Econ 807: Macroeconomic Theory and Policy

Assignment #4: Application

1. Consider an economy of 2-period lived overlapping generations of individuals with preferences given by:

$$V_t = \ln(c_t(1)) + \beta \ln(c_{t+1}(2))$$

where $c_j(i)$ represents consumption in the i^{th} period of life during date $t = 1, 2, ..., \infty$. Each generation has the same number of people N (population is constant). The young are born without capital, but are able to work and generate a real wage w_t; with this income they can consume $c_t(1)$ and invest in capital k_{t+1}. The old have no labour income; they must finance their retirement consumption out of capital income $r_{t+1}k_{t+1}$, where r_t represents the rental rate of capital at date t. Thus, individuals face the following sequence of budget constraints:

$$\begin{aligned} c_t(1) + k_{t+1} &= w_t; \\ c_{t+1}(2) &= r_{t+1}k_{t+1}. \end{aligned}$$

While individuals view factor prices (w_t, r_t) parametrically, these prices are determined endogenously in a competitive factor market. Assume that a representative firm owns a constant returns to scale production technology:

$$Y_t = AK_t^{\theta} H_t^{1-\theta}$$

where K_t and H_t represent measures of the aggregate capital and labour inputs, respectively. In equilibrium, $K_t = Nk_t$ and $H_t = N$, so that factor prices are determined by:

$$\begin{aligned} w_t &= (1-\theta)(K_t/N)^{\theta}; \\ r_t &= \theta(K_t/N)^{\theta-1}. \end{aligned}$$

   (a) Numerically solve for the steady state competitive equilibrium allocations $c(1), c(2), k$ and prices (w, r). Use $\theta = 1/3$ and choose A such that $Y = 1$ in the steady state.

   (b) Suppose now that the government taxes labour income at rate $\tau = 0.25$ in order to finance a lump-sum social security payment T to the old. In a steady state, the government budget constraint is given by: $NT = N\tau w$. Solve for the steady state allocations and prices. Compare the new equilibrium to the one without a social security program and explain any differences. What is the quantitative impact of this policy on real GDP? How is the well-being of the representative young person affected by such a program?

   (c) Consider an economy that is currently in a steady state without social security. Suppose that suddenly (and unexpectedly), the old

generation vote in a government that implements the social security payment T computed in part (b) above. Assume that this transfer is expected to remain in place for the indefinite future. Compute the equilibrium transition dynamics for allocations, prices and tax rates. Plot on a graph. Which generations benefit and which lose from the implementation of this fiscal policy?

2. Consider an economy populated with a unit mass of individuals with preferences defined over consumption and leisure: $U = \ln(c) + \lambda \ln(1-n)$, where n represents the fraction of time spent in the labour market. There are two types of people, distinguished by their labour-market skill (real wage); i.e. $w_H > w_L$. Let θ denote the fraction of high-skill workers. There is a government that taxes income at the rate τ and redistributes all tax proceeds in a lump-sum manner back to the population (i.e, this is a negative income tax). Calibrate the model in a sensible way. For three values of $\tau \in \{0, .25, .50\}$, compute the distributions for: (a) earned income; (b) after-tax income; (c) consumption; and (d) employment. Explain the effects and desirability of such a redistribution policy (from the perspective of the model agents).