Abstract

This paper explores the economic consequences of a social security system financed by a payroll tax in the context of a life-cycle economy in which some individuals are debt-constrained and where these debt constraints arise endogenously because of default options and a lack of commitment. In this environment, the structure of fiscal policy alters the incentive for default and consequently leads to adjustments in the supply of credit that may either loosen or tighten debt constraints. We investigate to what extent these credit market adjustments may affect the impact of policy. Our findings suggest that aggregate measures are not very sensitive to whether debt constraints are modeled endogenously or exogenously. However, there can be significant effects in the impact that policy has on different groups of individuals.


1 Introduction

Hubbard and Judd (1987) were among the first to examine the welfare implications of the U.S. social security system in the context of a life-cycle model with borrowing constraints. Because the U.S. social security system is financed primarily through a payroll tax, the burden of finance falls disproportionately on the young and middle-aged members of the population who are approaching, or are in, their peak earning years. The provision of social security is likely to reduce the desired private saving (increase desired borrowing) among individuals in these younger cohorts. But to the extent that individuals in this subgroup are debt-constrained, a proportional payroll tax reduces consumption expenditures dollar for dollar, and as a consequence, increases the welfare cost (reduces the benefit) of government program. Hubbard and Judd (1987) demonstrate that these effects can be quantitatively important.

The question that we pose in this paper is whether the welfare calculations undertaken by Hubbard and Judd (1987), who employ the industry standard of an exogenous non-negative net worth restriction, are sensitive to the way in which debt constraints are modeled. In particular, we consider embedding, within a simplified version of their environment, a theory of debt restrictions based on the existence of inalienable property rights; see, for example, Kehoe and Levine (1996). In this environment, net debtors (typically young individuals with bright futures) have the option of defaulting on unsecured debt at the cost of being subjected to wage garnishment and having some or all of their future assets seized by creditors. Consequently, defaulting today makes future participation in the financial market more difficult; and individuals must balance the cost and benefit of default. Likewise, creditors must incorporate these incentives into their supply schedule for loans, with the result that
some individuals may find themselves debt-constrained. There are good reasons to believe that the structure of these incentives respond to the nature of fiscal policy. Krueger and Perri (1998), for example, demonstrate how redistributive taxes may alter default incentives in a way that adversely affect the way in which private insurance markets function. In the present context, a more generous social security system may have similar effects by tightening debt constraints and exacerbating the welfare effects of policy.

Our preliminary computational results suggest that, at least for models with environments similar to the one employed here, the specification of non-negative net worth constraints is probably not a bad abstraction, especially if one is only concerned with measuring the aggregate impact of policy. On the other hand, we find that endogenizing credit market restrictions can have significant implications for the distribution of welfare effects across individuals; i.e., an exogenous specification of the borrowing restriction will tend to underestimate the welfare cost of social security for debt-constrained individuals, and overestimate the welfare cost for unconstrained individuals.

2 The Economic Environment

Consider an economy populated by overlapping generations of individuals who live for $J$ periods, indexed by $j = 1, 2, \ldots, J$. The population is assumed to grow at a constant rate $n$ per period, and we denote the share of age-$j$ individuals in the population by $\mu_j$, which is time-invariant and satisfies $\mu_j = (1 + n)^{-1} \mu_{j-1}$ for $j = 2, 3, \ldots, J$ and $\sum_{j=1}^{J} \mu_j = 1$. 
Individuals have preferences defined over deterministic sequences of consumption $c_j$; these preferences are represented by a utility function $U(c_1, c_2, \ldots, c_J)$ of the following form:

$$U = \sum_{j=1}^{J} \beta^{j-1} u(c_j),$$

where $u(\cdot)$, the per-period felicity function, has the usual properties, and $\beta > 0$ represents the subjective discount factor.

Individuals are endowed with one unit of time in each period of their life, which they supply inelastically. Individuals are able to transform one unit of time into $e_j$ efficiency units of labor. The ability level of individuals within age cohort $j$ varies according to a density function $f_j(x)$. Note that these human capital endowments evolve deterministically and are assumed to be observable by all parties. Although we do not wish to diminish the role of idiosyncratic uncertainty, the idea we model here is that the most important component of life-cycle variation in earnings-capabilities is largely forecastable.

Individuals are born with zero assets and will leave the economy with zero assets; there are no bequests. Let $w$ denote the (after-tax) price of one raw unit of labor and let $R$ denote the gross real rate of interest. As well let $a_j$ denote the net beginning-of-period asset position of an age-$j$ individual. Individuals face the following sequence of budget constraints:

$$c_j + a_{j+1} \leq we_j + Ra_j + T_j, \quad j = 1, 2, \ldots, J;$$

where $a_0 = 0$ and $T_j$ represents a lump-sum government transfer which we later interpret as a social security payment.

In the absence of any debt restrictions, the problem faced by individuals is to
maximize life-time utility (1) subject to the sequence of budget constraints (2). When $a_j < 0$, however, an age-$j$ individual may find it optimal to default on his debt obligations. An individual will default on his debt at age $s$ unless

$$V_s(a_s) \equiv \sum_{j=s}^{J} \beta^{j-s} u(c_j) \geq V_s^d, \quad s = 2, \ldots, J,$$

(3)

where $V_s$ and $V_s^d$ denote the continuation payoff accruing to an age-$s$ individual under alternative strategies of not defaulting or defaulting, respectively. This equation expresses that an individual will choose to default if the utility associated with that alternative is higher than that of honoring his debt. We will refer to this inequality as the individual rationality constraint.

The term $V_s^d$ in equation (3) will in general depend on a number of factors: first, it explicitly depends on the age of the individual as, quite naturally, the incentive to default varies over the life-time of an individual. Second, the relative value of defaulting will vary with an individual’s current asset position (the sign, of course, and the magnitude). One can also foresee that changes in fiscal policy, which redistribute resources across ages and/or generations, can also affect default incentives. Another important factor is the ability of creditors to garnish different types of income, which, in turn, depends on bankruptcy laws. Finally, history also plays an important role to determine the relative value of defaulting. For instance, depending on bankruptcy laws, it may be difficult for creditors to garnish an individual’s income if that individual has defaulted in the past since a fraction of his income is already being garnished by other creditors.

The legal structure we consider in this paper is one in which creditors can take actions to garnish a defaulter’s wages and/or lay claims to the defaulter’s property (either current or future). $0 \leq \pi^w \leq 1$ denote the fraction of wage income that a
creditor is feasibly (or legally) able to garnish; and let $0 \leq \pi^k \leq 1$ denote the fraction of physical and/or financial assets that the creditor can feasibly seize (exclusive of the value of the social security entitlement).

To illustrate the role of the individual rationality constraint (3), consider the problem faced by an age-$J$ individual who enters his last period of life with assets $a_J$ and has never defaulted in the past. If the asset position is positive, consuming all his income is his only option. If $a_J < 0$, the individual has to choose between (i) consuming his income minus the loan repayment $(we_J + Ra_J + T_J)$ or (ii) defaulting on his loan and consuming the fraction of his income that cannot be garnished by creditors $((1 - \pi^w)we_J + T_J)$. Defaulting is thus an optimal strategy unless

$$a_J \geq -\frac{\pi^wwe_J}{R}. \quad (4)$$

In other words, defaulting will be optimal if the loan repayment (principal plus interest) is larger than the amount of resources that creditors can legally garnish. Since no creditor will extend a loan to an individual knowing that he will later default on that loan, the age-$J$ individual rationality constraint effectively limits the set of asset positions that can be chosen at age $(J - 1)$; this constraint is given by (4).

The borrowing constraint (4) changes if an individual has a history of default. In this case, the individual would default on any loan extended to him in the previous period. This follows because property rights in the fraction of this individual’s wage earnings that can be garnished have already been assigned to previous creditors. In other words, the individual’s disposable wage income is not affected by his decision to defaults or not. As a result, defaulting is always a better option if the individual comes into the period with a negative asset position. Since no creditors will extend a loan to an individual with a history of default, such an individual is prohibited from
borrowing and faces a non-negative borrowing constraint from the time of default until the end of his life.\(^1\)

The same reasoning applies to earlier periods of an individual’s life. Since \(V^d_s\) does not vary with \(a_s\) and \(V_s(a_s)\) is strictly increasing in \(a_s\), there exists an asset position \(\bar{a}_s\) such that \(V_s(\bar{a}_s) = V^d_s\). It is also the case that an individual with a history of default, regardless of his age, will be unable to obtain any unsecured loan since creditors know that such an individual’s (disposable) income is entirely inalienable.

The above analysis emphasizes that the individual rationality constraint faced by an age-\((j + 1)\) individual translates into a restriction on the set of asset positions that can be chosen at age \(j\). Naturally, this borrowing constraint, or the maximum amount of debt that can be extended to an individual, depends on the same factors that affect the incentive to default discussed earlier. There are two distinct costs of defaulting on a loan: one is the loss of a fraction of current and future income, which can be thought of as a an income tax, the other is that an individual who defaults no longer has access to credit in future periods. This analysis thus suggests an interesting parallel between models with endogenous versus exogenous borrowing constraints. Essentially, individuals cannot choose an asset position that will put them in a situation where defaulting and facing exogenous no-borrowing constraints as well as “income taxes” in the future is a better option than paying back their loan.

In addition, this analysis provides a simple way of further analyzing the model. In particular, the continuation value of defaulting \(V_s\) is the solution to the following

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\(^1\)To reflect the fact that some credit is actually extended to individuals with a default history, a more realistic formulation could be that creditors can only take actions to garnish wages and lay claims on defaulters’ assets for a limited number of periods.
problem:

\[ V^d_s = \max_{j=s}^{J} \beta^{j-s}u(c_j) \]  \hspace{1cm} (5)

subject to

\[ c_s + a_{s+1} \leq (1 - \pi^w)we_s + T_s \]  \hspace{1cm} (6)

\[ c_j + a_{j+1} \leq (1 - \pi^w)we_j + (1 - \pi^h)Ra_j + T_j, \quad j = s + 1, \ldots, J \]  \hspace{1cm} (7)

\[ a_{j+1} \geq 0, \quad j = s, \ldots, J - 1. \]  \hspace{1cm} (8)

In a steady-state, the per capita capital stock is given by:

\[ K = (1 + n)^{-1} \sum_{j=1}^{J} \mu_j \int a_{j+1}(x)df_j(x), \]

and the per capita level of hours measured in efficiency units is given by:

\[ H = \sum_{j=1}^{J} \mu_j \int e_j(x)df_j(x). \]

Output is produced according to a constant returns to scale production technology

\[ Q = F(K, H), \]

where

\[ F(K, H) = AK^\alpha H^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0. \]

Equilibrium factor prices are determined competitively; i.e.,

\[ \hat{w} = F_H(K, H); \]

\[ R - 1 = F_K(K, H) - \delta, \]

where \( 0 \leq \delta \leq 1 \) is the rate at which physical capital depreciates. Note that the after-tax wage rate is equal to \( (1 - \tau)\hat{w} \). Goods-market clearing requires:

\[ C + (n + \delta)K = Q, \]
where $C = \sum_{j=1}^{J} \mu_j \int c_j(x) df_j(x)$. Finally, government budget-balance requires that all the proceed from the social security tax be redistributed to individuals in the form of a lump-sum transfer,

$$\tau \hat{w} H = \sum_{j=1}^{J} \mu_j \int T_j(x) df_j(x).$$

## 3 Calibration

### 3.1 Parameter Selection

The model’s parameters are those that describe: demographics ($n, J$, as well as the endowment process); preferences ($\beta, \sigma$); technology ($A, \alpha, \delta$); and the legal institution ($\zeta_w, \zeta_k$). The parameter values chosen for the benchmark economy are reported in Table 1.

<table>
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<tr>
<th>$e_1$</th>
<th>$\bar{x}$</th>
<th>$\bar{m}$</th>
<th>$m$</th>
<th>$s$</th>
<th>$\theta$</th>
<th>$n$</th>
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<tr>
<td>1.00</td>
<td>-5.50</td>
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<td>-1.50</td>
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<table>
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<th>$\beta$</th>
<th>$\sigma$</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\pi^w$</th>
<th>$\pi^k$</th>
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<tbody>
<tr>
<td>0.8179</td>
<td>2.00</td>
<td>3.00</td>
<td>0.35</td>
<td>0.6415</td>
<td>0.00</td>
<td>0.10</td>
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</table>

Individuals live for three periods ($J = 3$). We assume that $e_1$ is common across individuals and normalize it to unity. This specification is meant to reflect the observation that skill levels among the young are relatively homogeneous. However,
individuals face different prospects for the growth of their human capital; in particular, we assume that \( e_2 = e_1 + \exp(x) \), where \( x \) varies across the population according to a (truncated) normal probability density function

\[
f(x) = [2(\pi s)^{1/2}]^{-1} \exp \left[ -\frac{1}{2} \left( \frac{x - m}{s} \right)^2 \right],
\]

with \(-\infty < x \leq \pi < \infty\) and where \( m \) and \( s \) are parameters. In our computational experiments below, we assume that the \( x \) (and hence \( e_2 \)) are elements of a 100 point grid; i.e., \( e_2 \in \{\underline{e}, \ldots, \overline{e}\} = E \). For simplicity, we assume that this mid-life ability level decays at the common rate of \((1 - \theta)\), so that \( e_3 = \theta e_2 \) for each \( e_2 \in E \).

The parameters governing the life-cycle evolution of human capital \((m, \theta)\) are chosen so as to approximate the life-cycle earnings profile reported in Figure 1 of Huggett (1996). The upper and lower bounds on mid-life human capital endowments, together with the standard deviation parameter, were chosen to generate a reasonable dispersion in population earnings.

In this model with three-period lived individuals, we take one model-period to represent twenty years of adult life. In the literature, annual subjective discount factors range from 0.96 to over 1.00; we choose an intermediate value equal to 0.99, so that \( \beta = 0.99^{20} \). Annual rates of depreciation that are used in the literature fall between 0 and 0.10; again, we choose an intermediate value of 0.05, so that \( \delta = 1 - (1 - 0.05)^{20} \). The technology parameter \( A \) simply scales values and is chosen for this purpose. The parameter \( \alpha \) corresponds to capital’s share of total income and is typically chosen to be between 0.30 and 0.40; again, we choose an intermediate value. Following Hubbard and Judd (1987), the rate of population growth is chosen to be one percent per annum, so that \( n = 1.01^{20} - 1 \).
The parameter $\pi^w$ measures the degree to which creditors can recoup debt via wage garnishment, should a debtor choose to default. In the United States, Federal law stipulates that a minimum of 75% of wages or 30 times the Federal minimum hourly wage per week, whichever is higher, be exempt from garnishment. Some states (Texas, Pennsylvania, Alaska, South Dakota and Florida) prohibit wage garnishment entirely; see Fay, Hurst and White (1998, p. 16). Of course, legal stipulations are one thing and their enforcement is another. In practice, it is likely very difficult for creditors to enforce debt repayment through wage garnishment, even with a court order granting them title to some of the debtor’s wage income. The primary reason for this state of affairs likely resides in the fact that indentured servitude is legally prohibited in the United States.\footnote{This was not always the case; see Galenson (1981).} Consequently, debtors can escape wage garnishment by reallocating time to an activity that generates an excludable consumption flow (e.g., leisure or home production), or perhaps to some underground activity. These considerations lead us to set $\pi^w = 0$.

Debtors have a more difficult time protecting their physical and financial assets (relative to their human capital) from seizure by creditors. Kehoe and Levine (1996) effectively assume $\pi^k = 1.00$, so that creditors can seize all assets (in value up to the amount owed). This restriction effectively excludes defaulters from future participation in financial markets. In reality, the ability of creditors to seize property is limited. Even in the absence of personal bankruptcy laws, courts will typically limit the amount of assets that can be seized (especially if their seizure results in ‘below subsistence’ living standards). In addition, since the Bankruptcy Reform Act of 1978, generous exemptions for ‘rich debtors’ have been possible. This law allowed bankrupts to keep $7500 in homestead equity and $4000 in non-homestead property
from creditors. In 1994, these exemption levels were doubled. Furthermore, many states have adopted even more generous exemption levels (e.g., an unlimited homestead equity feature is not uncommon). Consequently, debtors can to some extent avoid asset seizure by allocating their assets and savings to these exempt categories. In one extreme case, we could set $\pi^k = 0$, but this would result in a non-negative net worth constraint since debtors could default with impunity. While this case represents the norm commonly adopted in the literature, it is also somewhat counterfactual; in reality, a considerable amount of unsecured debt is extended by creditors. For the benchmark economy studied here, it turns out that $\pi^k$ primarily determines the maximum level of the debt/income ratio that debtors can obtain. In principle, we should be able to calibrate $\pi^k$ to replicate some average of the debt/income ratios observed in the economy. At this writing, we do not have this information, so we choose $\pi^k = 0.10$, which results in a maximum debt/income (annual) ratio of 0.30. We also perform some sensitivity analysis with respect to this parameter.

### 3.2 Welfare Measures

A government policy regime essentially boils down to the choice of $\tau$. Let $c_j(x, \tau)$ for $j = 1, 2, 3$ be the equilibrium consumption allocation for a type $x$ individual under regime $\tau$ and define the function:

$$W(\lambda, \tau) = \int \sum_{j=1}^{3} \beta^{j-1} u \left( (1 + \lambda) c_j(x, \tau) \right) df(x).$$

We interpret $W(0, \tau)$ as the ranking that a ‘representative’ individual (behind the veil of ignorance) would attach to living in an economy under policy regime $\tau$.

Our benchmark economy is that of a laissez-faire regime; i.e., $\tau = 0$. Our ‘com-
pensating variation’ measure \( \lambda \) is computed according to:

\[
W(\lambda, \tau) = W(0, 0).
\]

That is, \( \lambda \) has the interpretation of being the fraction of per-period consumption that one would have to compensate the representative individual for moving from the laissez-faire policy regime to the policy regime \( \tau \).

Below, we also present compensating variations conditional on ability. In this case, redefine the function

\[
W(x, \lambda, \tau) = \sum_{j=1}^{3} \beta^{j-1} u\left((1 + \lambda)c_j(x, \tau)\right)
\]

and compute the type-contingent compensating variation \( \lambda(x) \) according to:

\[
W(x, \lambda, \tau) = W(x, 0, 0).
\]

### 3.3 Properties of the Calibrated Model

In Table 2, we report a number of statistics produced by our calibrated model across a range of parameter values for \( \pi^k \) (where \( \pi^k = 0.10 \) corresponds to the benchmark economy). In Table 2, **Capital/Output** refers to the annualized capital-output ratio; the **Interest Rate** refers to the annualized gross real rate of interest; **Debtors** refers to the fraction of the population that desires a negative net asset position; **Constrained Debtors** refers to the fraction of Debtors who would like to borrow more (and pay it back), but are debt-constrained in the sense described above; and **Debt/Output** represents the annualized ratio of outstanding net debt relative to output. **Welfare** represents the utility index defined earlier; and **Compensating Variation** tells us the fraction by which life-time consumption must be augmented in order to make the
Column 3 of Table 2 records some of the properties of the benchmark model. The model delivers a reasonable capital/output ratio of 3.30; in the United States, this ratio is around 3.00. The annual real rate of return on capital is 4.64%, which is in the neighborhood of the historical average for the U.S. economy. With a population growth rate of 1% per annum, the steady state capital stock is clearly below its Golden Rule level. The model economy features 7.4% of the adult population with negative net worth; this unsecured debt has a value close to 6% of annual GDP. We are not sure how these latter two numbers match up with the data. Finally, the model delivers an income Gini of 0.23, which is somewhat lower that what is observed in the data (estimates typically range between 0.30 and 0.40).
Figure 1 displays the life-cycle properties of consumption, earnings, and income across ability quintiles. Relative to the bottom four quintiles, the top quintile experiences very rapid consumption growth between the periods of youth and middle age. This rapid rise relative to the rest of the population is explained by the presence of the binding debt constraints that are more prevalent among this group of high-ability individuals.

Relative to the data, the consumption profiles lack the ‘hump-shaped’ behavior that is commonly reported in the literature. However, as pointed out by Jappelli and Modigliani (1998), family size displays a similar hump-shaped behavior and taking this fact into account would likely flatten out the measured age-consumption profile considerably. These authors also argue that the data support the age-saving profiles predicted by the life-cycle model, at least, when one uses a concept of income that takes into account the presence of pension arrangements.

Figure 2 displays consumption, earnings, income and the net asset position across the one hundred different ability types of individuals. The differences in earnings across ability types is very small for the first fifty types, but increases dramatically for the remaining types; this in an artifact of our lognormal specification for ability. As a consequence, for any given age cohort, consumption, earnings, income and net asset positions do not differ by very much at the bottom range of ability type. At the upper end of the ability scale, consumption rises rapidly with ability for the middle aged and for the old. For the young, high-ability types, consumption remains relatively low, which reflects the fact that these are the individuals who are debt-constrained; see also the bottom right panel of Figure 2.
Table 3: Partial Equilibrium Impact of Changes in $\pi^k$

<table>
<thead>
<tr>
<th></th>
<th>$\pi^k = 0.00$</th>
<th>$\pi^k = 0.05$</th>
<th>$\pi^k = 0.10$</th>
<th>$\pi^k = 0.15$</th>
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<td>Capital/Output</td>
<td>3.3607</td>
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<td>3.3038</td>
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<tr>
<td>Capital Stock</td>
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<td>Debtors</td>
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<td>0.0740</td>
<td>0.0740</td>
</tr>
<tr>
<td>Constrained Debtors</td>
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<td>0.9373</td>
<td>0.9373</td>
<td>0.8769</td>
</tr>
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<td>0.0305</td>
<td>0.0619</td>
<td>0.0944</td>
</tr>
<tr>
<td>Welfare</td>
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<td>0.0030</td>
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<td>$-0.0027$</td>
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</table>

3.3.2 Varying the Ability to Seize Assets

The parameter $\pi^k$ governs the ability of creditors to seize the assets of debtors in default and hence influences how tightly debt constraints bind. Table 3 displays the impact of changes in the ability to seize assets under the assumption that prices remain constant at their benchmark levels. Under this assumption, a change in $\pi^k$ only affects individuals’ ability to borrow without changing their desired (unconstrained) asset position. The case of $\pi^k = 0$ essentially corresponds to a situation where debt constraints take the form of an exogenous non-negative net worth constraint. As $\pi^k$ increases, the ability of creditors to seize assets improves and the amount of unsecured debt extended rises. Note, however, that the number of individuals wishing to borrow does not appear to be very sensitive to this parameter, at least for the range considered. When general equilibrium effects are ignored, increasing $\pi^k$ allows individuals to better smooth consumption and thus makes them better off.

Table 2 on page 14 reports the general equilibrium results from varying $\pi^k$ between 0.00 and 0.15. Increases in $\pi^k$, which increases levels of net indebtedness, imply lower
aggregate saving and hence a higher interest rate and a lower capital stock. An important effect of the lower capital stock is the depressing effect it has on real wages. In fact, we see that improvements in the technology for asset seizure actually lead to reduced levels of aggregate welfare; this is primarily due to the lower capital stock (and wages) associated with higher levels of indebtedness.

Figure 3 displays the heterogeneous impact that improvements in the ability to seize assets has across ability types. The top panel of Figure 3 reveals that increases in $\pi^k$ allow the young high-ability types to increase their loan levels substantially; the effect on the desired net asset positions of those with relatively low ability are negligible. While increasing $\pi^k$ reduces welfare for the ‘representative’ individual, the bottom panel of Figure 3 reveals that the welfare effects, when conditioning on type, are quite heterogeneous (the compensating variation is relative to the benchmark economy). When debt constraints bind tightly (say because $\pi^k$ is low), the majority of young persons are better off, because such a situation results in a higher capital stock and higher wages. Not surprisingly, those that are hurt by the tight debt constraints are the high ability types who wish to borrow against their higher future earnings, but are prevented from doing so since they cannot commit to repaying their debt when $\pi^k$ is low.

4 The Impact of Social Security and the Payroll Tax

Hubbard and Judd (1987) explore the welfare implications of a social security system financed with a payroll tax in the presence of an exogenous non-negative net worth constraint. Their primary conclusion is that the welfare benefit (cost) of social secu-
rity falls (increases) significantly when individuals cannot borrow. The goal of this section is to measure the extent to which these welfare calculations are sensitive to the specification of debt constraints. In particular, we wish to examine whether debt constraints that arise endogenously, and may therefore respond to government policy, exacerbate or mitigate the welfare implications of a social security program.

Following Hubbard and Judd (1987), the program under consideration involves a 6% payroll tax. The proceeds of this tax are distributed to members of the old generation in a lump-sum fashion. The analysis proceeds in a number of stages. We begin by investigating partial equilibrium effects; at this stage we fix factor prices at their benchmark levels. We then consider the economic impact of social security when debt constraints are exogenous (XDC case) in the sense that they remain fixed at their benchmark levels; subsequently, debt constraints are allowed to adjust to the new policy regime (EDC case). We then repeat this exercise, allowing for general equilibrium effects. The results are reported in Table 4.

4.1 Partial Equilibrium

Under a similar parameterization, but a considerably richer model, Hubbard and Judd (1987) report that the effect of a 6% payroll tax is to reduce the capital/output ratio by 39%.³ Our model also predicts a sizable, but somewhat more modest decrease of 10.5%. As far as aggregate quantities go, the results in Table 4 suggest that the size of error one is to make by abstracting from the endogeneity of the debt constraint is likely very small. When the debt constraint is endogenous, the capital/output ratio does not fall by as much, the number of borrowers increases, the number of

³See their Table 2, p. 638.
Table 4: Impact of Social Security

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Economy</th>
<th>Partial Equilibrium</th>
<th>General Equilibrium</th>
</tr>
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<tr>
<td></td>
<td>XDC</td>
<td>EDC</td>
<td>XDC</td>
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<tr>
<td>Capital/Output</td>
<td>3.3038</td>
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<td>Interest Rate</td>
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<td>1.0464</td>
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<td>Capital Stock</td>
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<td>Debtors</td>
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<td>0.0740</td>
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<td>Constrained Debtors</td>
<td>0.9373</td>
<td>1.0000</td>
<td>0.9391</td>
</tr>
<tr>
<td>Debt/Output</td>
<td>0.0620</td>
<td>0.0658</td>
<td>0.0612</td>
</tr>
<tr>
<td>Gini (Before Tax)</td>
<td>0.2347</td>
<td>0.2527</td>
<td>0.2528</td>
</tr>
<tr>
<td>Gini (After Tax)</td>
<td>0.2347</td>
<td>0.2199</td>
<td>0.2199</td>
</tr>
<tr>
<td>Welfare</td>
<td>0.8150</td>
<td>0.7518</td>
<td>0.7512</td>
</tr>
<tr>
<td>Compensating Variation</td>
<td>0.0000</td>
<td>0.0372</td>
<td>0.0375</td>
</tr>
</tbody>
</table>

Constrained borrowers rises by less, the level of net indebtedness falls by a greater amount, and aggregate welfare falls by more. All of these effects are attributable to the fact that the provision of social security leads to a tightening of debt constraints. The quantitative discrepancy between the two cases, however, is very small.

On the other hand, while the average impact on economic well-being is not much affected by the endogeneity of the debt constraint, the top panel of Figure 4 reveals that for the relatively few individuals who are debt constrained, the added welfare loss can be sizeable, with compensating variations increasing by over one percentage point for some individuals.
4.2 General Equilibrium

In a many-period version of the model developed here (but in the absence of debt constraints), Kotlikoff (1979) estimated that the general equilibrium effects of the U.S. social security system led to a decline in the steady-state capital stock anywhere between 10% and 20%. For a 6% payroll tax, Hubbard and Judd (1987) estimate a 46% decline in the capital/output ratio. With exogenous debt constraints, our model predicts a 23% decline in the capital stock and a 15.6% decline in the capital/output ratio. These results should increase one’s confidence in the ability of our simple model to approximate the quantitative impact of policy reforms on economic aggregates estimated within the context of a richer class of models that often feature many periods, bequests, and uninsured idiosyncratic uncertainty.

As is consistent with the results reported in Hubbard and Judd (1987), the adverse consequences of a payroll-tax financed social security program in the presence of debt constraints is greatly exacerbated when general equilibrium forces come into play. However, the differences that arise owing to the endogeneity of the debt constraints remain small in most cases. With endogenous debt constraints, the level of indebtedness in the economy does shrink considerably (the debt/output ratio is 30% lower in this case), owing to the tightening of credit market conditions. But the impact that this difference makes on the aggregate capital stock is rather small, with endogeneity resulting in a capital stock that is ‘only’ 1% larger. The average welfare differences that arise owing to endogeneity of the debt constraint are also quite small, although note that general equilibrium reverses the partial equilibrium results reported above;

4See their Table 3, p. 640. However, note that for an intertemporal elasticity of substitution parameter equal to 1/2 (σ = 2), their benchmark general equilibrium model features a capital/output ratio of 9.62, which is substantially higher than what is typically measured for the U.S. economy.
i.e., with endogenous debt constraints, welfare does not fall by as much under general equilibrium as it does when debt constraints are exogenous (the reverse is true under partial equilibrium).

Once again, however, we note that these aggregate implications mask a considerable amount of differences that occur at the individual level. The bottom panel of Figure 4 displays the compensating variations required across ability types under general equilibrium for both exogenous and endogenous debt constraints. Evidently, the endogeneity of the debt constraint leaves lower ability types better off relative to the case where debt constraints are assumed to be exogenous. These individuals are not directly affected by the increased tightness of the constraint and the resulting (relative) increase in capital implies a higher return for their labor. The high ability individuals also benefit from the increased return to labor, but these gains are swamped by the tighter debt constraints that afflict this set of individuals. At the aggregate level, this differential impact across ability types more or less washes out, leaving us with the conclusion that in terms of aggregate measures of welfare, endogenizing the debt constraint makes little difference.

5 Sensitivity Analysis

Several parameters affect individuals’ desire and ability to borrow and can influence the results of the previous section. In this section, we investigate how sensitive these results are to changes in the preference parameters $\sigma$ and $\beta$.

Figure 5 reports how the welfare calculations are affected for different specifications of the Intertemporal Elasticity of Substitution parameter; i.e., for $\sigma = 1.01$ and
\[ \sigma = 3.00. \] The Figure shows that increasing \( \sigma \) magnifies the welfare cost of social security. These welfare calculations, however, are equally affected whether one uses an endogenous or an exogenous borrowing constraint specification.

Similarly, Figure 6 reports how the welfare calculations are affected by changes in the specification of the discount factor; i.e., for \( \beta = 0.98 \) and \( \beta = 0.999 \). Once again, this Figure shows that whereas the magnitude of the welfare cost of social security is affected by the discount factor, the welfare cost of social security with endogenous relative to exogenous borrowing constraints is not sensitive to changes in this parameter.

6 Conclusions

This paper investigates whether the welfare calculations usually found in the literature with regard to the introduction of a social security system (eg. Hubbard and Judd, 1987) are sensitive to the specification of the borrowing constraint. We develop an overlapping generations model in which, in the spirit of Kehoe and Levine (1996), debt restrictions arise endogenously due to the existence of inalienable property rights. The legal structure we consider is one in which creditors lay claims to a fraction of a defaulter’s assets (either current or future). Although an improvement in the ability of creditors to seize assets helps debt constrained individuals to obtain credit, it also lowers the level of the capital stock as well as the wage rate, leading to lower aggregate welfare.

The results of this paper suggest that if one is only interested in the aggregate impact of social security, then an exogenous specification of the borrowing constraint
is not a bad approximation. These aggregate results, however, mask important heterogeneous effects at the individual level. Finally, we show that these results are not sensitive to the choice of preference parameters in our benchmark calibration.
References


Figure 1: Life-Cycle Properties of the Benchmark Model Across Ability Quintiles
Figure 2: Consumption, Earning, Income and Net Asset Position Across Ability Quintiles
Figure 3: Varying the Ability to Seize Assets ($\pi^k$)
Figure 4: Compensating Variation Required under a Social Security Regime

**Partial Equilibrium**

- **Exogenous Debt Constraint**
- **Endogenous Debt Constraint**

**General Equilibrium**

- **Exogenous Debt Constraint**
- **Endogenous Debt Constraint**
Figure 5: Compensation Variations for Different Values of $\sigma$

For $\sigma = 1.01$

For $\sigma = 3.00$
Figure 6: Compensation Variations for Different Values of $\beta$

$\beta = 0.980$

$\beta = 0.999$

Legend:
- PE XDC
- PE EDC
- GE XDC
- GE EDC

Ability Types