Aysmmetry in central bank inflation control

D. Andolfatto

April 2015

The model

Consider a two-period-lived OLG model. The young born at date t have preferences $U_t = c_{t+1}$. The young also have an endowment y and a storage technology where k_t units of investment at date t yields $xf(k_t)$ units of output at date $t+1$. Capital depreciates fully after yielding output.

There are two government assets, money and bonds. Let $M_t \geq 0$ denote the supply of money at date t. Let $B_t \in \mathbb{R}$ denote the supply of bonds at date t, where $B_t < 0$ means that the government is a net creditor to the public. Let $D_t = M_t + B_t$. The government budget constraint is given by

$$T_t = R^b_{t-1} B_{t-1} + R^m_{t-1} M_{t-1} - B_t - M_t$$

where R^m and R^b denote the gross nominal interest rates on money and bonds, respectively. I assume that the lump-sum tax is imposed on the old. We can ignore the GBC in what follows.

Let p_t denote the price-level at date t. Then a young agent at date t faces the following sequence of budget constraints,

$$\begin{aligned} p_t y &= M_t + B_t + p_t k_t \\ p_{t+1} c_{t+1} &= p_{t+1} x f(k_t) + R^b_t B_t + R^m_t M_t - T_{t+1} \end{aligned}$$

It is convenient to express these constraints in real terms,

$$\begin{aligned} y &= m_t + b_t + k_t \\ c_{t+1} &= x f(k_t) + R^b_t \left(\frac{p_t}{p_{t+1}} \right) b_t + R^m_t \left(\frac{p_t}{p_{t+1}} \right) m_t - \tau_{t+1} \end{aligned}$$

where $m_t = M_t/p_t$, $b_t = B_t/p_t$ and $\tau_t = T_t/p_t$. Now combine the two constraints above to form a single constraint,

$$c_{t+1} = xf(k_t) + R_t^b \left(\frac{p_t}{p_{t+1}} \right) [y - m_t - k_t] + R_t^m \left(\frac{p_t}{p_{t+1}} \right) m_t - \tau_{t+1} \tag{1}$$

which also happens to be the objective function for a young agent.

Young agents choose (m_t, k_t) to maximize (1) subject to the constraint $m_t \geq \sigma k_t$, which has the flavor of a reserve requirement. Let λ_t denote the Lagrange multiplier associated with this constraint. Then we have the following two FOCs,

$$\begin{aligned} -R_t^b \left(\frac{p_t}{p_{t+1}} \right) + R_t^m \left(\frac{p_t}{p_{t+1}} \right) + \lambda_t &= 0 \\ xf'(k_t) - R_t^b \left(\frac{p_t}{p_{t+1}} \right) - \sigma \lambda_t &= 0 \end{aligned}$$

Combine these two equations to form,

$$xf'(k_t) = \left[(1+\sigma) R_t^b - \sigma R_t^m \right] \left(\frac{p_t}{p_{t+1}} \right) \tag{2}$$

Condition (2) determines k_t as a function of interest rates and inflation. With k_t so determined, the demands for money and bonds are easily computed. There are two cases to consider. First, $\lambda_t > 0$ iff $R_t^b > R_t^m$, in which case $m_t = \sigma k_t$ and $b_t = y - m_t - k_t$. Second, $\lambda_t = 0$ iff $R_t^b = R_t^m$, in which case $m_t > \sigma k_t$ [excess cash reserves] and $d_t = m_t + b_t$ determined by $d_t = y - k_t$, with the composition of m_t and b_t indeterminate.

Monetary and fiscal policy

Assume that the fiscal authority chooses $\{D_t\}$ and that the monetary authority chooses $\theta_t \equiv M_t/D_t$. For now, I assume that the monetary authority chooses a fixed $R_t^m = R^m$. Note that I do not rule out $R^m < 1$ (negative nominal interest rate on money). Also note that since $M_t \geq 0$, $\theta_t \geq 0$. On the other hand, since $B_t < 0$ is possible, $\theta_t = M_t/(M_t + B_t) > 1$ is also possible.

I impose the following constraint on monetary policy,

$$M_t \leq \alpha D_t \tag{3}$$

for all t, where $0 < \alpha < \infty$. The force of this constraint is that it constrains the monetary authority so that, in the *long-run*, it cannot issue money faster than the rate at which the fiscal authority is issuing debt. In the short-run, it is possible to grow money faster than debt. Note too that $M_t \geq 0$ means there is a limit to how fast the monetary authority can shrink the supply of money relative to debt in the long-run.

Equilibrium

In any equilibrium, we have $p_t m_t = M_t$ and $p_t d_t = D_t$ for all t. And so, the inflation rate must satisfy,

$$\left(\frac{p_{t+1}}{p_t}\right) = \left(\frac{D_t}{D_{t+1}}\right)\left(\frac{d_{t+1}}{d_t}\right) \tag{4}$$

Combine (4) with (2) together with $k_t = y - d_t$ to get,

$$xf'(y - d_t) = \left[(1+\sigma)R_t^b - \sigma R^m\right]\left(\frac{D_t}{D_{t+1}}\right)\left(\frac{d_{t+1}}{d_t}\right) \tag{5}$$

Now, there are two cases to consider, one in which the reserve constraint binds and one in which it does not. For now, assume that the constraint binds, so that $m_t = \sigma k_t$. Since $m_t = \theta_t d_t$, it follows that $\sigma k_t = \theta_t d_t$. And since $k_t = y - d_t$, it follows that $\sigma(y - d_t) = \theta_t d_t$, which permits us to solve explicitly for

$$d_t = \left(\frac{\sigma}{\sigma + \theta_t}\right) y \tag{6}$$

Thus, monetary policy determines the real value of total debt (and the price-level, $p_t = D_t/d_t$). Since fiscal policy determines D_{t+1}/D_t, it follows that monetary and fiscal policy together interact with (5) to determine the equilibrium nominal interest rate $R_t^b > R^m$. For the case in which $R_t^b = R^m$, we drop condition (6), so that d_t is determined by (5).

Again, assuming $R_t^b > R^m$, combine (6) with (5) to form,

$$xf'\left(\left(\frac{\theta_t}{\sigma + \theta_t}\right) y\right) = \left[(1+\sigma)R_t^b - \sigma R^m\right]\left(\frac{1}{\eta_{t+1}}\right)\left(\frac{\sigma + \theta_t}{\sigma + \theta_{t+1}}\right) \tag{7}$$

where $\eta_{t+1} \equiv D_{t+1}/D_t$. Note that the equilibrium inflation rate is given by,

$$\left(\frac{p_{t+1}}{p_t}\right) = \eta_{t+1}\left(\frac{\sigma + \theta_{t+1}}{\sigma + \theta_t}\right) \tag{8}$$

Consider the stationary equilibrium where $\theta_t = \theta$ and $\eta_{t+1} = \eta$. In this case, condition (7) reduces to,

$$xf'\left(\left(\frac{\theta}{\sigma+\theta}\right)y\right) = \left[(1+\sigma)R^b - \sigma R^m\right]\left(\frac{1}{\eta}\right) \tag{9}$$

Proposition 1 *R^b is decreasing in θ, increasing in η, and increasing in R^m. The inflation rate is given by $\Pi = \eta$.*

Thus, in a stationary state, it is the fiscal authority that determines the inflation rate. This is because θ is constant in a stationary state, so that the money supply is driven by the supply of debt, i.e., $M_t = \theta D_t$ for all t, which, incidentally, satisfies the restriction (3) for any $\theta \leq \alpha$.

Note that condition (9) is just the Fisher equation with the LHS, the marginal product of capital, equal to the real interest rate. In this model, the monetary authority can choose the real interest rate through an appropriate open market operation. A permanent increase in θ reduces the real value of debt (increases the price level permanently), permitting capital investment to expand, which lowers the marginal product of capital. Alternatively, think of an open market purchase of bonds as increasing the price of bonds, lowering their yield, and increasing the demand for capital investment.

Fighting highflation

Inflation was running high in the 1970s. I assume this was because η was high (relative to the growth in demand for U.S. debt). Fed Chair Paul Volker had a lower inflation target in mind, say $\Pi^* < \eta$. If the monetary authority is in a dominant position, then one would expect the fiscal authority to capituate to its view and lower η to $\eta^* = \Pi^*$. This is all very fine, but why is the monetary authority likely to be in a dominant position in this scenario?

Well, suppose that the fiscal authority decides to take a hard line on debt growth, refusing to lower η. Condition (8) shows how the monetary authority can nevertheless bring inflation down to its preferred target, i.e.,

$$\Pi^* = \eta\left(\frac{\sigma+\theta_{t+1}}{\sigma+\theta_t}\right) \tag{10}$$

Condition (10) determines the path for θ_t necessary to achieve the inflation target,

$$\theta_{t+1} = [(\Pi^*/\eta) - 1]\,\sigma + (\Pi^*/\eta)\,\theta_t$$

which implies that the monetary authority must promise to monetize a smaller and smaller fraction of the outstanding debt; i.e., $\theta_t > \theta_{t+1} > \theta_{t+2} > ... > 0$.

Now, notice that if the fiscal authority is willing to stand its ground for long enough, it has to win this battle. This is because the money supply must eventually go to zero under this monetary policy. Once M_t hits zero, the price-level path is determined by D_t (now consisting entirely of debt in the hands of the public). This is a game that the fiscal authority *can* win.

Well, except for one thing. Recall Proposition 1, which implies that as θ_t declines over time, the real rate of interest rises over time (since inflation is fixed at Π^*, the nominal interest rate rises one-for-one with the real interest rate). At the same time, the level of capital investment is declining to zero. The economy essentially enters into a prolonged and increasingly intense recession. If the fiscal authority wins this game, the result is secular stagnation, except with a very high real rate of interest.

Of course, both the monetary and fiscal authorities will be blamed for the recession arising from this conflict. But there is good reason to suppose that the monetary authority has the upper hand economically. This is because as the real interest rate rises, so too will the debt-service cost on the part of the fiscal authority who, will now have to raise taxes, cut spending, or both. At some point, the debt-service cost becomes too much to bear and the fiscal authority capitulates to the monetary authority. Of course, if all parties had rational expectations and an understanding of who has the bargaining power, capitulation would occur in short order, avoiding the cost of conflict. Either way, Paul Volker wins.

Fighting lowflation

Fast forward to today. The situation is now reversed, with many central banks missing on their inflation target from below. It should be easy, shouldn't it, to just do the opposite of what Paul Volker did. I want to argue here that it is not so easy. The situation is not symmetric because in this case, it is the fiscal authority that has the bargaining power.

And so, suppose we begin in a steady-state with inflation below target, $\eta < \Pi^*$. To raise inflation to target, the central bank sends θ_t on an upward path (see condition (10)), monetizing larger and larger fractions of government debt. By Proposition 1, the effect of this policy is to lower the real interest rate. Since inflation is targeted at Π^*, the nominal interest rate declines as well. The central bank's resolve here is *lowering* the fiscal authority's debt-service cost. Why should the fiscal authority capitulate in this situation? The central bank has no leverage over the fiscal authority in this case. But it is maintaining the inflation target, at least. What happens if the central bank continues to do so? One of two things must happen.

1. The nominal interest rate hits its floor, R^m. In this case, the composition of the central bank's balance sheet is irrelevant. All that matters is the growth rate of nominal debt, η. Inflation jumps back down to $\eta < \Pi^*$. Central bank loses.

2. The size of the central bank's balance sheet hits its upper limit, condition (3). At this point, the central bank's balance sheet cannot increase faster than the rate of treasury-issuance. Since the fiscal authority determines the latter, it also determines the inflation rate $\eta < \Pi^*$. Central bank loses.