CEU Money and Banking
Assignment 4: International Monetary Systems

The due date for this assignment is Wednesday, October 29.

1. Consider a world consisting of 2-period-lived overlapping generations (and an initial old generation). The population is constant. A representative young agent cares for consumption when young and old (c_y, c_o), according to utility function,

$$U(c_y, c_o) = \ln(c_y) + \beta \ln(c_o).$$

   Each young agent is endowed with $y > 0$ units of non-storable output (when young). Derive the demand functions (c_y^d, c_o^d) and $q^d = y - c_y^d$ for a given real rate of return on money R (the inverse of the inflation rate). Use a diagram to depict the optimal choice for two different values of R (i.e., $R_L < R_H$). Why does the demand for real money balances not depend on inflation here? Hint: wealth and substitution effects.

2. Now, imagine that there are two economies just like the one described in question 1. Each economy has its own money supply, labelled M^a and M^b. Assume that these money supplies are held constant over time and that people view the two monies as perfect substitutes. Let $e_t = v_t^b / v_t^a$ denote the nominal exchange rate. In a stationary equilibrium, $R^a = R^b = 1$ and there is a nominal exchange rate indeterminacy. Assume that the two countries "agree" to fix the exchange rate at $e_t = e$. Moreover, imagine that country a starts expanding its money supply at some constant rate $\mu^a > 1$ to finance its government spending g^a. Assume that country b keeps its money supply constant and has no government spending. Hence, the government budget constraint for country a is,

$$g^a = [1 - 1/\mu^a]\, v_t^a M_t^a.$$

   Note that the demand for real money balances is the same in each country; i.e., $q^a = q^b = q$ (and moreover, is independent of the inflation rate, by question 1). The market-clearing condition is given by,

$$v_t^a \left[M_t^a + e M^b \right] = 2q.$$

   (a) Combine these latter two equations and explain why as $t \to \infty$, we have,

$$g^a = [1 - 1/\mu^a]\, 2q.$$

   (b) Explain why half of the government expenditure in country a is being financed by country b people. Depict this result using two diagrams side-by-side for each country. On each diagram, depict the resource constraint, the equilibrium budget constraint, the golden rule allocation, and the equilibrium allocation.

   (c) If you were a resident of country b, explain why you might want to see your government abandon the fixed exchange rate system and impose foreign currency controls.

3. Now, consider the same model except with $g^a = 0$ (and constant money supplies). Imagine introducing a third set of agents with preferences and endowments just like the others (there is an equal mass of each type of agent). Label these agents c types; their demand for real money balances is also equal to $q^c = q$. But now, imagine that (for some unexplained reason), type a and b agents only want to transact in their domestic currency, while type c agents view the two currencies as perfect substitutes. Hence, $q^a = q$, $q^b = q$, and $q^c = q = q_a^c + q_b^c$; where q_i^c denotes the real money balances held by type c agents in terms of country i money. Demonstrate that any exchange rate within the range,

$$\left(\frac{1}{2}\right)\left(\frac{M^a}{M^b}\right) \leq e \leq 2\left(\frac{M^a}{M^b}\right)$$

is an equilibrium exchange. Hint: the market-clearing conditons are now given by,

$$\begin{aligned} v_t^b M^b &= q + q_b^c; \\ v_t^a M^a &= q + q_a^c. \end{aligned}$$