Econ 807: Macroeconomic Theory and Policy
Assignment 2

Due: March 01, 2001.

1. Explain why the domestic investment spending of a small open economy with access to world financial markets is unlikely to respond strongly to a transitory reduction in aggregate income (say, owing to a widespread crop failure). How is such an economy likely to respond to this sort of shock? You can answer this question using a diagram.

2. A person has preferences for current and future consumption given by $U = \ln c_1 + \beta c_2$, where $\beta > 0$. As well, the person owns and operates a production technology $y = A_t k_t$, where $A_t > 0$ and k_t represents capital in place at the beginning of period t. (Assume that k_1 is given and that capital depreciates fully after one period). The productivity parameter A_t follows a two-state Markov process; i.e., let $\Pr[A_{t+1} = A^j \mid A_t = A^j] = \pi > 1/2$, where $A^j \in \{A^L, A^H\}$, $A^L < A^H$.

   (a) Let $R(A^j)$ denote the (gross) expected return to investment when $A_t = A^j$. Show that $R(A^H) > R(A^L)$ and explain.

   (b) Solve for the optimal investment choice and explain how it depends on the current state of technology $A_t = A^j$ (there are two effects to consider: a wealth effect and an expectations effect).

3. From Exercise 2.2 of Stokey, Lucas and Prescott (1989), we can derive the optimal growth path for capital in a deterministic growth model as:

$$k_{t+1} = \alpha\beta \left[\frac{1 - (\alpha\beta)^{T-t}}{1 - (\alpha\beta)^{T-t+1}} \right] k_t^{\alpha}$$

   for $t = 0, 1, ..., T-1$, and with $k_{T+1} = 0$, $k_0 > 0$ given. Simulate out time paths for the optimal trajectory for different combinations of the parameters T, α, β and k_0.

4. Consider an economy that consists of a group of people (with unit mass) who differ in their current period income; i.e., let y_1 be distributed according to the cumulative distribution function $G(a) = \Pr[y_1 \leq a]$ defined over some bounded interval $Z \subset R_+$. Let $\overline{y}_1 = \int_Z y dG(y)$. Individuals can use their endowment y_1 for either consumption (c_1) or investment (x). Suppose that x represents an investment in human capital; let $y_2 = wx$ denote the return on the investment, where $w > 0$ is a parameter. There is no financial market. Preferences are given by $U = \ln c_1 + \beta \ln c_2$.

   (a) Solve for the level of investment $x^d(y_1)$ undertaken by each person. In this economy, we find that $Cor(y_1, y_2) > 0$. Explain.

   (b) Suppose that there is a government that is concerned about earnings inequality (and how this inequality persists through time). In an attempt to remedy the disparity in incomes, the government announces a negative income tax (NIT) scheme that will be implemented in the future period. A NIT is a flat tax rate τ on earnings together with a universal lump-sum transfer payment T. For an arbitrary pair of (τ, T), derive the investment function $x^d(y_1, \tau, T)$. Find the comparative static results $dx^d/d\tau$ and dx^d/dT. Explain.(Note: assume that $x^d > 0$).

(c) Assume that the government chooses some τ; treat this as a parameter. The revenues raised by this tax are used to finance the lump-sum transfer T, so that the government budget constraint is given by:

$$\tau w \int_Z x^d(y_1, \tau, T) dG(y_1) = T.$$

We can use the equation above to solve for the equilibrium level of T,

$$T = \left[\frac{(1-\tau) w \tau \beta \overline{y}_1}{1 + \beta(1-\tau)} \right].$$

Derive this expression and plug it into the investment function $x^d(y_1, \tau, T)$ in order to derive:

$$x^D(y_1, \tau) = \left(\frac{\beta}{1+\beta} \right) \left[y_1 - \frac{\tau}{1 + \beta(1-\tau)} \overline{y}_1 \right].$$

CONJECTURE: *The expression* $d \ln x^D(y_1, \tau)/d\tau$ *is a decreasing function of* y_1. See whether or not this is true (I think that it is) and explain the economic significance of the result.

(d) I seem to find that by increasing the generosity of the transfer scheme (i.e., increasing τ), the level of future per capita GDP falls; i.e.,

$$\overline{y}_2 = w \left(1 - \frac{\tau}{1 + \beta(1-\tau)} \right) \left(\frac{\beta}{1+\beta} \right) \overline{y}_1.$$

Derive this expression and explain the result.

(e) The distribution of future before-tax earnings y_2 is proportional to the distribution of $x^D(y_1, \tau)$. What happens to the degree of inequality in future before-tax earnings [where inequality is measured by $Var(wx^D(y_1, \tau))$] when the government implements its redistribution scheme? Explain.

(f) Bonus marks: How does the transfer policy affect the distribution of after-tax income $(1-\tau)y_2 + T$? If you cannot derive an analytical solution, you may want to code up a quantitative version of this model, suitably parameterized, and examine what happens to the distribution of after-tax income as you increase τ. At the same time, examine the quantitative effect on the distribution of before-tax income.

5. A representative individual has preferences for consumption and leisure given by: $U_t = c_t + \frac{\psi}{1-\eta}(1-n_t)^{1-\eta}$. This person owns and operates a production technology $y_t = e^{z_t} n_t^{\theta}$. Write a FORTRAN or GAUSS program to solve for a piecewise linear approximation of the true solution function $n_t = n(z_t)$.