A model of U.S. monetary policy before and after the great recession*

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Abstract

I study a simple dynamic general equilibrium monetary model to interpret key macroeconomic developments in the U.S. economy both prior to and after the Great Recession. In normal times, when the Fed’s policy rate is above the zero lower bound, the Fed can control inflation and countercyclical monetary policy works in a textbook manner. When a shock drives the policy rate to the zero lower bound, the economy enters a liquidity trap scenario in which open market purchases of government securities have no real or nominal effects, apart from expanding the supply of excess reserves in the banking sector. In a liquidity trap, the Fed loses control of inflation, which is now determined by the fiscal authority. It is possible for the Fed in this case to lower inflation by selling off sufficient quantities of its security holdings and by refusing to monetize debt. It is not, however, possible for the Fed to raise inflation without fiscal accommodation. The Fed can, in this case, do no better than to keep its policy rate as low as possible into the indefinite future.

1 Introduction


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of five hundred basis points. Since 2008, the policy rate is effectively at twenty-five basis points—the interest paid on excess reserves. Prior to 2008, the Fed’s balance sheet stood at less than one trillion dollars. Since 2008, its security holdings and liabilities have experienced a fourfold increase. Most of these liabilities exist as excess reserves in the banking system. Prior to 2008, excess reserves were essentially zero.

Since 2009, the U.S. economy recovered steadily, if somewhat more slowly than desired. After peaking at over ten percent in 2009, the civilian unemployment rate is now close to five and half percent. Despite the fourfold increase in base money supply, PCE inflation has undershot the Fed’s two percent target throughout much of the recovery phase. With inflation at about fifty basis points below target and the labor market continuing to improve, the Fed stands prepared to raise its policy rate as economic conditions dictate. Speculation over the date at which “lift off” will occur is rampant in the financial pages of newspapers as is concern over the wisdom of possibly raising rates prematurely.

People have many questions concerning the economic developments just described. Why did interest rates plummet so dramatically in 2008? Why did the massive increase in base money appear to have no noticable effect on the price-level or inflation? Does the fact that most of this new money sits as excess reserves in the banking system portend an impending inflationary episode—an event that the Fed will have trouble controlling? Or will inflation continue to drift lower as interest rates remain low, replicating the experience of Japan over the past two decades?

Below, I answer these (and other) questions from the perspective of a simple dynamic general equilibrium model. The exercise is primarily pedagogical in nature. In what follows, I develop a simple overlapping generations model with three assets: money, bonds and capital. In general, money is dominated in rate of return but is held nevertheless to meet a legal reserve requirement. This reserve requirement binds when the nominal interest rate on bonds exceeds the interest paid on money. Excess reserves are willingly held when the nominal interest rates on bonds and money are equated. When this latter condition holds, the economy is in a “liquidity trap.” Open market purchases of bonds have no real or nominal effects, apart from increasing excess reserves in the banking system.

I demonstrate how the model can be used to interpret the effect of open
market operations in “normal” times–defined to be episodes in which money is dominated in rate of return and excess reserves are zero. An open market purchase of securities in this case has the effect of expanding the supply of scarce reserves in the economy, making it easier for banks and other entities to fulfill their reserve requirements. The policy rate (the interest rate on bonds) declines and the price-level rises. As desired capital spending expands, banks increase their loan activity. There is no effect on long-run inflation if inflation is anchored by an appropriate fiscal policy.

I then consider a negative “aggregate demand” shock–technically, a news shock that leads agents to revise downward their forecasts over the future productivity of (or after-tax return on) contemporaneous capital spending. *Ceteris paribus*, the effect of such a shock is to induce a portfolio substitution away from capital and into government securities (money and bonds), placing downward pressure on bond yields and the price-level. An open market purchase at this point places additional downward pressure on bond yields, but upward pressure on the price-level. Additional cash reserves loosen the regulatory constraint, stimulating investment. In this way, the monetary authority stabilizes both real economic activity and the price-level. I argue that this is essentially how the Fed strives to achieve its dual mandate in normal times.

When a negative “aggregate demand” shock is sufficiently severe, the consequent decline in desired investment spending places significant downward pressure on bond yields as investors substitute out of capital and into government securities. The Fed can try its usual countercyclical measures at this point, but is ultimately stymied when its policy rate falls to its lower bound. Additional open market operations at this point have no effect. I argue that, to a first approximation at least, this is the reason why the Fed’s post 2008 “quantitative easing” (QE) programs likely had very little economic impact apart from expanding the supply of excess reserves in the banking system.\footnote{The three QE programs to date are: QE1 (Dec. 2008 to Mar. 2010), QE2 (Nov. 2010 to Jun. 2011) and QE3 (Sep. 2012 to Oct. 2014).}

While conventional open market operations are inoperative in a liquidity trap situation, the Fed may still influence real economic activity through the interest it pays on excess reserves—the so-called IOER rate. While this rate is subject to the usual zero lower bound constraint (barring a situation in which all cash is replaced with electronic money), the Fed can still raise...
the IOER. The effect of raising the interest rate on reserves in a liquidity trap is very different from the effect it might have in normal times. In normal times, paying interest on reserves reduces the “tax” associated with holding low-yield cash and so promotes capital investment (bank lending). In a liquidity trap, increasing the interest rate on reserves increases the yield on government debt in general and causes a portfolio substitution effect—out of private capital and into government securities.

I also demonstrate how the Fed loses direct control of inflation in a liquidity trap. Inflation is now controlled by the fiscal authority, in particular, by the rate at which total nominal debt is expected to rise. If the fiscal authority passively supplies debt to meet market demand, the model implies a real indeterminacy arises: the economy can get “stuck” at any number of subnormal levels of economic activity, depending on which self-fulfilling inflation rate transpires (willingly accommodated by the fiscal authority). Determinancy is restored when the fiscal authority anchors the inflation rate by expanding the supply of debt on its own schedule and not in accordance to market demands.

If the inflation rate is indeterminate, or if it is anchored at an excessively high rate, the Fed could regain control of inflation through asset sales that would once again render reserves scarce. In this case, inflation is dictated by the path of the monetary base, which is under Fed control. If, on the other hand, inflation is anchored at an excessively low rate in a liquidity trap scenario, then the Fed is essentially powerless to achieve its inflation mandate. Purchasing government debt at a faster rate than it is issued is not sustainable. I argue that such a scenario, while unlikely, is not entirely implausible given the current U.S. political and fiscal situation. If the situation does transpire, then it could conceivably postpone the Fed’s “lift off” date far into the indefinite future.

2 The model economy

2.1 Preferences and technology

In what follows, I describe a variant of Samuelson’s (1957) overlapping generations model. Time is discrete and the horizon is infinite, \( t = 1, 2, \ldots, \infty \). At
each date $t \geq 1$, a unit mass of young agents enter the economy and a unit mass of old agents leave the economy. Apart from an initial unit mass of old agents (who live for one period only) each generation of young agents lives for two consecutive periods. The total population is therefore fixed across time and is at every date $t$ divided evenly between the young and old. A young person at date $t$ becomes an old person at date $t + 1$.

Agents of every generation $t \geq 1$ are endowed with $y$ units of output when young and zero units of output when old. Individuals are assumed to value consumption in their old age only. Consequently, the young face a trivial consumption-saving decision—it will always be optimal for them to save their entire income. The simplified consumption-saving choice will permit me to focus on portfolio allocation decisions—the mechanism I wish to emphasize below. For simplicity, I also assume that preferences are linear.

Each young agent has access to an investment opportunity where $k_t$ units of output invested at date $t$ yields $xf(k_t)$ units of output at date $t + 1$, where $x > 0$ is an exogenous productivity parameter that governs the expected return to capital spending. Assume that the production function $f$ satisfies $f''(k) < 0 < f'(k)$, that is, higher levels of investment generate higher levels of future output, but with diminishing returns to scale. As well, assume that $f'(0) = \infty$ so that some investment will always be optimal and that $xf'(y) < 1$ so that the marginal return to capital spending at very high levels of investment is less than the population growth rate. Finally, assume that capital depreciates fully after it is used in production.

2.2 Welfare

The competitive equilibrium of this economy is autarkic, i.e., $k_t = y$ for all $t$. Because $xf'(y) = 0 < 1$, the economy is dynamically inefficient (the competitive equilibrium real interest rate is less than the population growth rate). As such, there is a welfare-enhancing role for government debt. As is well known, the Golden rule allocation is implementable as a competitive monetary equilibrium with a perpetually fixed stock of government debt (although, as we shall see below, one needs to worry about the stability properties of such an equilibrium).

The policy of maintaining a fixed quantity of nominal debt continues to remain optimal here even if (say) $x$ were to follow a stochastic process
because of my assumption of linear (risk-neutral) preferences. Generalizing the model to nonlinear preferences would in this case imply a role for state-contingent interventions essentially for the purpose of completing a missing intergenerational insurance market. I am reluctant to generalize the analysis in the manner, however, because the main points I wish to stress can be expounded much more cleanly in a linear world. Extending the model to nonlinear preferences and modeling aggregate uncertainty explicitly is, in my view, better left as an exercise to undertake after the properties of this simpler environment are understood.

Apart from the existence of government debt then, the analysis below offers no welfare rationale for the policies examined. For example, I assume the existence of two forms of government debt, money and bonds, even though the model provides no reason for two distinct forms of debt. Clearly then, the approach I am taking is a short-cut that will, I hope, nevertheless prove useful for positive, rather than normative, purposes.

2.3 Government policy

There are two nominal assets, money $M_t$ and bonds $B_t$, each issued by the government. Bonds yield a gross nominal one-period (from $t$ to $t+1$) yield denoted by $R^b_t$. I assume that money can potentially earn interest at rate $R^m_t$ (think of this as interest paid on reserves). For simplicity, I set government purchases to zero. The interest and principal owing on maturing government debt $R^m_{t-1}M_{t-1} + R^b_{t-1}B_{t-1}$ must be financed out of combination of new debt and a lump-sum tax $T_t$; i.e.,

$$R^m_{t-1}M_{t-1} + R^b_{t-1}B_{t-1} = T_t + M_t + B_t$$  \hspace{1cm} (1)

Let $D_t$ denote the nominal value of the government’s total outstanding debt at date $t$, i.e., $D_t = M_t + B_t$. In what follows, I assume that the fiscal authority determines the path of $D_t$ and $T_t$ and I assume that the monetary authority determines the path of interest rates $R^m_t, R^b_t$ along with the composition of the total debt $\theta_t = M_t / D_t$. Condition (1) shows explicitly how monetary and fiscal policy are interlinked through the government budget constraint.

Fiscal policy operates as follows. First, I assume that the fiscal authority
grows the nominal debt at a fixed rate $\mu$, so that,

$$D_t = \mu D_{t-1}$$

(2)

with the initial debt $D_0 > 0$ endowed to the initial old. With interest rates and debt-composition determined by monetary policy, I assume that the fiscal authority passively adjusts the lump-sum tax $T_t$ to satisfy the government budget constraint (1).

As it will prove convenient to express variables in real terms, let $p_t$ denote the date $t$ price-level and define $\tau_t = T_t/p_t$, $d_t = D_t/p_t$. Using (2) and $\theta_t = M_t/D_t$, rewrite the government budget constraint (1) as follows,

$$\tau_t = \left[ \frac{R_{t-1}^m \theta_{t-1} + R_{t-1}^b (1 - \theta_{t-1})}{\mu} - 1 \right] d_t$$

(3)

In what follows, I assume that the tax $\tau_t$ (or transfer, if negative) falls entirely on the old at date $t$.\footnote{The assumption that the lump-sum tax/transfer falls solely on the old is not innocuous. Among other things, it will imply that helicopter drops of nominal assets are neutral. This is because only the old possess nominal assets at the time of a monetary injection so that lump-sum transfers of money to them ends up increasing everyone’s money balances in proportion to their holdings.}

Since money and bonds share identical risk and liquidity characteristics in the set-up considered here, to motivate a demand for money when it is dominated in rate of return (i.e., when $R_t^b > R_t^m$) I assume that individuals are subject to a legal minimum reserve requirement. I will specify the exact nature of this reserve requirement when I describe individual decision-making.

With fiscal (and regulatory) policy set in the manner described above, I turn attention to investigating the properties of alternative monetary policies. In all of the monetary policies I consider below, I assume that that interest on reserves is set exogenously to some level $R_t^m$. In most models, it is assumed that money exists in the form of zero-interest cash, so that $R_t^m = 1$. In the analysis below, $R_t^m \geq 1$ is permitted, which suggests interpreting the relevant money supply as electronic central bank reserves.\footnote{I do not make a distinction between cash and central bank reserves in this paper, although it would be interesting to extend the analysis along this dimension.}

I consider three different monetary policy regimes. First, I model an interest rate peg $R_t^b = R_t^b \geq R_t^m$, where $\theta$ is determined by market forces.
Second, I model a money-to-debt ratio peg \( \theta_t = \theta \), where \( R^b \) is determined by market forces. Third, I consider a more general interest rate rule along the lines of Taylor (1993).

### 2.4 Decision-making

A young person is endowed with \( y \) units of real income. Since consumption is not valued when young, all income is saved, with savings divided between the three available assets: money \( (m_t) \), bonds \( (b_t) \), and capital \( (k_t) \). Thus,

\[
y = m_t + b_t + k_t
\]

(4)

where \( m_t, b_t \) denote real money and bond holdings, respectively. Given a portfolio choice, future (old age) consumption is given by,

\[
c_{t+1} = x f(k_t) + R^b_t(p_t/p_{t+1}) b_t + R^m_t(p_t/p_{t+1}) m_t - \tau_{t+1}
\]

(5)

Following Smith (1991), I assume that individuals must hold a minimum amount of cash reserves against their capital holdings; in particular,

\[
m_t \geq \sigma k_t
\]

(6)

where \( 0 < \sigma < 1 \) is an exogenous policy parameter. Much of what follows will depend on whether the reserve requirement constraint is binding or not.

The reserve requirement (6) may seem peculiar because it appears to require agents to hold reserves against assets rather than liabilities. But as pointed out by Smith (1991), it is possible to map this specification into something that looks more realistic by reinterpreting the model in an appropriate way. Suppose, for example, that after acquiring the portfolio \( y = m_t + b_t + k_t \), the young find it convenient to “deposit” \( p_t[m_t + k_t] \) dollars in a “bank” (consisting of a coalition of young agents). The bank issues liabilities of equivalent value, i.e., \( p_t[m_t + k_t] \) dollars that are redeemable for a future monetary value \( p_{t+1}[x f(k_t) + R^m p_t m_t] \) dollars. A more realistic reserve requirement specifies that a minimum fraction \( \xi \) of bank liabilities \( p_t[m_t + k_t] \) need to be held as cash, i.e., \( p_t m_t \geq \xi p_t[m_t + k_t] \). If we define \( \sigma = \xi/(1 - \xi) \), then this more realistic reserve requirement corresponds exactly to (6). The representation (4), (5) and (6) then simply consolidates the balance sheet of banks and their depositors.\(^4\)

\(^4\)I did not assume here that the young deposit their entire endowment with the bank
Let us now move on to characterize optimal behavior. Substitute (4) into (5) and form the expression,

\[ W_t = x f(k_t) + R^b_t(p_t/p_{t+1}) [y - m_t - k_t] + R^m_t(p_t/p_{t+1}) m_t - \tau_{t+1} + \lambda_t [m_t - \sigma k_t] \]

where \( \lambda_t \geq 0 \) is the Lagrange multiplier associated with the reserve requirement. Maximizing \( W_t \) (expected future wealth/consumption) with respect to \( m_t \) and \( k_t \) yields the restrictions,

\[ \lambda_t = (R^b_t - R^m_t) (p_t/p_{t+1}) \]
\[ R^b_t(p_t/p_{t+1}) = x f'(k_t) - \sigma \lambda_t \]

Condition (7) makes clear that the reserve requirement will bind tightly (\( \lambda_t > 0 \)) if and only if bonds strictly dominate money in rate of return (\( R^b_t > R^m_t \)). If money cannot earn interest (\( R^m_t \geq 1 \)) and if money does not earn interest (\( R^m_t = 1 \)); then one could say that the reserve requirement binds tightly only when the economy is away from the zero-lower-bound (ZLB).

Condition (8) implicitly defines the demand for investment. This condition shows that the expected rate of return on capital spending exceeds (equals) the return on bonds when the reserve requirement binds (is slack). That is, when the reserve requirement binds, agents would like to expand their capital spending, since the return from doing so is higher than investing in bonds. But doing so means accumulating additional low-return cash. Hence, the reserve requirement serves as a tax on capital spending, and condition (8) equates the “after-tax” returns on capital and bonds.

Combine (7) and (8) to form,

\[ x f'(k_t) = [(1 + \sigma) R^b_t - \sigma R^m_t] (p_t/p_{t+1}) \]

Condition (9) characterizes investment demand \( k_t \). This condition holds whether or not the reserve requirement binds. What is left to determine is the demand for government assets. If \( R^b_t > R^m_t \), then the demand for real because it would have had the effect of rendering the demand for real money balances exogenous (when binding), i.e., \( m_t = \sigma y \). This defect is easily rectified, however, if we assume that the young value consumption so that deposits do not correspond to \( y \).

It would also be of some interest to experiment with other specifications, for example, requirements that some minimal amount of bonds also be held in reserve. The effect of an open market operation in this case would depend on which set of reserve constraints are binding.

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money balances is given by \( m_t = \sigma k_t \). That is, the demand for reserves is proportional to the demand for investment. The demand for bonds can then be determined residually from condition (4) as \( b_t = y - m_t - k_t \).

When the reserve requirement is slack, money and bonds are viewed as perfect substitutes in individual wealth portfolios. With \( k_t \) determined by (9), the demand for government assets is well-defined and given by \( d_t = y_t - k_t \). But the individual demand for money and bonds is indeterminate. That is, any combination of \( m_t, b_t \) satisfying \( m_t \geq \sigma k_t \) and \( m_t + b_t = d_t \) is consistent with individual optimization. The implication here is that the demand for money and bonds will, in this case, accommodate themselves to the respective supply of money and bonds without the need for any price adjustment.

**Proposition 1** The investment demand function \( k_t \) characterized by (9) is increasing in: (i) the expected return to capital investment \( x \); (ii) the expected rate of inflation \( (p_{t+1}/p_t) \); and (iii) the interest rate on reserves \( R^m \).

Investment demand is decreasing in the nominal yield on bonds \( R^b \).

The proof of this proposition follows immediately from (9). Intuitively, an increase in \( x \) increases the expected productivity of capital and so stimulates capital spending. An increase in the expected rate of inflation reduces the real interest rate on competing nominal assets, stimulating a portfolio substitution away from these assets and into capital. It is worth emphasizing the effect on investment demand from an increase in “the” interest rate. Proposition 1 asserts that the answer depends on exactly which interest rate one is referring to. An increase in the interest rate on bonds has the effect here of reducing investment demand—agents substitute out of capital and into higher-yielding government securities. An increase in the interest rate on reserves, however, has the effect of stimulating investment demand. An increase in the interest rate on reserves lowers the cost of holding reserves, so agents are motivated to expand their holdings of reserves, which then permits capital spending to increase.\(^5\)

\(^5\)Friedman (1960) advocated paying interest on (required) reserves to alleviate the implicit tax associated with a binding reserve requirement.
3 Equilibrium

In any equilibrium, we have $M_t = p_t m_t$, $B_t = p_t b_t$ and $D_t = p_t d_t$. Because $D_t = \mu D_{t-1}$, the expected rate of inflation must satisfy,

$$\Pi_{t+1} \equiv \left( \frac{p_{t+1}}{p_t} \right) = \left( \frac{D_{t+1}}{D_t} \right) \left( \frac{d_t}{d_{t+1}} \right) = \mu \left( \frac{d_t}{d_{t+1}} \right)$$  (10)

Now, combine (10) with (9) together with $k_t = y - d_t$ to form,

$$xf'(y - d_t) = \left[ \frac{(1 + \sigma) R^b_t - \sigma R^m_t}{\mu} \right] \left( \frac{d_{t+1}}{d_t} \right)$$  (11)

Recall that $\theta_t \equiv M_t / D_t = m_t / d_t$. If the reserve requirement binds ($R^b_t > R^m$), then $\theta_t = \sigma (y - d_t) / d_t$ which, when expressed in terms of $d_t$ becomes,

$$d_t = \left( \frac{\sigma}{\theta_t + \sigma} \right) y$$  (12)

If the reserve requirement is slack ($R^b_t = R^m$), then condition (12) can be ignored (since the composition of debt $\theta_t$ is irrelevant in this case). From the government budget constraint (3) we have,

$$\tau_t = \left[ \frac{R^m_t \theta_{t-1} + R^b_{t-1}(1 - \theta_{t-1})}{\mu} - 1 \right] d_t$$  (13)

3.1 Interest rate peg

The first type of monetary policy I want to study is an interest rate peg: $R^b_t = R^b > R^m$. An equilibrium in this case consists of bounded sequences for $d_t$, $\tau_t$ and $\theta_t$ that satisfy (11), (12) and (13) for all $t \geq 1$. A stationary equilibrium is an equilibrium that satisfies $(d_t, \tau_t, \theta_t) = (d, \tau, \theta)$ for all $t$.

Note that the equilibrium here has a recursive structure. That is, condition (11) determines $\{d_t\}_{t=1}^\infty$. With $d_t$ so determined, condition (12) determines the sequence of open-market operations $\{\theta_t\}_{t=1}^\infty$ that are necessary to support the fixed interest rate regime. With $\{d_t, \theta_t\}$ so determined, condition (13) then determines the lump-sum tax $\tau_t$ that is necessary to balance the government budget.
Define \( A^{-1} \equiv [(1 + \sigma)R^b - \sigma R^m] / \mu > 0 \) and rewrite (11) as,

\[
d_{t+1} = Axf'(y - d_t)d_t \equiv P(d_t)
\]

(14)

From (14), we see that conditional on a policy \((R^b, R^m, \mu)\), we see that two stationary equilibria are possible, one of which is degenerate \((d = 0)\), the other which satisfies \( 1 = Axf'(y - d^*) \) with \( 0 < d^* < \infty \) (point A in Figure 1). Given the strict concavity of \( f \), the non-degenerate stationary equilibrium is unique.\(^6\)

Let me now investigate the stability properties of these two stationary states. First, note that \( P'(d) = Ax\left[ f'(y - d) - f''(y - d)d \right] \geq 0 \), with \( P'(0) = 0 \) and \( P'(d) > 0 \) for \( d > 0 \). Thus, \( P(d) \) is increasing monotonically in \( d \). Second, note that \( \lim_{d \to 0} P(d)/d = P'(0) = 0 \) and \( \lim_{d \to y} P(d)/d = P'(y) = \infty \), so that \( P(d) \) takes the general shape displayed in Figure 1, crossing the 45\(^\circ\) line twice: once at the origin and once at point A.

The properties of \( P(d) \) are familiar in OLG models of fiat money, where money is the only asset and whose nominal return is pegged (usually to zero). Under an interest rate peg then, there exists a continuum of non-stationary equilibria indexed by an arbitrary initial condition \( d_1 \in (0, d) \) with the property that \( d_t \to 0 \).\(^7\) Equilibria of this form are “hyperinflations” where the value of nominal government debt eventually approaches zero.\(^8\) Since \( d_1 = D_1/p_1 \), the multiplicity of nonstationary equilibria implies that the initial price-level is indeterminate.

For what it’s worth, the non-degenerate steady-state \( 0 < k^* < y \) is characterized by,

\[
x f'(k^*) = \left[ \frac{(1 + \sigma)R^b - \sigma R^m}{\mu} \right]
\]

(15)

Although \( k^* \) is unstable under the interest-rate target rule, we can still make statements on how it depends on parameters.

**Proposition 2** If \( R^b > R^m \), then \( k^* \) is increasing in \( x, \mu \) and \( R^m \), and is decreasing in \( R^b \).

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\(^6\)In a related model, Sargent and Wallace (1985) assert the existence of a continuum of stationary equilibria satisfying a restriction similar to (14)—see their equation 6, pg. 283. The same indeterminacy exists here if \( A \) is left free, in which case, policy is assumed to adjust passively to private sector expectations and behavior. 

\(^7\)Paths with the property \( d_t \to 1 \) are ruled out as equilibria because they violate feasibility \( d_t \leq y \) for all \( t \). 

\(^8\)Thus, a hyperinflation is possible even with a contracting supply of money \((\mu < 1)\).
To prove this, define \( g(k) \equiv f'(k)(y - k) \) and note that \( g'(k) = (y - k)f''(k) - f'(k)k < 0 \). Intuitively, an increase in \( x \) increases the return on (hence the demand for) capital. An increase in the inflation rate \( \mu \) lowers the real return on government bonds, inducing a portfolio substitution away from bonds and into capital (and there into money as well, to meet the reserve requirement). An increase in the interest rate on reserves, however, has the effect of stimulating capital spending here because it lowers the “tax” on holding money.

Under the policy regime described here, different policy rates \( R^b \) are associated with different money-to-debt ratios \( \theta \). In particular, from condition \((12) \theta = \sigma k/(y - k)\), so that \( \theta \) is increasing in \( k \). From Proposition 2 then, a higher \( R^b \) is associated with a lower \( \theta \) (a “tighter” monetary policy). As well, since \( p_t = D_t/(y - k) \), a higher \( R^b \) is associated with a lower price-level, although the long-run inflation rate remains pinned by \( \mu \).

While the non-degenerate steady-state is unstable under this policy regime, it turns out to be stable under the policy regime I consider next.

### 3.2 Money-to-debt ratio peg

The second type of monetary policy I want to study is a money-to-debt peg: \( \theta_t = \theta > 0 \). An equilibrium in this case consists of bounded sequences for \( d_t, \tau_t \) and \( R^b_t \) that satisfy \((11), (12) \) and \((13)\) for all \( t \geq 1 \). A stationary equilibrium is an equilibrium that satisfies \((d_t, \tau_t, R^b_t) = (d, \tau, R^b)\) for all \( t \).

When the reserve requirement binds \((R^b_t > R^m)\), condition \((12)\) determines the real quantity of government debt \( d = (\sigma/(\theta + \sigma))y \) and the equilibrium level of capital spending \( k = y - d = (\theta/(\theta + \sigma))y \). The implication of this is that the price-level is now determinate, \( p_t = D_t/d \) for all \( t \geq 1 \). Moreover, because \( d_t = d \) for all \( t \geq 1 \), there are no non-stationary equilibria. The policy of pegging \( \theta \) instead of \( R^b \) results in a unique equilibrium that is also a stationary equilibrium (as long as \( R^b > R^m \)).

Use \((11)\) and \((12)\) together with \( \theta_t = \theta \) to derive this expression for the equilibrium bond yield,

\[
R^b = (1 + \sigma)^{-1} \left[ \mu x f' \left( \frac{\theta}{\theta + \sigma} \right) y + \sigma R^m \right] > R^m
\]  

**Proposition 3** If \( R^b > R^m \), then the equilibrium nominal bond yield \( R^b \) is...
strictly increasing in $x$ and strictly decreasing in $\theta$. The equilibrium level of capital spending $k$ is increasing in $x$ and $\theta$.

The proposition is easily validated by inspecting (16). The intuition is straightforward. An increase in $x$ leads to an upward revision in the forecasted return to capital spending—that is, there is an increase in the demand for investment at any given interest rate. Agents are motivated to substitute out of bonds and into capital. But policy here pins down the real value of the outstanding supply of bonds. The decline in bond demand must therefore be fully absorbed as a decline in the price of bonds—that is, the bond yield must rise.

An increase in $\theta$ corresponds to a (permanent) open market operation that expands the supply of cash relative to bonds. The added supply of reserves permits agents to expand capital spending. But as capital spending expands, the rate of return to capital declines (the marginal product of capital is diminishing). As capital investment becomes relatively unattractive at the margin, agents are induced to substitute into bonds, increasing their price (lowering their yield).

Proposition 3 together with (16) implies that there exists a number $\hat{x} > 0$ such that,

$$R^b = \mu \hat{x} f' \left( \left( \frac{\theta}{\theta + \sigma} \right) y \right) = R^m$$

(17)

When $R^b = R^m$, the reserve requirement is slack. Thus, for a given configuration of policy parameters $(\theta, \mu, R^m)$, a sufficiently bad shock ($x < \hat{x}$) will drive bond yields to their lower bound (the interest rate paid on reserves). For $x \leq \hat{x}$, the stationary value of real debt ($d$) is no longer determined by (12)–it is instead determined by (11),

$$R^m = \mu x f' (y - d)$$

(18)

with associated price-level $p_t = D_t / d$. Note that when the interest rate is driven to its lower bound, the policy regime effectively switches to the interest rate peg regime described earlier with all its associated indeterminacies. Condition (18) determines $d$ (and $k$) independently of $\theta$. In other words, open market operations that swap money for bonds do not matter, not even for the price-level.\footnote{I remind the reader that by an “open market operation,” I mean a swap of bonds}
Proposition 4 If $R^b = R^m$, then an increase in $\theta$ (an expansionary open-market operation) has no effect on the capital spending $k = y - d$ or the price-level $p_t = D_t/(y - k)$. The only effect is to increase excess reserves $m - \sigma k > 0$, where $m = \theta d$.

4 Discussion

The results above demonstrate that the comparative statics of both policy regimes above are identical. The only difference is whether we want to think of monetary policy as targeting an interest rate, permitting the money-to-debt ratio to accommodate itself to the chosen rate, or whether we want to think of monetary policy as choosing the composition of government debt, permitting the yield on government bonds to clear the bond market. When money is dominated in rate of return, the model delivers standard “textbook” results in terms of the consequences of monetary policy (actions that affect the policy rate $R^b$). When shocks drive the economy to a region in the parameter space where the zero lower bound is in effect ($R^b = R^m$), the model delivers classic “liquidity trap” effects (e.g., Krugman, 1998). Let me now use the model to interpret the U.S. macroeconomy and monetary policy before and after 2008.

4.1 Typical recession and policy response

One way to generate a business cycle here is to assume that $x$ is subject to change over time. My preferred interpretation of $x$ is that it constitutes a “news shock” (Beaudry and Portier, 2007) realized at date $t$ but which affects productivity at date $t + 1$. A decline in $x_t$ at date has the effect of reducing the demand for investment at date $t$ without changing the supply of output at date $t$, i.e., in this model, the real GDP is fixed at $Y_t = y + x_{t-1}f(k_{t-1})$. As such, a decline in $x_t$ looks like a negative “aggregate demand shock” associated with a growing pessimistic outlook relating to the return to capital for reserves for a given level of debt $D_t$. If the open market operation consists instead of financing a given ratio of additional debt $D_t + \Delta D_t$, then there would be a price-level effect although, in this model, a surprise injection of nominal debt is neutral.
investment.\textsuperscript{10}

Prior to 2008, the Fed’s policy rate \((R^b_t)\) was above the zero lower bound \((R^m = 1)\). Consider the economic contractions that occurred in the early 1990s and early 2000s. Both of these recessionary episodes were associated with diminished expectations, which I want to think of here as a sequence of progressively lower realizations of \(x\). By Proposition 3, the effect of lower \(x\) is to decrease investment demand, hence decrease capital spending which, in turn, leads to lower output. With long-run inflation anchored by the fiscal authority, such shocks can only have transitory effects on inflation, but they can have permanent effects on the price-level. Absent an intervention, the effect of a lower \(x\) is to cause a decline in the price-level, which reflects an increase in the real demand for government securities \(d = y - k\). A sequence of “bad news shocks” would therefore generate a deflationary episode \(p_t > p_{t+1} > p_{t+2}\), even though expected inflation remains anchored at \(\mu\).

Since \(R^b > R^m\), there is scope for a monetary intervention that either lowers \(R^b\) directly, or lowers \(R^b\) indirectly through open market operations that expand the size of the Fed’s balance sheet (i.e., an increase in \(\theta\)). By Proposition 3, the effect of “loosening” monetary policy in this way is to stimulate capital spending, thereby preventing output from falling as much as it would have absent the intervention. As well, the same intervention has the effect of stabilizing the price level.

Incidentally, it is of some interest, I think, to ask what “causes” the interest rate to decline in a recession. To many observers, it appears that the Fed is causing the interest rate to decline, either directly through its policy rate, or indirectly through its open market operations. As the analysis above suggests, such a view is only partially correct. Consider, for example, the competitive equilibrium real interest rate in this economy absent any government \(r = x f''(y) < 1\). In this case, a decline in \(x\) will cause the interest to decline because the supply of saving is fixed at \(y\) and because a lower \(x\) implies a lower demand for capital. In other words, there are natural market forces at work pushing the interest rate lower in a recession that are independent of Fed actions. The only question, really, is whether the Fed wants to accommodate these market forces or not. If it does not, the contraction in investment spending will be greater than it otherwise would.

\textsuperscript{10}Note that for our positive analysis, it matters not whether expectations are rational or not. Pessimism here manifests itself in exactly the same way, regardless of its source.
be. In this sense, the Fed is not “causing” the interest rate to decline— it is simply accommodating market forces that “want” a lower interest rate.

4.2 The great recession and quantitative easing

The economic contraction of 2008 is unusual in at least two respects. First, it was unusually severe and second, the market yield on U.S. treasuries fell to the interest rate on reserves. Consider Figure 2, which plots the 3-month Treasury yield and the ratio of base money to government debt held by the public. Note that the bond yield began to decline well before the start of the recession. This is consistent with deteriorating expectations (a decline in $x$) weakening investment demand and making bonds relatively more attractive. As the economic outlook continued to deteriorate throughout 2008, the economy contracted and yields continued to decline. With the failure of the Lehman Brothers in the fall of 2008 and the economy on the verge of crisis, the U.S. Federal reserve announced the first of its large scale asset purchase programs (LSAPs) known as QE1. In the context of our model, one can interpret “quantitative easing” as a sharp increase in $\theta$. With these events, the yield on short term treasuries declined essentially to their lower bound $R^b \leq R^m$ (see Figure 2, late 2008)—an effect that is consistent with the prediction of the model (see Proposition 2).

When $R^b = R^m$, Proposition 3 asserts that any further loosening of monetary policy (in the sense of increasing $\theta$) is completely innocuous—increasing the supply of base money does not even influence the price-level, a prediction which is consistent with the evidence (see Figure 3). When $R^b = R^m$, the economy is in a “liquidity trap.” That is, the economy is satiated with liquidity and any further attempts to inject liquidity (withdraw bonds) will only lead to excess cash reserves being willingly held (as they are, at this point, perfect substitutes for the bonds they replace in private wealth portfolios). The evidence presented in Figure 3 seems not inconsistent with this prediction—most of the increase in the supply of base money since late 2008 is in fact being held as excess reserves in the banking system.

Another striking development in 2008 was the sharp decline in the “money multiplier”—the ratio of a broad money aggregate relative to the monetary base; see Figure 4, which plots M1 (roughly currency in circulation plus demand deposit liabilities) relative to the monetary base. The model developed
above is not rich enough to make a sharp distinction between currency in circulation $M_t^c$ and bank reserves $M_t^b$, where $M_t = M_t^c + M_t^b$, so let me just assume that $M_t^c = \xi_t M_t$, where $0 < \xi_t < 1$ is exogenous. Suppose that some exogenous fraction $0 < \alpha_t < 1$ of the economy’s capital stock is intermediated by banks, so that demand deposit liabilities in the model equal $p_t \alpha_t k_t$. In this case, M1 is given by,

$$M_1 = \xi_t M_t + p_t \alpha_t k_t$$

Market-clearing requires $p_t = D_t / d_t$ which, when substituted into the expression above yields, after some manipulation,

$$\left[ \frac{M_1}{M_t} \right] = \xi_t + \alpha_t \left( \frac{k_t}{y - k_t} \right) \left( \frac{1}{\theta_t} \right)$$

(19)

Thus, holding fixed the parameters $\xi_t, \alpha_t$ and $\theta_t$, an exogenous “bad news shock” (a sudden decline in $x$) is predicted (by Proposition 2) to cause a sharp decline in the money multiplier. The intuition is simple: the contraction in investment demand leads to a proportional decline in bank financing. Incidentally, since $p_t = D_t / (y - k_t)$, the same shock induces a decline in the price-level (ceteris paribus) which did in fact occur and which arguably would have been much more severe had $D_t$ not expanded at nearly the same time. Once the economy is at the zero lower bound ($R_b = R_m$), the theory predicts that monetary policy in the form of $\theta_t$ has no real or nominal effects except, as condition (19) reveals, on the money multiplier. The observed decline in the U.S. money multiplier since 2008 can then be explained as the consequence of the Fed’s continues quantitative easing programs at the zero lower bound.

Of course, if “monetizing” a greater fraction of government debt is as innocuous as Proposition 3 suggests, then what rationalizes the Fed’s QE2 and QE3 programs? One answer is that the conditions stated in Proposition 3 are extreme: they describe a circumstance in which government bonds are literally perfect substitutes for interest-bearing cash reserves. In reality, the Fed’s LSAP programs have included non-traditional securities, for example, higher-yielding longer-dated government bonds as well as agency debt.\(^{11}\)

Technically then, one might expect some effect, but one that is likely to be

\(^{11}\)Agency debt consists mainly of new (not legacy) AAA-rated mortgage-backed-securities issued by Fannie Mae and Freddy Mac.
small given the historically low yields that presently characterize these non-traditional securities. If so, then this would explain the trouble economists have generally had in identifying the quantitative effects of the Fed’s LSAP programs (e.g., Thornton, 2014).

4.3 Why is inflation so low?

Figure 5 plots the PCE inflation rate, the short-term nominal interest rate, and the real GDP growth rate since 2007. According to this data, economic growth has returned to pre-recession levels, the nominal interest rate is close to zero, and yet the inflation rate remains stubbornly below the Fed’s 2% target. According to standard Phillips curve reasoning, accelerating growth should cause inflation to go up, not down. Is there a way to rationalize this observation?

In the specification of policy above, I assumed that the fiscal authority mechanically chooses to grow its nominal debt at rate $\mu$. While this policy alone does not pin down the price-level, it does pin down the expected growth path of the price-level; i.e., it determines the expected rate of inflation. For the case in which $R_b = R_m$, monetary and fiscal policy together then determine the real rate of return on government debt $R_m/\mu$ which, through the Fisher equation (15) then determines the equilibrium level of capital spending, i.e.,

$$xf'(k) = \frac{R_m}{\mu} \quad (20)$$

An alternative specification of fiscal policy is that it permits its nominal debt to grow passively at the rate at which it is demanded. In conventional infinitely-lived agent models, the real interest rate $r = xf'(k)$ is determined independently of monetary policy. In such a scenario, an improvement in the economic outlook (an increase in $x$) has the effect of increasing the real rate of interest. Since the nominal interest rate $R_m$ is determined by policy, the Fisher equation (20) implies that the inflation rate must decline to satisfy the no-arbitrage condition equating the risk-adjusted real returns on capital and bonds. Thus, this is one possible explanation for why inflation declines as the economy improves. Andolfatto and Williamson (2014) describe a similar mechanism triggered by financial sector “healing” that relaxes debt constraints following a crisis. In both cases, the critical assumption is that the
fiscal authority grows its nominal debt to accommodate the market-clearing inflation rate.

This alternative specification of fiscal policy in my OLG setting, however, introduces a real indeterminacy along the lines of Wallace and Sargent (1985). Technically, any \((k, \mu)\) pair satisfying \(0 < k \leq y\) is consistent with equilibrium. Since growth in the demand for nominal debt depends, in part, on how the price-level is expected to grow, we have a situation in which private sector inflation expectations can be self-fulfilling, with the fiscal authority expanding the supply of nominal debt to accommodate whatever inflation rate people choose to focus on. This indeterminacy implies that the economy may get “stuck” at a level of real GDP that is too high or too low relative to some criterion that policymakers judge desirable.\textsuperscript{12} The notion that the economy might get stuck in a suboptimal equilibrium is a key insight in Keynes (1936).\textsuperscript{13} Farmer (2014) is an important modern proponent of this view. Thus, even if the economy returns to its long-run real growth rate (in this model, zero), the economy may remained mired in a “secular stagnation” where economic activity is depressed relative to its potential.

A final observation in regard to the relationship between inflation and interest rates is that (20) is consistent with a perpetually negative real of interest and strictly positive rate of inflation. This type of relationship is not consistent in conventional infinitely-lived agent models. In this latter class of models, the real interest rate is strictly positive and invariant to policy. Consequently, a Friedman rule monetary policy \((R^m = 1)\) must imply deflation \((\mu < 1)\). Conventional models modified to permit transactional debt that (owing to a shortage of good collateral assets) incorporate a liquidity premium can, however, accommodate the evidence (again, see Andolfatto and Williamson 2014 for an example).

\textsuperscript{12}One such measure is the Congressional Budget Office concept of “potential GDP;” see http://www.cbo.gov/sites/default/files/03-16-gdp.pdf

\textsuperscript{13}Keynes (1936, Chapter 18) states “In particular, it is an outstanding characteristic of the economic system in which we live that, whilst it is subject to severe fluctuations in respect of output and employment, it is not violently unstable. Indeed it seems capable of remaining in a chronic condition of sub-normal activity for a considerable period without any marked tendency either towards recovery or towards complete collapse.”
4.4 Monetary policy going forward

 Monetary policy in the United States since the end of the Great Recession has been characterized by a policy rate driven essentially by the interest on excess reserves $R^m = 1.0025$ and a balance sheet that is over four times larger than it was prior to the financial crisis, with most Fed liabilities existing as excess reserves in the banking system. As the real economy continues to improve, albeit at a slower pace that many have hoped, and with inflation only fifty basis points below target, the Fed is preparing for lift off—the date at which circumstances warrant increasing the policy rate. These circumstances evident include continued improvement in the labor market and evidence that PCE inflation is unmistakably making its way back to its two percent target. Ultimately, the plan (or desire) is to “normalize” monetary policy, which the Fed describes in the following way:

Monetary policy normalization refers to the steps the Federal Open Market Committee (FOMC) will take to remove the substantial monetary accommodation that it has provided to the economy during and in the aftermath of the financial crisis that began in 2007. Specifically, monetary policy normalization refers to steps to raise the federal funds rate and other short-term interest rates to more normal levels and to reduce the size of the Federal Reserve’s securities holdings and to return them mostly to Treasury securities, so as to promote the Federal Reserve’s statutory mandate of maximum employment and price stability. The Committee plans to continue to use the federal funds rate as its key policy rate during the normalization process and to continue to set a target range for the funds rate when it begins to remove policy accommodation and for some time thereafter. When the Committee begins to normalize policy, it will raise the target range for the federal funds rate. This tightening of policy will be transmitted to other short-term interest rates and affect broader financial conditions in the economy.

How close is the U.S. to “normal?” By some metrics, for example, the 5.5% civilian unemployment rate, the U.S. economy seems not too far from normal. On the other hand, the expected real rate of return on short-maturity U.S. debt is negative 2%, substantially below its historical average of 2%.

As a practical matter, it is difficult to determine conclusively whether “normality” has been achieved. However, the model developed above can help shed some light on this question by providing a set of diagnostics. Think of the federal funds rate as $R_b$, which was over five percent prior to the crisis (see Figure 5). Also prior to the crisis, interest on excess reserves was zero, $R_m = 0$, and excess reserves were zero as well. This state of affairs accords well with our theory which predicts zero excess reserves when $R_b > R_m$.

A combination of depressed economic conditions (lower $x$) together with a highly expansionary monetary policy (higher $\theta$) then drove the traditional policy rate down to $R_m$, which was raised in 2008 from zero to twenty-five basis points. The diagnostic is this: if the economy has indeed returned back to normal (in the sense of $x$ returning to its pre-crisis level), why hasn’t the price-level inflation increased in proportion to the expansion in base money supply?

The fact that the price-level continues to grow at even less than the targeted inflation rate, together with the fact that large quantities of excess reserves continue to be held in the banking sector, suggests that economic conditions have not returned back to normal—at least, along some important dimensions. Given a fixed $R_m/\mu$, condition (20) suggests that the tell-tale sign of a normalizing economy (an increase in $x$) should be a significant increase in the level of capital spending (with a corresponding expansion in bank lending, to the extent that investment is bank-financed). Theoretically, the effect of a normalized $x$ would also be to create a brief, but sharp, increase in the price-level, even as long-term inflation expectations remain anchored at $\mu$. The Fed is arguably primed to “lift off” once it sees strong evidence of this type of price-level movement.

\textsuperscript{16}In fact, the federal funds rate and the yield on very short-term treasuries is presently below $R_m$, a phenomenon that is evidently a by-product of the fact that government-sponsored-agencies like Fannie Mae and Freddy Mac are not permitted to earn interest on their reserve accounts.
4.5 What could possibly go wrong?

While the quoted passage above alludes to the idea of reducing the size of the Fed’s security holdings, there seems little desire to embark on this path in the early stages of lift off.\textsuperscript{17} Thus, for at least the foreseeable future, the Fed is essentially limited to using $R^m$ as its policy rate.\textsuperscript{18} I’d like to conclude by using the theory above to speculate on how things might go wrong as the Fed prepares to normalize.

The first thing to note is that, theoretically at least, the stationary equilibrium associated with an interest rate peg is unstable and induces price-level indeterminacy. Pegging the money-to-debt ratio restores stability and determinacy, but only when $R^b > R^m$, which is unlikely to happen any time soon. On the plus side, it is not clear how seriously to take this theoretical concern. The Japanese economy, for example, seems to avoided hyperinflation despite decades of zero nominal interest rates.

Moreover, hyperinflationary outcomes can theoretically be avoided by assuming that interest rate policy depends on macroeconomic conditions along the lines described by Taylor (1993). Consider, for example, a Taylor rule given by

$$\ln R^m_t = \max \{ \phi \ln \Pi_t + (1 - \phi) \ln \Pi^* + \ln r_t, 0 \}$$

(21)

where $r_t = x f'(k_t)$ is a measure of the real rate of interest and $\phi > 0$ is a parameter that governs how strongly the policy rate (here, interest on reserves) adjusts to deviations in inflation from target $\Pi^*$. The “max” operator restricts the policy rate from falling below a zero lower bound.\textsuperscript{19}

Consistent with the literature that studies Taylor rules, I assume that the fiscal authority passively accommodates inflation expectations, so that $\mu_{t+1} = \Pi^e_{t+1}$, where $\Pi^e_{t+1}$ denotes the expected inflation rate. Along a perfect

\textsuperscript{17}The lack of desire to sell securities seems to be driven by the fear that any such announcement might lead to a sell-off in the bond market, disrupting financial markets and hindering the recovery. See Neely (2014) for a description of the 2013 “taper tantrum” event.

\textsuperscript{18}At the date of lift off, the Fed will in fact use an “overnight reverse repo” (ON RRP) interest rate $R^b \leq R^o \leq R^m$ to induce the federal funds rate higher. In the event that the federal funds rate does not respond as desired, the Fed is likely to increase the IOER rate in its attempt to maintain “monetary policy control.”

\textsuperscript{19}Technically, I could allow negative nominal interest rates. All that is important here is that a lower bound exists not too far below zero.
foresight path $\Pi_{t+1}^f = \Pi_{t+1} = \mu_{t+1}$. As explained earlier, this specification of fiscal policy introduces a real indeterminacy, which can be resolved in a couple of ways. First, we could assume that $0 < k_t < y$ is determined exogenously, in which case inflation expectations are determined by the Fisher equation (20), i.e.,

$$\ln \mu_{t+1} = \ln R_t^n - \ln [xf'(k_t)] \quad (22)$$

Second, we could assume that inflation expectations are formed exogenously, in which case condition (22) determines the equilibrium level of capital spending. In either case, combine (22) with (21), invoking $\Pi_t = \mu_t$, and assuming $k_t = k$ to form

$$\ln \mu_{t+1} = \begin{cases} \phi \ln \mu_t + (1 - \phi) \ln \Pi^* & \text{if } \ln R_t^n > 0 \\ - \ln [xf'(k)] & \text{if } \ln R_t^n = 0 \end{cases} \quad (23)$$

The behavior for inflation described by (23) depends critically on whether the parameter $\phi$ is greater or less than unity. If $0 < \phi < 1$, then there is a unique steady-state inflation rate that corresponds to the target rate $\Pi^*$. Moreover, along the perfect foresight path, the inflation rate approaches the target rate monotonically from any initial condition $\mu_0 = p_0/p_{-1}$, with $p_0 = D_0/(y - k_0)$ where $0 < D_0 < \infty$ is determined exogenously by the fiscal authority.\footnote{That is, I assume that the fiscal authority chooses the initial supply of nominal debt, but thereafter supplies nominal debt perfectly elastically to accommodate market demand.} If $\phi > 1$, then there are two steady-states, one of which is the one just described. The second steady-state occurs when the nominal interest is at its zero lower bound, in which case the equilibrium inflation rate falls perpetually short of its target. As stressed by Benhabib, Schmitt-Grohé and Uribe (2001), this latter low-inflation equilibrium is stable and the “intended” equilibrium unstable when the “Taylor principle” holds, i.e., $\phi > 1$.

Thus, I invite the reader to imagine that the U.S. economy is presently in the low-inflation equilibrium just described. In this steady-state, $1 < \mu = 1/[xf'(k)] < 1.02 = \Pi^*$. If the fiscal authority has not anchored $\mu$, then any combination of $(\mu, k)$ satisfying $\mu = 1/[xf'(k)]$ is consistent with equilibrium. In particular, a “secular stagnation” outcome is possible in which the level of economic activity (measured here by $k$) is less than “normal”—even if $x$ has returned to normal. In this world, an improvement in the economic environment brought about (say) an increase in $x$ has the effect of increasing...
the real interest rate (assuming that \( k \) either remains the same or does not expand so far as to keep the marginal product of capital at its initial level). The effect of this development is to put downward pressure on the inflation rate. According to the Taylor rule, the prescription for a decline in inflation is to reduce the nominal interest rate aggressively or, barring this possibility, to keep it at zero for the indefinite future. Moreover, while a decline in \( x \) will have the effect of raising the inflation rate, the associated reduction in output is likely to warrant keeping the interest rate at zero in this case as well. In this manner, policymakers appear to be “stuck” in Japanese-style low-inflation and low-interest rate equilibrium.

4.6 Monetary vs fiscal policy

In the analysis above, I assumed that the fiscal authority chose \( \mu \) (either actively, or passively to accommodate inflation expectations). Let me now generalize this specification to highlight an interesting asymmetry that arises for an inflation-targeting central bank.

Let \( M_t = \mu_t M_{t-1} \) and \( D_t = \delta_tD_{t-1} \) where \( \mu_t \) and \( \delta_t \) are chosen by the monetary and fiscal authorities, respectively. Recall that \( \theta_t = M_t/D_t \) and

\[
\begin{align*}
p_t m_t &= M_t \quad (24) \\
p_t d_t &= D_t \quad (25)
\end{align*}
\]

for all \( t \), so that

\[
\Pi_{t+1} = \frac{M_{t+1}}{M_t} \frac{m_t}{m_{t+1}} = \frac{\theta_{t+1} D_{t+1}}{\theta_tD_t} \frac{m_t}{m_{t+1}} = \frac{D_{t+1}}{D_t} \frac{d_t}{d_{t+1}} \quad (26)
\]

Suppose that the monetary authority sets \( \mu_t = \mu \) for all \( t \). Then

\[
\mu = \left[ \delta_t \frac{\theta_{t+1}}{\theta_t} \right] \Rightarrow \theta_{t+1} = (\mu/\delta_t)\theta_t \quad (27)
\]

Now, suppose that the fiscal authority accommodates the monetary authority’s inflation target and sets \( \delta_t = \mu \). Then in a stationary equilibrium, we have \( \Pi = \mu \).

But suppose that the fiscal authority is not so accommodating. I want to consider two cases. In the first case, the fiscal authority wants to issue
debt at a rate faster than the central bank is willing to monetize it, \( \delta_t \geq \mu \) for all \( t \). In the second case, the opposite is true, so that \( \delta_t \leq \mu \) for all \( t \). The economic implications of these two cases depend on whether \( R^b_t > R^m \) or \( R^b_t = R^m \). Thus, there are four cases to consider.

### 4.7 Case: \( R^b_t > R^m \)

From (11)–(13), the equilibrium \( \{R^b_t, d_t, \tau_t\} \) in this case must satisfy

\[
xf'(y - d_t) = \left[ \frac{(1 + \sigma)R^b_t - \sigma R^m}{\delta} \right] \left( \frac{d_{t+1}}{d_t} \right) \tag{28}
\]

\[
d_t = \left( \frac{\sigma}{\theta_t + \sigma} \right) y \tag{29}
\]

\[
\tau_t = \left[ \frac{R^m \theta_{t-1} + R^b_{t-1}(1 - \theta_{t-1})}{\delta} \right] d_t \tag{30}
\]

Suppose the fiscal authority attempts \( \delta_t = \delta > \mu \). Then condition (27) implies that \( \theta_t \searrow 0 \) as \( t \to \infty \) which, by condition (29) implies \( d_t \nrightarrow y \) as \( t \to \infty \). Condition (28) then implies that the real interest approaches \( xf'(0) \) in the limit, as capital investment is driven to zero. While the carrying cost of the debt is very high, condition (30) shows how a lump-sum tax equal to the carrying cost can support arbitrary debt-service costs. Finally, we can use (26) to ascertain the equilibrium path of inflation

\[
\Pi_{t+1} = \mu \frac{m_t}{m_{t+1}} = \delta \frac{\theta_{t+1}}{\theta_t} \frac{m_t}{m_{t+1}} = \delta \frac{d_t}{d_{t+1}}
\]

In the limit, as \( d_t \) approaches \( y \), the inflation rate is determined by the fiscal authority’s choice of \( \delta > \mu \). Granted, this economy is a disaster. Government bonds have displaced money. Because money is essential for capital spending in this model, capital spending evaporates. I conclude that while an outcome like this is theoretically possible, it is unlikely to be sustainable in reality.

Suppose instead that the fiscal authority attempts \( \delta_t = \delta < \mu \). Then condition (27) implies that \( \theta_t \leq 1 \) grows monotonically until it reaches its upper bound (since \( M_t \leq D_t \) for all \( t \)). When the upper bound is reached, the central bank is monetizing the entire debt. But at this point, the central bank can monetize no more debt than the fiscal authority is willing to issue,
so \( \mu \) is forced down to \( \delta \). As in the previous case, the fiscal authority can determine the inflation rate. Along the transition path, the nominal and real interest rates are declining with inflation \( \Pi_{t+1} \searrow \delta \) as \( t \to \infty \).

4.8 Case \( R^b_t = R^m \)

5 Conclusions

In a world where \( R^b > R^m \) and reserves are scarce, the Fed has the power to control inflation through its control of the money base \( M_t = \theta_t D_t \). That is, even if the fiscal authority grows its debt, the monetary authority can always choose how much of that debt to monetize through its choice of \( \theta_t \). When \( R^b = R^m \), however, the Fed no longer has direct control over the inflation rate because money and bonds are viewed as close substitutes. What matters for inflation in this case is the rate of growth of total nominal debt and not its composition. In this latter case, the Fed might exert indirect influence over the inflation to the extent that the fiscal authority accommodates its debt-management policy in response to the Fed’s policy rate.

In addition, when \( R^b = R^m \) and \( \mu \) is not anchored by the fiscal authority, a real indeterminacy arises. In this case, Keynesian “animal spirits” are left to determine the equilibrium levels of output and inflation. One solution to this indeterminacy is to insist that the fiscal authority anchor inflation expectations by (say) committing to grow its nominal debt at an appropriate rate.\(^{21}\) With inflation expectations anchored in this manner, the Fed would be free to choose a rule for its policy rate that it thinks most appropriate.

What if the fiscal authority (i) fails to anchor \( \mu \), (ii) anchors \( \mu \) at a level judged too high, or (iii) anchors \( \mu \) at a level judged too low?

In case (i), the Fed could engage in asset sales (drain reserves) to the point where reserves are once again scarce \( (R^b > R^m) \). In this case, control over the monetary base implies control over the price-level and inflation. The solution to case (ii) is the same as case (i). Case (iii) is more problematic and not a case that is frequently studied since historically, the idea of governments reluctant to issue debt seems hard to take seriously.

\(^{21}\) “Appropriate” in the context of the United States would include taking into account the growth in \( \text{world} \) demand for Treasury debt.
Nevertheless, case (iii) constitutes a plausible scenario in today’s economy. While the rate of growth of U.S. Treasury debt since 2008 has been rapid, yields remain remarkably low. The most obvious way to square these two observations is to suppose that the growth in the demand for treasury debt has exceeded the growth in supply. A part of this world wide demand for U.S. treasury debt no doubt stems from relatively poor domestic conditions around the globe together with distrust over domestic sovereigns, leaving investors seeking safety relatively few options. This, coupled with the prospect that U.S. political forces may result in curtailments of future U.S. government budget deficits suggests the possibility of a continued world wide bond shortage. In the context of the model above, it may result in a lower \( \mu \), making inflation depart even further below the Fed’s inflation target. Interestingly, there is little the Fed can do in this case, apart from keeping its policy rate a low as it can (possibly negative) and hoping for continued economic recovery (increases in \( x \)).
6 Figures

Figure 1
Equilibrium Dynamics (Interest Rate Peg)
Figure 2

Treasury Yield and Money-to-Debt Ratio
United States

Source: FRED/Thomson Reuters
Figure 3

Money Supply and the Price-Level

[Graph showing Money Supply and the Price-Level with monetary base and PCT lines]

Monetary Base  

PCT
Figure 4

Source: Federal Reserve Bank of St. Louis
Shaded areas indicate US recessions - 2015 research.stlouisfed.org
Figure 5

Output, Inflation and Interest Rates
United States
7 References


18. Samuelson


