Fiat Money and Coordination: A Perverse Coexistence of Private Notes and Fiat Money

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OLG model

• 2-period-lived agents, agents care only for consumption when old, $E_t u(c_{t+1})$

• Young are endowed with $y$ units of output (can be consumed or invested)

• Young are paired with another individual of the same generation and one of the pair is randomly selected to be a fabricator or worker

• The fabricator can transform inputs $(x_t^w, x_t^f)$, $x_i \leq y$ for each individual $i = w, f$, in the match, into future output according to technology

$$d_{t+1} = \alpha 2 \min \{x_t^w, x_t^f\}; \quad \alpha > 1$$
• Individuals are paired again in the second period, fabricator and worker (a different pairing from the first period)

• The produced good $d_t$ can only be used for consumption

• A Pareto optimal allocation (that maximizes output) is given by $(x^w_t, x^f_t) = (y, y)$ and $d_t = \alpha 2y$ (there are other P.O. allocations with lower levels of output)

• An equal division of the output implies $c^w_t = c^f_t = \alpha y$ (other surplus-dividing rules are possible)
Equilibria

- Assume that fabricators possess commitment and equal division of produced output

- Bryant suggests that there is one equilibrium—the P.O. allocation described above

- Intuition: young do not value $y$ so a (weakly) dominant strategy is supply $y$ inelastically
  - while not essential, fabricators could issue bearer notes at date $t$ redeemable for $d_{t+1}$, which they use to pay for worker input $x_t^w$
Multiple equilibria

- The claim of uniqueness seems tenuous to me

- Imagine, for example, that the young value their own endowment even just a little bit; i.e., modify preferences such that

\[ \varepsilon(y - x_t^i) + E_t u(c_{t+1}^i) \text{ with } 0 < \varepsilon < 1 \]

- There appears to be a continuum of (symmetric steady state) equilibria

  - i.e., any \((x^w, x^y) = (x, x)\) with \(0 \leq x \leq y\)

  - note: multiplicity may not survive SPE refinement if fabricator moves first (someone should check)
Outside money

• I want to search for a steady-state monetary equilibrium, where money earns a gross real return $R < \alpha$ (Notice that $R$ now seems to play the role of $\varepsilon$ in my earlier formulation—agents have an outside option)

• So, imagine that a young agent expects all other young to supply $0 \leq x' \leq y$
  
  – since $\alpha > R$, best reply is $x = x'$ and to use remainder $(y - x)$ to purchase money (from old)

• Main point appears to be that the availability of outside money may lead to suboptimal equilibria that would not otherwise exist (hence, the “perversity”)
Sunspot equilibria

• Even if there is no coordination failure today, the possibility of future coordination failure can generate a demand for outside money

• Introduce a random (i.i.d.) variable $\Sigma$ that takes on one of two values ($1=$ pessimistic, $2=$ optimistic)

  – let $0 < \eta < 1$ denote the probability of the pessimistic state

• Sunspot $\Sigma$ is observed by all, when young are just about to make their investment choices
• In a stationary sunspot equilibrium, there are four possible rates of return on money $R_{ij}$ where $i$ is sunspot value this period and $j$ is sunspot value next period.

• For $S = 1$, the choice problem is

$$\max_{q_1} \{ \eta u (x(y - q_1) + R_{11}q_1) + (1 - \eta)u (x(y - q_1) + R_{12}q_1) \}$$

• For $S = 2$, the choice problem is

$$\max_{q_2} \{ \eta u (x(y - q_2) + R_{21}q_2) + (1 - \eta)u (x(y - q_2) + R_{22}q_1) \}$$
• When $S = 1$, assume that $q_1^d = y$ (corner solution)

• When $S = 2$, assume solution is interior, so that

$$(R_{21} - x)u'(x(y - q_2^d) + R_{21}q_2^d) = (x - R_{22})(1 - \eta)u'(x(y - q_2^d) + R_{22}q_2^d)$$

• Let $M$ be nominal stock of money; then market-clearing requires

$$v_t^1 M = q_1^d \text{ and } v_t^2 M = q_2^d$$

which implies...

$$R_{11} = (v_{t+1}^1/v_t^1) = 1$$
$$R_{22} = (v_{t+1}^2/v_t^2) = 1$$
$$R_{12} = (v_{t+1}^2/v_t^1) = (q_2^d/y) < 1$$
$$R_{21} = (v_{t+1}^1/v_t^2) = (y/q_2^d) > 1$$
• Note: $R_{12} < 1$ implies corner solution in $S = 1$ is optimal (conjecture verified)

• Plug in equilibrium rates of return into FOC...

$$
\left( \frac{y}{q_2} - x \right) \eta u' (x(y - q_2) + y) = (x - 1)(1 - \eta) u' (x(y - q_2) + q_2)
$$

• As $q_2 \searrow 0$, LHS $\nearrow \infty$ and RHS $\searrow (x - 1)(1 - \eta) u' (xy) > 0$

• As $q_2 \nearrow y$, LHS $\searrow (1 - x) \eta u' (y) < 0$ and RHS $\nearrow (x - 1)(1 - \eta) u' (y) > 0$

• By continuity of $u'(.)$, there is a $0 < q_2 < y$ satisfying the above equation