Monetary Economics

Chapter 9: Settlement

Prof. Aleksander Berentsen

University of Basel

Structure of this chapter

1. **Introduction**
2. The environment
3. Fictionless settlement
4. Settlement and liquidity
5. Settlement and default risk
6. Settlement and monetary policy
7. Conclusion

Introduction

- In this chapter, we consider economies where monetary exchange and credit coexist, and where debt must be settled with money.
- That is, a debt obligation cannot be settled by simply producing out.
- The fact that money is required to settle debt can generate liquidity problems in credit markets.
- These liquidity problems will affect the value of money and the price of existing debt, which in turn can distort the allocation of resources.
- Hence, liquidity problems in credit markets can spill over into product markets.
- This line of reasoning has been used to justify the need for an elastic supply of currency, which is one of the founding principles of the establishment of the Federal Reserve System.

Structure of this chapter

1. Introduction
2. **The environment**
3. Fictionless settlement
4. Settlement and liquidity
5. Settlement and default risk
6. Settlement and monetary policy
7. Conclusion

The environment

We make a departure from the benchmark model. We now divide a period into **four** subperiods:

- Morning (CM_1): agents produce and consume the general good and trade money holdings in a competitive market.
- Day (DM): bilateral matching and exchange of the search good.
- Night (CM_2): trade of the general good and money holding in a competitive market.
- Late night: agents sellte their debts, production and consumption are not feasible.

Agents live for only four subperiods.

The environment - Buyers

- Half of the buyers, so-called **early producers**, can only produce in the CM_1
- The other half can only produce in the CM_2. They are **late producers**.
- This structure implies that buyers who are unable to produce at the beginning of their lives cannot accumulate money to trade in the DM.
- Late-producing buyers are able to repay any debt issued in the DM by producing for money in the CM_2.
- There is no asymmetry of information regarding the buyer's ability to repay his debt: a buyer's type is common-knowledge in a match.
- They can produce the general good, but have no desire to consume it.
- IOUs can be counterfeited by buyers at no cost.

The environment - Buyers

Buyers' preferences are described by the instantaneous utility function

$$U^b(q, y) = u(q) - y,$$

where y is the buyer's production of the general good and q is the consumption of the search good.

The environment - Sellers

- Sellers are born at the beginning of the day subperiod, and die at the end of the morning of the subsequent period:
  - In any morning, the economy is populated with young buyers and old sellers.
  - In all other subperiods, the economy is populated with buyers and sellers who are born in the same period.
- In the DM, sellers can only authenticate the IOU issued by the buyers they are matched with, and will accept only non-counterfeited IOUs.
- Sellers are able to produce the search good in the DM but have no desire to consume it.
- They are unable to produce the general good but want to consume it.

The environment - Sellers

The preferences for the seller are given by

$$U^s(q, x) = -c(q) + x,$$

where x is the seller's consumption of the general good and q is the amount of the search good that is produced.

Note that agents *do not* discount utility across subperiods over their lifetime.

The environment

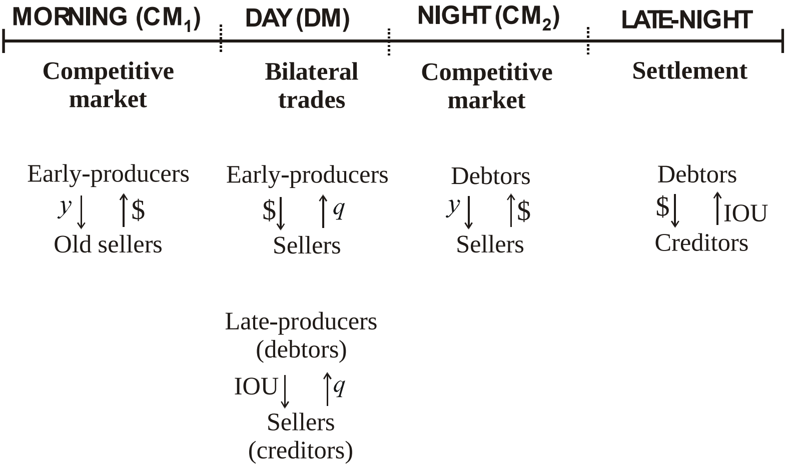

Figure 9.1: Timing and pattern of trade

The environment

- For simplicity, we eliminate any search-matching frictions by setting the matching probability σ to one.
- We focus on stationary equilibria.
- Since money is traded for general goods in competitive markets in the two different subperiods, we distinguish two prices for money:
  - ϕ_1 is the price of money in terms of general goods in the CM_1.
  - ϕ_2 is the price of money in the CM_2.

Structure of this chapter

1. Introduction
2. The environment
3. **Fictionless settlement**
4. Settlement and liquidity
5. Settlement and default risk
6. Settlement and monetary policy
7. Conclusion

Frictionless settlement

There are no frictions in the settlement phase:

- all debtors and creditors arrive simultaneously at a central meeting place in the late-night subperiod, and
- all debts are settled instantaneously.

Consider a match in the DM, between a buyer who is an early-producer and a seller. This buyer produced general goods in the morning to get m units of money, which he spends in a bilateral match in the day for q^m units of the search good.

The seller's participation constraint is

$$-c(q^m) + \max(\phi_1, \phi_2)m \geq 0.$$

A seller values a unit of money at $\max(\phi_1, \phi_2)$ because he has the option to spend his money either in the CM_2, at the price ϕ_2, or in the following CM_1, at the price ϕ_1.

Note that $\max(\phi_1, \phi_2) = \phi_2$: If $\phi_2 < \phi_1$, then sellers will spend their money in the following CM_1, but this outcome is inconsistent with the clearing of the CM_2, since late-producers need to acquire money at night to settle their debts.

Note also that an early-producing buyer has no incentive to accumulate money in the CM_1 *and* issue debt in the DM because sellers prefer (weakly) to receive money that they can spend in CM_2.

Frictionless settlement

Match between an early-producer and a seller

Hence, a buyer with money in the DM makes the following take-it-or-leave-it offer to the seller,

$$\max_{m,q^m} \left[u\left(q^m\right) - \phi_1 m\right], \tag{9.1}$$

$$\text{s.t. } c(q^m) = \phi_2 m \tag{9.2}$$

Substituting m from (9.2) into (9.1) and taking the first-order condition for q^m, we obtain

$$\frac{u'\left(q^m\right)}{c'\left(q^m\right)} = \frac{\phi_1}{\phi_2}. \tag{9.3}$$

From (9.3), $q^m = q^*$ if and only if $\phi_1 = \phi_2$.

If $\phi_2 > \phi_1$, then $q^m > q^*$. The demand for money from early-producers in the CM_1 is then

$$m = \frac{c(q^m)}{\phi_2}. \quad (9.4)$$

The supply of money in the CM_1 comes from old sellers who hold the entire stock of money, M.
Since there is a measure $1/2$ of early-producing buyers, equilibrium in the CM_1 money market implies that $M = m/2$ and, from (9.4), q^m satisfies

$$c(q^m) = 2M\phi_2. \quad (9.5)$$

Frictionless settlement

Match between a late-producer and a seller

- A late-producing buyer must issue an IOU to pay for the search good.
- The buyer repays the debt by producing output for money in the CM_2.
- The terms of trade in the match are determined by a take-it-or-leave-it offer (q^b, b) by the buyer, where q^b is the amount of search good produced by the seller and b is the amount of dollars that the buyer commits to repay.

The buyer's offer is given by the solution to

$$\max_{q^b,b}\left[u(q^b)-\phi_2 b\right] \tag{9.6}$$

$$\text{s.t.} \ -c(q^b)+\phi_1 b=0. \tag{9.7}$$

The seller values the buyer's debt at the price ϕ_1 since he spends the money obtained in the late-night settlement subperiod the next morning.

The solution to the buyer's problem (9.6)–(9.7) is

$$\frac{u'(q^b)}{c'(q^b)} = \frac{\phi_2}{\phi_1}. \tag{9.8}$$

From (9.8), $q^b = q^*$ if and only if $\phi_1 = \phi_2$.

Frictionless settlement

Match between a late-producer and a seller

If $\phi_1 < \phi_2$, then $q^b < q^*$. From (9.7), the amount of nominal debt issued by the buyer in the match is

$$b = \frac{c(q^b)}{\phi_1}. \tag{9.9}$$

Frictionless settlement

Match between a late-producer and a seller

Consider the equilibrium in the CM_2. If $\phi_2 > \phi_1$, then sellers holding money at the beginning of the CM_2 will spend all of it so that at the end of the night all of the money is held by the late-producing buyers, i.e., $b/2 = M$. If $\phi_2 = \phi_1$, then sellers holding money are indifferent between spending it in the CM_2 or in the following CM_1. In this case, $b/2 \leq M$. In summary,

$$b \left\{ \begin{array}{c} = \\ \leq \end{array} \right\} 2M \quad \text{if} \quad \phi_2 \left\{ \begin{array}{c} > \\ = \end{array} \right\} \phi_1. \tag{9.10}$$

Frictionless settlement

Steady-state equilibrium

If $\phi_1 = \phi_2$, then from (9.3) and (9.8) $q^m = q^b = q^*$. And from (9.5), $\phi_1 = \phi_2 = c(q^*)/2M$. From (9.9), $b = 2M$, which is consistent with (9.10).

Therefore, $q^m = q^b = q^*$, $b = 2M$ and $\phi_1 = \phi_2 = c(q^*)/2M$ is an equilibrium. In this equilibrium, the price of money is the same in the CM_1 and CM_2, and the efficient quantity of the search good q^* is traded in all matches.

Structure of this chapter

1. Introduction
2. The environment
3. Fictionless settlement
4. **Settlement and liquidity**
5. Settlement and default risk
6. Settlement and monetary policy
7. Conclusion

Settlement and liquidity

Settlement frictions are captured by having debtors and creditors arrive and leave the late-night settlement period at different times:

- All of the creditors—who are sellers—and a fraction α of debtors—who are late-producing buyers—arrive at a central meeting place at the beginning of the settlement period.
- Then a fraction δ of the creditors depart, after which the remaining $(1 - \alpha)$ debtors arrive.
- Finally, the remaining $(1 - \delta)$ creditors and all of the debtors leave the settlement period.
- At this point all of the buyers die, and all of the sellers move into the morning of the next period.

Settlement and liquidity

These arrival and departure frictions will create a need for a resale market for debt during the late-night settlement period.

We will sometimes refer to creditors (debtors) as being early-leaving (-arriving) and late-leaving (-arriving)

We denote ρ as the value of one-dollar of debt in terms of money in this market.

Settlement and liquidity

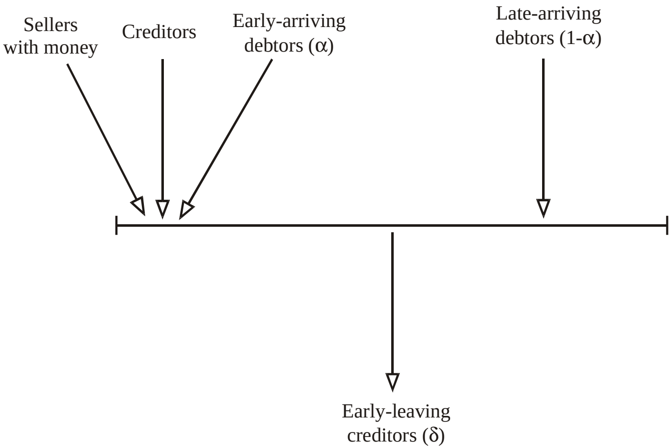

Figure 9.2: Frictions in the settlement phase

Settlement and liquidity

Match between an early-producer and a seller

- The DM bargaining problem of the buyer must now take into account the possibility that a seller who receives money for producing search goods during the DM may want to use some of it to purchase debt in the settlement period.
  - A seller who receives one unit of money in a bilateral match during the DM can spend it in the CM_2 for ϕ_2 units of the general good, or he can buy $1/\rho$ IOUs in the settlement period and then purchase ϕ_1/ρ units of the general good in the following CM_1.
- In equilibrium, sellers must be willing to spend some of their money in the CM_2 in order to allow late-producing buyers to acquire money to settle their debt in the late-night subperiod.
- The inequality $\phi_2 \geq \phi_1/\rho$ is required for equilibrium in the CM_2

The seller's participation constraint is still given by $c(q^m) = \phi_2 m$.

The early-producing buyer's bargaining problem is the same as the setup with frictionless settlement:

$$\max_{m,q^m} \left[u\left(q^m\right) - \phi_1 m\right],$$
$$\text{s.t. } c(q^m) = \phi_2 m$$

and the quantity produced, q^m, satisfies again

$$c(q^m) = 2M\phi_2.$$

- Creditor sellers may have to sell their IOUs at a discount if they need to leave the settlement phase before their debtors arrive.
- Let ϖ denote the expected value to the seller of a one-dollar IOU expressed in dollars.

The buyer's bargaining problem can be represented by

$$\max_{q^b, b} \left[u(q^b) - \phi_2 b \right] \tag{9.11}$$

$$\text{s.t.} \quad -c(q^b) + \varpi \phi_1 b = 0, \tag{9.12}$$

where ϖ satisfies

$$\varpi = \delta \left[\alpha + (1-\alpha)\rho \right] + (1-\delta) \left[\frac{\alpha}{\rho} + (1-\alpha) \right]. \tag{9.13}$$

Equation (9.13) has the following interpretation:

- With probability δ, a seller holding a one-dollar IOU must leave the settlement place early.
- If his debtor has already arrived, an event which occurs with probability α, the IOU is redeemed for one dollar. Otherwise, the IOU is sold at the price ρ.
- With probability $1-\delta$, the seller with a one-dollar IOU does not need to leave early. Therefore, the IOU that he holds is redeemed for one dollar, independent of the arrival time of his debtor.
- However, if the debtor of a seller arrives early, an event which occurs with probability α, the creditor can use the dollar he receives to buy $1/\rho$ IOUs that will be redeemed for $1/\rho$ dollars at the end of the settlement phase.

Debtor arrives... / Creditor leaves...	early (α)	late ($1-\alpha$)
early (δ)	1	ρ
late ($1-\delta$)	$1/\rho$	1

Table 1: Value of $1 IOU in the settlement period (no default)

The solution to the late-producing buyer's bargaining problem (9.11)–(9.12) is given by

$$\frac{u'(q^b)}{c'(q^b)} = \frac{\phi_2}{\varpi\phi_1}. \tag{9.14}$$

The quantities traded in the DM in exchange for IOUs are efficient if $\phi_2 = \varpi\phi_1$. From (9.12), the quantity of debt issued by buyers in the DM is

$$b = \frac{c(q^b)}{\varpi\phi_1}. \tag{9.15}$$

- Denote Δ as the funds that each seller with money—and there is a measure $1/2$ of such sellers—retains at night so that he can purchase second-hand IOUs in the late-night settlement period.
- The total amount of money supplied in the CM_2 is equal to the total stock, M, minus money held by sellers to purchase existing IOUs in the settlement period, $\frac{\Delta}{2}$.
- The demand for money comes from buyers who need to settle their debt, equal to $b/2$.

Hence, equilibrium in the CM_2 requires that

$$\frac{b}{2} + \frac{\Delta}{2} = M. \tag{9.16}$$

- If $\phi_2 > \phi_1/\rho$, then sellers who hold money at the beginning of the night prefer to spend it in the CM_2 rather than the following CM_1.
- If, however, $\phi_2 = \phi_1/\rho$, then sellers are indifferent between spending money in the CM_2 or in the next CM_1.

To summarize,

$$\Delta \begin{cases} = 0 & \text{if} \quad \phi_2 > \frac{\phi_1}{\rho} \\ \geq 0 & \text{if} \quad \phi_2 = \frac{\phi_1}{\rho} \end{cases} . \tag{9.17}$$

- Note that $\rho \leq 1$.
  - $\rho > 1$ would imply that anyone who purchases the IOU will get a strictly negative net payoff.
- There are two possible sources for the supply of funds to purchase existing IOUs in the settlement period.
  - First, there are the creditors who are repaid early and leave late, who hold in total $(1-\delta)\alpha b/2$ units of money.
  - Second, there are sellers who received money during the DM and supply $\Delta/2$ units of money in the settlement period.

- The demand for funds from early-leaving creditors is $\rho\delta(1-\alpha)b/2$.
- If the supply of funds, $(1-\delta)\alpha\frac{b}{2}+\frac{\Delta}{2}$, is greater than the volume of second-hand IOUs to be purchased, $\delta(1-\alpha)\frac{b}{2}$, then buyers of those IOUs will bid up the price until it reaches $\rho = 1$.
- Otherwise, the price of second-hand IOUs will adjust so that the supply of funds, $(1-\delta)\alpha\frac{b}{2}+\frac{\Delta}{2}$, is equal to the demand, $\delta(1-\alpha)b\rho/2$.

To summarize, the market-clearing price of second-hand debt, ρ, satisfies

$$\rho = \begin{cases} 1 & \text{if } (1-\delta)\alpha\frac{b}{2} + \frac{\Delta}{2} \geq \delta(1-\alpha)\frac{b}{2} \\ \frac{(1-\delta)\alpha b+\Delta}{\delta(1-\alpha)b} & \text{otherwise} \end{cases} \quad . \quad . \tag{9.18}$$

If the supply of funds is large enough to redeem the IOUs of early-leaving creditors at face value, then the price of existing debt is one. If there is a shortage of funds, then existing debt will be sold at a discount.

Definition

A steady-state equilibrium is a list $(\phi_1, \phi_2, \rho, q^m, q^b, b, \Delta)$ that satisfies (9.3)–(9.5) and (9.14)–(9.18).

We distinguish between two types of equilibria: one where $\rho = 1$ and one where $\rho < 1$.

If $\rho = 1$, then there is no liquidity shortage in the settlement period

- Existing IOUs are sold at par, $\rho = 1$.
- From (9.13), the expected value of a one dollar IOU in the DM is one, i.e., $\varpi = 1$.

As a result, the equilibrium conditions are identical to those of the economy without any frictions in the settlement period, i.e.,

- $q^m = q^b = q^*$,
- $\phi_1 = \phi_2 = c(q^*)/2M$,
- $b = 2M$,
- $\Delta = 0$.

Note that from (9.18), $\rho = 1$ requires that $(1-\delta)\alpha/\delta(1-\alpha) \geq 1$ or, equivalently, $\alpha \geq \delta$.

Intuitively, there is no liquidity shortage if the measure of debtors who arrive early in the settlement place, α, is larger than the measure of creditors who leave early, δ.

Creditors who are repaid by early-arriving debtors can use this money to purchase the IOUs of creditors who need to sell them, the earlier-leaving creditors.

If $p < 1$, it means that existing debit is sold at a discount in the settlement period.
From (9.18), we have

$$\rho = \frac{(1-\delta)\alpha b + \Delta}{\delta(1-\alpha)b}.$$

The equilibrium is liquidity-constrained: the amount of money available at the settlement period just prior to the departure of the early-leaving creditors is insufficient to clear debts at their par value.

An important result is that if $\rho < 1$, then $\Delta > 0$, which implies that sellers with money provide additional liquidity in the settlement period by only spending a fraction of their money balances in the CM_2.

When $\rho < 1$ and $\Delta > 0$, condition (9.17) implies that $\phi_2 = \phi_1/\rho$, which means that sellers with money are indifferent between spending money in the CM_2 or the following CM_1.

Let's turn to the effect that the liquidity shortage has on the equilibrium allocation. From (9.13), $\varpi < \frac{1}{\rho}$ and, hence, $\frac{\phi_2}{\varpi\phi_1} > \frac{\rho\phi_2}{\phi_1} = 1$. Together with the fact that $\phi_2 > \phi_1$, (9.3) and (9.14) imply that

$$\frac{u'(q^m)}{c'(q^m)} = \frac{\phi_1}{\phi_2} < 1 < \frac{u'(q^b)}{c'(q^b)} = \frac{\phi_2}{\varpi\phi_1}.$$

The quantities traded in the DM must satisfy $q^b < q^* < q^m$: Buyers who trade with money in the DM receive more output than those who trade with credit.

Intuitively, a seller who is paid with money can use it to buy interest-bearing debt in the settlement period; in contrast, a seller paid with debt is facing the risk of having to sell his IOUs at a discount in the settlement period.

Settlement and liquidity

Steady-state equilibrium: $p < 1$

The liquidity shortage during the settlement period affects the allocation of resources by making money more valuable in the CM_2 than in the CM_1. Indeed, since unsettled debts are sold at a discount during the settlement period, there is an additional demand for liquidity at night.

The fact that money is more valuable in the CM_2 allows early-producing buyers to consume more, whereas the consumption of late-producing buyers falls.

Structure of this chapter

1. Introduction
2. The environment
3. Fictionless settlement
4. Settlement and liquidity
5. **Settlement and default risk**
6. Settlement and monetary policy
7. Conclusion

Settlement and default risk

- We now introduce an idiosyncratic risk of late-producing buyers defaulting on their debt: a debtor is able to produce at night with probability ϱ and unable to produce with probability $1 - \varrho$.
- Assume that a debtor does not know if he will default before the night period: buyers and sellers have symmetric information in their bilateral matches in the DM.
- We assume that debtors who are unable to produce and, therefore, default on their debt do not show up at the settlement period.
- This implies that early-leaving creditors who sell their IOUs do not know whether or not these IOUs will be repaid.

The bargaining problem for an early-producing buyer is still given by (9.1)–(9.2):

$$\max_{m,q^m} \left[u\left(q^m\right) - \phi_1 m \right],$$
$$\text{s.t. } c(q^m) = \phi_2 m$$

since the risk of default is irrelevant for transactions conducted with money.

The bargaining problem for a late-producing buyer, however, is now given by

$$\max_{q^b,b}\left[u(q^b)-\varrho\phi_2 b\right] \tag{9.19}$$

$$\text{s.t.}\quad -c(q^b)+\varpi\phi_1 b=0. \tag{9.20}$$

- From (9.19), the buyer receives q^b from the seller, and is able to produce at night with probability ϱ, in which case he can repay his debt.
- From (9.20), the seller who receives a promise of b dollars can expect to get ϖb dollars at the end of the period, which can be spent the following morning, where ϖ, the expected value of a one-dollar IOU, now reflects not only any settlement frictions but also the possibility of default.

The solution to problem (9.19)–(9.20) implies that

$$\frac{u'(q^b)}{c'(q^b)} = \frac{\varrho\phi_2}{\varpi\phi_1}. \tag{9.21}$$

- In the absence of any settlement frictions, it will be the case that $\varpi = \varrho$.
- Then, (9.21) is identical to (9.8), and the outcome is similar to the one of the economy without default risk.
- The default risk is simply reflected in the (higher) amount of money that the buyer commits to repay, and the quantity of output traded in bilateral matches remains efficient.

Settlement and default risk
Clearing of the day market

The seller must assess the probability that an existing IOU will be repaid, conditional on the fact that the debtor did not arrive early. This probability is,

$$
\begin{aligned}
\Pr\left[\text{no default}\,|\text{no early arrival}\right] &= \frac{\Pr\left[\text{no default} \cap \text{no early arrival}\right]}{\Pr\left[\text{no early arrival}\right]} \\
&= \frac{\varrho(1-\alpha)}{1-\varrho+\varrho(1-\alpha)} \\
&= \frac{\varrho(1-\alpha)}{1-\varrho\alpha}.
\end{aligned}
$$

There are three possible events for an IOU:

- An IOU is not repaid with probability $1-\varrho$
- It is repaid early with probability $\varrho\alpha$
- It is repaid late with probability $\varrho(1-\alpha)$

The maximum price an agent is willing to pay for a unit face value of existing IOU is ρ^*, i.e.,

$$\rho^* = \frac{\varrho(1-\alpha)}{1-\varrho\alpha}. \tag{9.22}$$

The expected value of a one-dollar IOU in the DM, when the possibility of settlement frictions exists, is

$$\varpi = \varrho\alpha\left(\delta + (1-\delta)\frac{\rho^*}{\rho}\right) + \varrho(1-\alpha)(1-\delta) + \delta(1-\varrho\alpha)\rho, \tag{9.23}$$

or, equivalently from (9.22),

$$\varpi = \varrho\left(\delta\alpha + (1-\delta)\alpha\frac{\rho^*}{\rho} + (1-\delta)(1-\alpha) + \delta(1-\alpha)\frac{\rho}{\rho^*}\right). \tag{9.24}$$

Debtor arrives... / **Creditor leaves...**	**early** ($\varrho\alpha$)	**late** ($\varrho(1-\alpha)$)	**never** $(1-\varrho)$
early (δ)	1	ρ	ρ
late ($1-\delta$)	ρ^*/ρ	1	0

Table 2: Expected value of $1 IOU in the settlement period (Default)

Following the same reasoning as in the previous section, the clearing of the CM_2 requires

$$\frac{\Delta}{2} + \frac{\varrho b}{2} = M, \tag{9.25}$$

Here, only a fraction ϱ of the late-producing buyers are able to produce in the CM_2 in order to repay their debt.

If $\phi_2 > \frac{\rho^*}{\rho}\phi_1$, the sellers with money at the end of the DM strictly prefer buying in the upcoming CM_2. As a result, the supply of funds from a seller who holds money at the beginning of the CM_2, Δ, satisfies

$$\Delta \begin{cases} = 0 & \text{if} \quad \phi_2 > \frac{\rho^*}{\rho}\phi_1 \\ \in [0, 2M] & \text{if} \quad \phi_2 = \frac{\rho^*}{\rho}\phi_1 \end{cases} . \tag{9.26}$$

Here, an existing IOU is redeemed with probability ρ^*, while in the previous section this probability was one.

The market-clearing price, ρ, satisfies

$$\rho = \begin{cases} \rho^* & \text{if } \frac{(1-\delta)\alpha\varrho b}{2} + \frac{\Delta}{2} \geq \frac{\delta(1-\varrho\alpha)b\rho^*}{2} \\ \frac{(1-\delta)\alpha\varrho b+\Delta}{\delta(1-\varrho\alpha)b} & \text{otherwise} \end{cases} \tag{9.27}$$

If the supply of funds is large enough—the left side of the inequality on the top line—to redeem the IOUs of early-leaving creditors at their actuarial price—the right side of the inequality—then the price of second-hand debt is ρ^*.

If there is a shortage of funds, then existing debt will have to be sold at a discount for the market to clear. Substituting $1-\varrho\alpha$ by its expression given by (9.22) into (9.27) and rearranging, we get

$$\frac{\rho}{\rho^*} = \begin{cases} 1 & \text{if } \frac{(1-\delta)\alpha\varrho b}{2} + \frac{\Delta}{2} \geq \frac{\delta(1-\alpha)\varrho b}{2} \\ \frac{(1-\delta)\alpha\varrho b+\Delta}{\delta(1-\alpha)\varrho b} & \text{otherwise} \end{cases}. \tag{9.28}$$

Definition

An equilibrium of the model with default risk is a list $(\phi_1, \phi_2, \rho, q^m, q^b, b, \Delta)$ that satisfies (9.1)–(9.2), (9.20), (9.21), (9.25), (9.26), and (9.28).

- The probability of no-default, ϱ, affects the equilibrium conditions only through the variables ϱb, $\frac{\rho}{\rho^*}$, and $\frac{\varpi}{\varrho}$.
- For example, $\frac{\rho}{\rho^*}$ given by (9.28) coincides with ρ given by (9.18) when ϱb is replaced by b.
- Hence, $(\phi_1, \phi_2, q^m, q^b, \Delta)$ coincide with their values in the no-default economy.
- The value of money and the quantities traded in the DM are not affected by the probability of default, which is taken into account in the price of bonds and the transfer of bonds in the DM

The equilibrium is not liquidity-constrained whenever the supply of liquidity from the late-leaving creditors who had their IOUs redeemed by early-arriving debtors, $\varrho\alpha(1-\delta)b/2$, is greater than the demand of liquidity from early-leaving creditors, $\delta(1-\alpha\varrho)\rho^* b/2$.

From (9.22), the condition $\varrho\alpha(1-\delta) \geq \delta(1-\alpha\varrho)\rho^*$ is equivalent to $\alpha \geq \delta$. This is precisely the condition we had in the absence of default risk.

In summary, the presence of a default risk does not make it more likely that the settlement frictions will generate a shortage of liquidity and hence a misallocation of resources.

Structure of this chapter

1. Introduction
2. The environment
3. Fictionless settlement
4. Settlement and liquidity
5. Settlement and default risk
6. **Settlement and monetary policy**
7. Conclusion

Settlement and monetary policy

- When liquidity is "plentiful" in the settlement subperiod, the efficient allocation can be implemented as an equilibrium, and this is independent of default probabilities.
- If, however, there is a liquidity shortage, then the allocation is no longer efficient, i.e., $q^b < q^* < q^m$.
- Is it possible for monetary policy to improve matters in this situation?
- We assume that there is no default risk, i.e., $\varrho = 1$, because, as we have seen, the default risk is simply internalized in the pricing mechanism.
- When there is a liquidity shortage—which occurs when the fraction of creditors who depart early, δ, is greater than the fraction of debtors who arrive early, α—the market clearing price for debt in the settlement period, ρ, will be less than one, and this ultimately leads to inefficient levels of production in the DM.

Settlement and monetary policy

- Suppose that there exists a monetary authority, or central bank, that can provide "liquidity" to the settlement period.
- The central bank purchases $\Delta^{cb} \leq \delta\left(1-\alpha\right)\frac{b}{2}$ amount of IOUs from early-leaving creditors in exchange for fiat money.
- When the late-arriving debtors come to the settlement period, the central bank will exchange the IOUs for fiat money.
- This operation is neutral for the stock of fiat money.
- Recall that the supply of funds by creditors who are paid early and stay late is $(1-\delta)\alpha b/2$ and that the face value of bonds of the creditors who leave early and whose issuers arrive late is $\delta(1-\alpha)b/2$.

If

$$(1-\delta)\alpha\frac{b}{2}+\Delta^{\text{cb}}\geq\delta\left(1-\alpha\right)\frac{b}{2},$$

then the liquidity problem is solved: the supply of funds by late-leaving creditors and the central bank is enough to satisfy the demand of funds by early-leaving creditors. In this case IOUs are traded at their face value, $\rho = 1$, and sellers spend all their money in the CM_2 so that $\frac{b}{2} = M$.

Consequently, in order to implement an efficient outcome as an equilibrium when there is a liquidity shortage in the absence of a central bank, the supply of liquidity by the central bank must satisfy

$$(\delta - \alpha) M \leq \Delta^{\text{cb}} \leq \delta (1 - \alpha) M.$$

The supply of funds by the central bank is large enough to cover the difference between the IOUs supplied by early-leaving creditors and the demand of IOUs that comes from late-leaving creditors, $(\delta - \alpha) M$, but it is not larger than the liquidity needs of early-leaving creditors, $\delta (1 - \alpha) M$.

Settlement and monetary policy

This temporary supply of liquidity by the monetary authority resembles either a discount window policy or an open-market operation.

- As an open-market operation, the central bank purchases $(\delta - \alpha) M$ units of bonds before the early-leaving creditors depart and sells the bonds back after the late-arriving debtors arrive.
- As a discount window policy, the central bank stands ready to purchase existing IOUs at their par value, with the understanding that the IOUs have to be repurchased at their par value by the late-arriving debtors before the settlement period ends.

Settlement and monetary policy

The increase in the money supply that results from the open-market operation or discount window policy is not inflationary, since the IOUs purchased by the monetary authority are all redeemed within the period so that the stock of money remains constant across periods.

This policy is consistent with the real bills doctrine, which says that the stock of money should be allowed to fluctuate to meet the needs of trade by means of self-liquidating loans.

Settlement and monetary policy

A central bank is not necessarily needed to overcome the liquidity problem. Suppose that a late-leaving creditor, say a *clearinghouse*, purchases the debt of early-leaving creditors with his own IOUs, with the understanding that the IOUs of the clearinghouse can be exchanged for money in the next morning.

- When the late-arriving debtors arrive, the clearinghouse will exchange the debt that it holds for money.
- In the next morning, the clearinghouse can repurchase its debt with money.

Hence, as long as the clearinghouse is able to repurchase the debt it has issued, the liquidity problem that arises due to the settlement frictions can be overcome by private agents.

Structure of this chapter

1. Introduction
2. The environment
3. Fictionless settlement
4. Settlement and liquidity
5. Settlement and default risk
6. Settlement and monetary policy
7. **Conclusion**

Conclusion

- We constructed a model where fiat money and credit coexist, and debt is settled with money.
- In the absence of settlement frictions, the socially-efficient allocation can be achieved.
- If there is a lack of synchronization between creditors and debtors during the settlement phase, then a market for existing debt opens, where agents with funds buy the IOUs held by agents with liquidity needs.
- If the supply of funds is sufficiently large, then existing IOUs are traded at their face value, and the allocation is socially efficient.

Conclusion

- If, however, there is a shortage of funds, then IOUs are traded at a discount and allocations are distorted. These settlement frictions generate a role for a provision of liquidity by the central bank.
- The central bank provides liquidity to creditors with liquidity shocks by trading existing debt for fiat money. Such open-market or discount window operations have no effect on the supply of money over time since within each period the injection of money is reversed when the debt is redeemed.
- The presence of a default risk does not increase the need for liquidity.