'(c)$, one can rewrite $G_R$ as:

$$G_R = -\int_0^w u''(c)xydF(x) > 0.$$ It then follows that: $\partial \hat{q}/\partial R = -G_R/G_q > 0$. ■

**Proof of Proposition 1**

Refer to the proof of Lemma 2 for the case when $R \geq R_0$.

Here is the proof of the second part of the Proposition (when $R < R_0$).

Let $H(d, R) \equiv \int_w u'(c(x))[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]dF(x)$

$$+ \int_w \{u'(c_m)[-\theta R/(1 + \theta) + R_0] - u'(c_n(x))x/(1 + \theta)\} dF(x)$$

$$+ [u(c(w)) - (1 - \pi)u(c_n(w)) - \pi u(c_m)] f(w)\partial w/\partial d;$$

where $c(x) = \theta R(y - d)/(1 + \theta) + R_0d + x(y - d)/(1 + \theta)$, $c_m = [\theta R(y - d)/(1 + \theta) + R_0d]/\pi$, $c_n(x) = [x(y - d)/(1 + \theta)]/(1 - \pi)$, and $w = [(1 - \pi)/\pi] [\theta R + (1 + \theta)R_0d/(y - d)]$.

Differentiating $H(d, R)$ with respect to $d$, I get:

$$H_d = \int_w u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]^2 dF(x)$$

$$+ \int_w \{u''(c_m)[x/(1 + \theta)]^2/(1 - \pi) + u''(c_m)[-\theta R/(1 + \theta) + R_0]^2/\pi\} dF(x)$$

$$+ \{u'(c(w))[-\theta R/(1 + \theta) + R_0 - w/(1 + \theta)]$$

$$- [u'(c_m)[-\theta R/(1 + \theta) + R_0] - u'(c_n(w))w/(1 + \theta)]\} f(w)\partial w/\partial d$$

$$+ [u'(c(w)) - \pi u'(c_m) - (1 - \pi)u'(c_n(w))] f'(w)(\partial w/\partial d)^2$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f'(w)(\partial^2 w/\partial d^2).$$
Using \( c(w) = c_n(w) = c_m \), one can simplify \( H_d \) as follows:

\[
H_d = \int_w^\infty u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]^2 dF(x) \\
+ \int_w \{u''(c_n)[x/(1 + \theta)]^2/(1 - \pi) + u''(c_m)[-\theta R/(1 + \theta) + R_0]^2/\pi \} dF(x) < 0.
\]

Differentiating \( H(d, R) \) with respect to \( R \), one gets:

\[
H_R = \int_w^\infty u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]\theta(y - d)/(1 + \theta)dF(x) \\
+ \int_w \{u''(c_m)[-\theta R/(1 + \theta) + R_0 - w/(1 + \theta)] + u'(c_m)(-\theta/(1 + \theta))\}dF(x) \\
+ \{u'(c(w)[-\theta R/(1 + \theta) + R_0 - w/(1 + \theta)] + u'(c_m)(-\theta R/(1 + \theta) + R_0))\}f(w)\partial w/\partial R \\
+ [1/c(w) - \pi/c_n - (1 - \pi)/c_n(w)] f(w)(\partial w/\partial d)(\partial w/\partial R) \\
+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))]/u(c_m)\}f(w)(\partial^2 w/\partial d\partial R) \\
= [-\theta/(1 + \theta)] \{ \int_w^\infty [u''(c(c - R_0 y)) + u'(c)] dF(x) \\
+ \int_w [u''(c_m)(c_m - R_0 y)/\pi) + u'(c_m)] dF(x) \}.
\]

Since \( u(c) = \ln c \) has the property that \( u''(c) = -u'(c) \), \( H_R \) can be expressed: \( H_R = \theta/(1 + \theta)!R_0 y \left[ \int_0^\infty u''(c)dF(x) + (1/\pi) \int_w u''(c_m)dF(x) \right] < 0.\]

Remember that \( w = [(1 - \pi)/\pi] \{ R\theta + R_0(1 + \theta)d/(y - d) \}.\]

\[
\partial w/\partial R = \theta(1 - \pi)\pi \left[ 1 + \theta^{-1}R_0(1 + \theta)y/(y - d)^2\partial d/\partial R \right] \\
= \theta(1 - \pi)\pi \left[ 1 - [\theta^{-1}R_0(1 + \theta)gH_R]/[(y - d)^2 H_d] \right] \\
= \theta(1 - \pi)\pi \times \\
\left\{ 1 - (R_0 y)^2/(y - d)^2 \left[ \int_w u''(c)dF(x) + (1/\pi) \int_w u''(c_m)dF(x) \right]/H_D \right\} \\
= \theta(1 - \pi)\pi(1 - A/B); \\
\]

where \( A \equiv (R_0 y)^2 \left[ \int_w u''(c)dF(x) + (1/\pi) \int_w u''(c_m)dF(x) \right] \), and \( B \equiv (y - d)^2 H_D.\)
\[ B = \int_{w}^{w} u''(c) \left( -\theta R / (1 + \theta) + R_0 - x / (1 + \theta) \right) + R_0 - x / (1 + \theta) \right)^2 (y - d)^2 dF(x) \]

\[ + \int_{w}^{w} \left\{ u''(c_n) \left[ x / (1 + \theta) \right]^2 / (1 - \pi) + u''(c_m) \left[ -\theta R / (1 + \theta) + R_0 \right]^2 / \pi \right\} (y - d)^2 dF(x) \]

\[ = \int_{w}^{w} u''(c) [c - R_0 y]^2 dF(x) + \int_{w}^{w} \left\{ -(1 - \pi) + u''(c_m) [c_m - R_0 y / \pi]^2 / \pi \right\} dF(x) \]

\[ = -\left\{ \int_{w}^{w} (1 - R_0 y / c)^2 dF(x) + (1 - \pi) \int_{w}^{w} dF(x) + \int_{w}^{w} (1 - R_0 y / \pi / c_m) dF(x) \right\} \]

\[ = -\left\{ 1 - \int_{w}^{w} (2R_0 y / c) dF(x) - \int_{w}^{w} (2R_0 y / c_m) dF(x) \right\} + \int_{w}^{w} (R_0 y / c)^2 dF(x) + (1 / \pi) \int_{w}^{w} (R_0 y / c_m)^2 dF(x) \}

\[ = -\left\{ 1 - \int_{w}^{w} (2R_0 y / c) dF(x) - \int_{w}^{w} (2R_0 y / c_m) dF(x) - A \right\} = -1 + C + A; \]

where \( C \equiv \int_{w}^{w} (2R_0 y / c) dF(x) + \int_{w}^{w} (2R_0 y / c_m) dF(x). \)

From the first order condition \( H(d, R) = 0, \) one has:

\[ \int_{w}^{w} u'(c) \left[ R \theta k - R_0 (1 + \theta) k + x k \right] dF(x) + \int_{w}^{w} \left\{ u'(c_n)(x k) + u'(c_m) \left[ R \theta k - R_0 (1 + \theta) k \right] \right\} dF(x) = 0; \]

or \( \int_{w}^{w} u'(c) (c - R_0 y) dF(x) + \int_{w}^{w} \left\{ (1 - \pi) + \pi u'(c_m)(c_m - R_0 y / k) \right\} dF(x) = 0; \)

or \( \int_{w}^{w} u'(c) (R_0 y) dF(x) + \int_{w}^{w} \left\{ u'(c_m)(R_0 y) \right\} dF(x) = 1; \)

or \( C = 2. \)

Now one has: \( \partial \tilde{w} / \partial R = [\theta (1 - \pi) / \pi] / (A + 1). \) Remember that \( B = -1 + C + A < 0 \Rightarrow 1 + A < 0. \) It then follows that \( \partial \tilde{w} / \partial R < 0. \)