Money, Output, and the Nominal National Debt

Bruce Champ and Scott Freeman (AER 1990)
OLG model

- Diamond (1965) version of Samuelson (1958) OLG model

- Let $N_t = nN_{t-1}$ population of young

- Representative young agent has preferences $U(c_{1t}) + E_t c_{2t}$

- Young endowed with 1 unit of labor; supplied inelastically at wage $w_t$

- Standard neoclassical production function $F(K_{t-1}, L_t)$; output can be either consumed or invested
- note: in this economy, $L_t = N_t$

- Define $k_t \equiv K_t / N_t$ and $f(k_t) \equiv F(k_t, n)$

- Capital depreciates fully after use in production

- Initial old are endowed with $M_0$ units of money

- Money supply dynamics $M_t = z_t M_{t-1}$

- New money is injected as lump-sum transfers to old: $a_t = (z_t - 1) M_{t-1} / N_{t-1}$
• Money supply growth rate is stochastic

$$z_t = \left[ \frac{z_t^*}{1 - \varepsilon_t} \right]$$

where $z_t^*$ is a positive r.v. and $\varepsilon_t$ is a zero-mean r.v. with $\varepsilon_t < 1$ for all $t$

• Realizations $z_t^*$ are known by all at $t - 1$; realizations $\varepsilon_t$ are known at $t$

• Consequently, former is anticipated ("news") while latter is a surprise ("innovation")

• Young are required to hold at least $\gamma$ units of money (real balances)
Let $p_t$ denote the price-level.

Government purchases $g_t$ per young person.

Let $B_t$ denote government debt (nominal)—one period maturity and pays the gross nominal interest rate $R_t$ at $t+1$.

Initial old own $B_0$ nominal units of debt and so are owed $R_0B_0$ dollars in period 1.

Government budget constraint...

\[ N_t p_t g_t + (R_{t-1} - 1)B_{t-1} = B_t - B_{t-1} + p_t N_t \tau_t \]  \hspace{1cm} (1)

\[ M_t - M_{t-1} = N_{t-1} \alpha_t \]  \hspace{1cm} (2)
• So, assumption here is that all new money is used to finance lump-sum transfers

• While lump-sum taxes and new debt are used to finance purchases and carrying cost of debt

• This specification is chosen because, evidently, “all the revenue effects of an expansion in fiat money will come solely from the effects of inflation on the real value of national debt”

• Note: we can return later and consider alternative specifications to check robustness of conclusions
Equilibrium conditions

- Rational expectations equilibrium defined in the usual way

- Let \((x_t, w_t)\) denote rental and wage rates for capital and labor, respectively; then firm profit maximization and competitive factor markets implies

\[
\begin{align*}
r_t &= f'(k_{t-1}) \\
w_t &= \frac{f(k_{t-1}) - f'(k_{t-1}) k_{t-1}}{n} \tag{3}
\end{align*}
\]

- The young earn a real wage \(w_t\) and pay a lump-sum tax \(\tau_t\)

- Three ways to save: capital \((k_t)\), bonds, \((b_t/p_t)\), and money \((m_t/p_t)\)
• Budget constraints (for all agents, apart from initial old)...

\[ c_{1t} = w_t - \tau_t - k_t - b_t/p_t - m_t/p_t \]  \hspace{1cm} (5)

\[ c_{2t} = r_t k_t + R_t b_t/p_{t+1} + m_t/p_{t+1} + a_{t+1}/p_{t+1} \]  \hspace{1cm} (6)

• and the “cash constraint”...

\[ m_t/p_t \geq \gamma \]  \hspace{1cm} (7)

• The decision problem may be stated as follows

\[
\max U \left( w_t - \tau_t - k_t - b_t/p_t - m_t/p_t \right) \\
+ E_t \left[ r_{t+1} k_t + R_t b_t/p_{t+1} + m_t/p_{t+1} + a_{t+1}/p_{t+1} \right] \\
+ \lambda_t \left[ m_t/p_t - \gamma \right]
\]
• FOCs...

\[
U'(c_{1t}) = r_{t+1} \quad (8)
\]

\[
U'(c_{1t}) = E_t \left[ \frac{R_t}{\Pi_t} \right] \quad (9)
\]

\[
U'(c_{1t}) = E_t \left[ \frac{1}{\Pi_t} \right] + \lambda_t \quad (10)
\]

• Authors restrict attention to equilibria in which \( R_t > 1 \) (so that cash constraint binds; i.e., \( \lambda_t > 0 \))

• Since the cash constraint binds, the equilibrium price-level is easily determined by combining (7) with the market-clearing condition \( N_t m_t = M_t \);
i.e.,

\[ p_t = \left[ \frac{M_t}{\gamma N_t} \right] \] (11)

- Expression above corresponds to simple QTM (treat it with caution—in particular, asset price does not depend on expectations of future variables)

- Another market-clearing condition requires \( N_t b_t = B_t \)

- Note that conditions (8) and (9) imply (also using 3),

\[ r_{t+1} = R_t E_t \Pi_t^{-1} = f'(k_t) \] (12)

so that the expected real return on government bonds must be equal to the real rate of return on capital (re: quasilinear preferences)
• Using (11), we know

\[ \Pi_t = \frac{M_{t+1} N_t}{M_t N_{t+1}} \]

\[ \Pi_t^{-1} = \left[ \frac{1 - \varepsilon_{t+1}}{\hat{z}^*_{t+1}} \right] n \]

\[ E_t \Pi_t^{-1} = \frac{n}{\hat{z}^*_{t+1}} \]

• Nominal interest rate on (nominally risk-free bonds) must therefore satisfy

\[ R_t = f'(k_t) \frac{\hat{z}^*_{t+1}}{n} \] (13)

• Note: if \( R_t > 1 \) (as assumed), then \( f'(k_t) > n/\hat{z}^*_{t+1} \) (money is dominated in real rate of return)
The real effects of inflation (monetary policy shocks)

- Not inflation *per se*; rather, the effects of monetary policy shocks (anticipated and unanticipated)

- Proposition 1: Anticipated monetary policy is neutral (real variables are independent of $z_{t+1}^\ast$)

- Proposition 2: Unanticipated monetary policy is not neutral (real variables depend on $\varepsilon_t$)
• Write the GBC (1) as follows

$$\frac{B_t}{p_t N_t} = \frac{R_{t-1} B_{t-1}}{p_t N_t} + g_t - \tau_t$$  \hspace{1cm} (14)

• Define $d_t \equiv R_{t-1} B_{t-1}/(p_t N_t)$ as the “burden of the national debt” passed on to the young

• Consider first-period budget constraint

$$c_{1t} = w_t - \tau_t - k_t - b_t/p_t - m_t/p_t$$

• We know that the real wage is determined by (4); write this as $w(k_{t-1})$
• From market-clearing, $b_t = B_t/N_t$ and $m_t = M_t/N_t$, which, together with the binding cash constraint, implies

$$c_{1t} = w(k_{t-1}) - \tau_t - k_t - \frac{B_t}{p_t N_t} - \gamma$$

• Combine this with (14) to derive

$$c_{1t} = w(k_{t-1}) - k_t - d_t - g_t - \gamma$$

• Now, (8) and (12) imply

$$U'(w(k_{t-1}) - k_t - d_t - g_t - \gamma) = f'(k_t) \quad (15)$$

• Lemma: Condition (15) implies that $k_t$ is a decreasing function of $d_t, g_t$, and an increasing function of $k_{t-1}$ (show this formally as an exercise)
The effect of $z_{t+1}^*$ and $\varepsilon_t$ on $d_t$...

$$d_t = \frac{R_{t-1}B_{t-1}}{p_tN_t}$$

$$= \left[ \frac{R_{t-1}B_{t-1}}{p_{t-1}N_{t-1}} \right] \left[ \frac{p_{t-1}N_{t-1}}{p_tN_t} \right] = \left[ \frac{R_{t-1}B_{t-1}}{p_{t-1}N_{t-1}} \right] \left[ \frac{p_t-1}{p_t} \right] \left[ \frac{1}{n} \right]$$

- Now use (11) to derive

$$\left[ \frac{p_{t-1}}{p_t} \right] = \left[ \frac{M_{t-1}}{M_t} \right] n = \left[ \frac{1 - \varepsilon_t}{z_t^*} \right] n$$

- Substituting the latter expression into the former

$$d_t = \left[ \frac{R_{t-1}B_{t-1}}{p_{t-1}N_{t-1}} \right] \left[ \frac{1 - \varepsilon_t}{z_t^*} \right]$$
Now use (13) to substitute out for $R_{t-1}$ and (11) to substitute out for $p_{t-1}$:

$$d_t = f'(k_{t-1}) \frac{z^*_t}{n} \left[ \frac{\gamma n B_{t-1}}{M_{t-1}} \right] \left[ \frac{1 - \varepsilon_t}{z^*_t} \right]$$

$$= f'(k_{t-1}) \gamma \left[ \frac{B_{t-1}}{M_{t-1}} \right] [1 - \varepsilon_t]$$

This proves the propositions.

Note that an unanticipated positive innovation in the money stock (a surprise inflation, or surprise jump in the price-level) acts like a partial default on the real value of the outstanding stock of nominal debt (i.e., $d_t$ declines).
• By the lemma above, this has the effect of expanding capital investment (the effective tax burden on the young declines, so they can afford to expand consumption and investment)

• The real interest rate declines (so does nominal interest rate, if expected inflation remains unchanged); future real wages, and future real GDP rises
A stripped down version of the model

- Constant population $N_t = 1$ for all $t$

- Representative young agent has preferences $E_t c_{2t}$

- Young endowed with $y$ unit of output

- Storage technology $k_t \rightarrow f(k_t)$

- No government purchases $g_t = 0$ for all $t$

- Everything else is the same
• Budget constraints

\[ 0 = y - \tau_t - k_t - \frac{b_t}{p_t} - \frac{m_t}{p_t} \]

\[ c_{2t} = f(k_t) + \frac{R_t b_t}{\prod_t p_t} + \frac{R_t m_t}{\prod_t p_t} + \frac{a_{t+1}}{p_{t+1}} \]

• If \( R_t > 1 \), then cash constraint will bind; assume this is so \( m_t/p_t = \gamma \)

\[ \frac{b_t}{p_t} = y - \tau_t - k_t - \gamma \]

\[ c_{2t} = f(k_t) + \frac{R_t b_t}{\prod_t p_t} + \frac{R_t \gamma}{\prod_t} + \frac{a_{t+1}}{p_{t+1}} \]

• So now, the young face a simple portfolio choice between \( k_t \) and \( b_t/p_t \)
• Combine these two constraints and formulate the choice problem...

\[
\max E_t \left\{ f(k_t) + \frac{R_t}{\Pi_t} (y - \tau_t - k_t - \gamma) + \frac{R_t}{\Pi_t} \gamma + \frac{a_{t+1}}{p_{t+1}} \right\}
\]

\[
f'(k_t) = R_t E_t \Pi_t^{-1}
\]  \hspace{1cm} (16)

• Note: compare (16) and (12) – same

• Price-level determination is the same (11); consequently, nominal interest rate is determined in the same way too

\[
R_t = f'(k_t) z^*_{t+1}
\]
• From (1) we can derive the real debt per young person as before

\[ \frac{B_t}{p_t} = d_t - \tau_t \]

• Combining with the young’s first-period budget constraint (and \( b_t = B_t \))

\[
\frac{b_t}{p_t} = y - \tau_t - k_t - \gamma \\
\frac{d_t - \tau_t}{p_t} = y - \tau_t - k_t - \gamma \\
d_t = y - k_t - \gamma
\]

• Lemma: \( k_t \) is a decreasing function of \( d_t \)

• As for \( d_t \), it can be shown to have the same properties as before (invariant to anticipated inflation, but decreasing in a surprise inflation)
Sensitivity analysis

• “Unanticipated inflation redistributes wealth from the current old to future generations. The current young react by increasing investment regardless of the method of financing the burden of past debt.”

  – if government reduces new debt, then the young replaces bonds with capital in their portfolios

  – if government reduces taxes, then young have more disposable income to save

  – the irrelevance of finance here hinges on quasilinearity

• The failure of the Ricardian equivalence theorem here is crucial for the nonneutrality of government debt
Anticipated inflation has no effect on real activity through changes in the real national debt because the anticipated real return on nominal bonds is tied through arbitrage to the real return on capital. This implication follows from two special features of the model—the demand for money is fixed and additions to the fiat money stock are distributed back to agents.”

If the demand for money were not fixed, real money balances would respond to anticipated inflation with effects on real capital and output suggested by Tobin (1965).”

If the seigniorage from the expansion of the fiat money stock were not returned to agents but used to help finance government expenditures or to retire the debt, an anticipated expansion of the fiat money stock would have the same qualitative effects as an unanticipated expansion, but of smaller magnitude.”
Elastic money demand (example)

- The cash constraint $m_t/p_t \geq \gamma$ is rather severe

- Perhaps a bit more realistic to assume $m_t \geq \gamma p_t k_t$ (can now interpret $\gamma$ as a legal cash-reserve ratio)

- As before, if $R_t > 1$, then this cash constraint must bind so that $m_t/p_t = \gamma k_t$

- This implies that the demand for capital investment and real money balances move in proportion