Econ 807: Macroeconomic Theory and Policy

Assignment #3

Due Date: March 21, 2001

1. Consider the optimal growth model:

$$\max_{\{k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln(k_t^{\alpha} - k_{t+1})$$

with $k_0 > 0$ given. We know that the solution is given by $k_{t+1} = \alpha\beta k_t^{\alpha}$.

(a) Compute the steady-state capital stock k^*. For $k_0 = 0.01k^*$, plot the optimal trajectory for M periods, where M is given by the value that satisfies $|\ln k_M - \ln k^*| < 0.01$.

(b) Write a program that solves for the optimal capital stock trajectory $\{k_{t+1}\}_{t=0}^{M}$ using the 'overshooting' algorithm described in class. Plot your numerical solution against the true solution on a graph.

(c) Write a program that solves for the optimal investment rule $g(k) = \alpha\beta k^{\alpha}$ using the 'monotone operator' algorithm described in class. Plot your numerical solution against the true solution on a graph.

2. Consider the following simple labour market search model. A firm that is currently matched with a worker generates a cash flow $(1-\xi)y_t$, where y_t is the output produced and ξ is the share of output accruing to the worker. The firm-worker match splits up with probability δ at the end of the period (after the cash flow has been generated), in which case the firm shuts down forever. Output (productivity) fluctuates from one period to the next according to a first-order Markov process. Let β^{-1} denote the gross real rate of interest. In this case, we can write the capital value of the firm as follows:

$$J(y) = (1-\xi)y + \beta(1-\delta)E\left[J(y') \mid y\right],$$

where E denotes the expectation operator associated with the realization of 'next' period's output y' conditional on information y.

(a) Let $\delta = 0.15, \xi = 0.65$ and $\beta = 0.99$. Suppose that y_t takes on three possible values: $y^H > y^M > y^L$, with $y^H = 1.01, y^M = 1.00$, $y^L = 0.99$. Let $\pi_{ij} = \Pr[y_{t+1} = y^j \mid y_t = y^i]$ for $i, j = H, M, L$. Let $\pi_{ij} = \pi$ for $i = j$ and $\pi_{ij} = \frac{1}{2}(1-\pi)$ for $i \neq j$. Compute the value function $J(y)$ for the following two cases: $\pi = 1/3$ and $\pi = 2/3$.

(b) Vacant firms match with an unemployed worker at the beginning of the period with probability $q(v_t)$, where v_t denotes the number of vacant jobs (recruiting intensity). Let $\kappa = 0.25E[y]$ denote the resource cost of maintaining a vacancy, where $E[y]$ denotes the unconditional expectation of output. Free-entry in job creation drives the capital value of a vacancy to zero, so that v is determined by:

$$q(v)J(y) = \kappa.$$

Let $q(v) = v^{\lambda - 1}$ and solve for $v(y)$ assuming that $\lambda = 0.60$.

(c) The evolution of employment in the model is given by:

$$n_{t+1} = (1 - \delta)\left[n_t + q(v_t)v_t\right]$$

with $n_0 > 0$ given. The average level of employment in this economy is given by:

$$n^* = \delta^{-1}(1 - \delta)\mu(v^*)^{\lambda}$$

where $v^* = q^{-1}(\kappa / E[J(y)])$. Beginning with $n_0 = n^*$, plot the impulse-response function for employment following a 1% negative shock to productivity (for both cases: $\pi = 1/3$ and $\pi = 2/3$).

(d) For $\pi = 1/3$, simulate employment and the shock for 1000 periods. Plot the autocorrelation function (8 lags) for employment growth and productivity growth. Explain any differences.