Optimal Debt Contracts

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1 Introduction

As an introduction, you should read "Why is There Debt," by Lacker (1991). As Lacker notes, the striking feature of a debt contract is that debt payments are fixed over a wide range of circumstances, although occasionally, as in a default, less than a full payment is made. Finding models in which people agree to a debt contract, although they are allowed to agree on any possible contingent repayment schedule, has proven surprisingly difficult. The idea explored here is that the opportunity for borrowers to hide their future resources sharply constrains the degree to which a loan repayment can be made contingent on the borrower's future resources.

2 A Basic Model

The model I present here differs from Lacker (1991) in form but not in substance. In many ways, it appears to be the standard workhorse model in the literature. The primary deficiency is that it is essentially a static model.

There are two agents: a manager and an investor (both agents are risk-neutral). There is a project that requires the services of a manager. The project requires an investment of capital k. The manager has zero wealth (i.e., k needs to be financed by the investor). The project yields a stochastic return $y \in \{y_1, ..., y_N\} \equiv Y$. Capital depreciates fully after the return is realized. Let $0 \leq \pi(y) \leq 1$ denote the probability that the project returns output y. Assume that this is a positive NPV project; i.e.,

$$\sum_{y \in Y} \pi(y) y > k. \tag{1}$$

Both agents have an outside option, the value of which is normalized to zero. Assume that all bargaining power resides with the manager.

Let us first imagine that y is verifiable (contracts contingent on y can be enforced). Let $R(y)$ denote the payment made to the creditor in state y. Feasibility requires that:

$$R(y) \leq y, \ \forall \ y \in Y. \tag{2}$$

The manager's state-contingent return is then given by $y - R(y)$. The investor's participation constraint (PC) is given by:

$$\sum_{y \in Y} \pi(y)R(y) - k \geq 0. \tag{3}$$

As the manager has all the bargaining power here, the only restrictions that are placed on $R(y)$ are given by (2) and PC (3) holding with equality. There are obviously many solutions here, and a standard debt contract (SDC) is one of them.

Exercise 1 *Construct (characterize) an optimal debt contract for the environment described above.*

While the SDC constructed here is optimal, it is not essential. We would like to identify properties of the environment that might render a SDC essential (i.e., the uniquely optimal contract).

To this end, let us now assume that y is observed by the manager only. Moreover, let me assume that the manager can only lie by hiding output (he cannot lie by claiming to have more output than he actually has).[1] By the revelation principle, we can restrict attention to allocations that are made contingent on the manager's report $\hat{y}$ of the true state; and focus on allocations in which the manager has an incentive to tell the truth.

Exercise 2 *Demonstrate that the only incentive-compatible (IC) allocation is a debt contract with $R(y) = y_1$. Moreover, explain why the only equilibrium is autarky if $y_1 < k$.*

Thus, if $y_1 \geq k$, we see that the only IC allocation features a constant payout; and in this sense, resembles debt. In fact, this is a risk-free debt contract. Many debt contracts are virtually risk-free, but it seems that most debt arrangements recognize at least a remote possibility of default. This possibility is important, even if the probability is small, because a borrower may be tempted to simulate default.

The question here is whether the environment might be modified in some way to allow for a constant payment in most, but not all, states of the world. To do so, Lacker introduces the notion of collateral. He is motivated by the observation that in most debt contracts, the debtor is usually required to **surrender something** distinct from the originally promised payment. In reality,

[1] This assumption is called "partial provability" in the literature. For example, I can prove to you that I can play the piano by playing it well; I cannot, however, prove that I cannot play the piano by playing it poorly.

this something frequently constitutes a specific capital good (your house, car, tools, etc.). But in fact, this something can be interpreted much more broadly. It might, for example, mean surrendering future access to credit markets (a loss of reputation); or losing your job (being fired for incompetence); or even surrending some aspect of your "self-esteem." What this suggests is that the role of collateral is primarily to increase the pain of default (and that this can be done in many ways). Because the details are not important for our purpose here, why not just assume that the default state entails a nontransferable utility cost to the debtor.

Here, I would like to follow Diamond (1984) by assuming that parties have access to a punishment technology. In particular, let $\phi(y)$ denote a state-contingent utility cost that can be applied to any party. I assume that parties can commit to this punishment (it will not work otherwise).

2.1 A 2-state example

Assume that $y \in \{0, x\}$, with $x > 0$. Let π denote the probability of zero output (which is suitably interpreted as project failure). Feasibility requires $R(0) \leq 0$ and $R(x) \leq x$. Assuming that $R(0) = 0$, to induce participation on the part of the investor, we clearly require:

$$R(x) = \left[\frac{k}{1-\pi} \right].$$

If the manager could be relied upon to tell the truth (i.e., if y was verifiable), then the manager's expected payoff would equal:

$$\begin{aligned} V^* &= (1-\pi)\left[x - R(x)\right] + \pi\left[0 - R(0)\right]; \\ &= (1-\pi)x - k; \end{aligned}$$

which is strictly greater than zero by condition (1).

But what if the manager cannot be relied upon to tell the truth? It is easy to see that the first-best allocation above is not IC. That is, while the manager will tell the truth when $y = 0$ (he has no other choice), he will be induced to tell a lie when $y = x$ (by reporting $\hat{y} = 0$). If there are no repercussions associated with reporting $\hat{y} = 0$, then the only equilibrium here is autarky (explain why).

Consider then the following "penal code." If the manager reports $\hat{y} = x$, he suffers no utility cost; i.e., choose $\phi(x) = 0$. If the manager reports $\hat{y} = 0$, he suffers utility cost $\phi(0) > 0$. Notice that one unfortunate consequence of this penal code is that the manager will end up being punished if output turns out to be low even though he is telling the truth. Nevertheless, the question here is whether *ex ante* utility may be improved upon by instituting such a penal code.

IC requires that:

$$x - R(x) \geq x - R(0) - \phi(0);$$

or,

$$\phi(0) \geq R(x).$$

Since it makes no sense to inflict unecessary punishment, it will be optimal to achieve IC by setting $\phi(0) = R(x)$. In this case, the expected payoff to the manager is given by:

$$\begin{aligned} V^0 &= (1-\pi)\left[x - R(x)\right] + \pi\left[0 - R(0) - \phi(0)\right]; \\ &= (1-\pi)x - \left(\frac{1}{1-\pi}\right)k. \end{aligned}$$

Clearly, $V^0 < V^*$. Still, if $V^0 > 0$, then this contract improves upon autarky.

2.2 A 3-state example

To speak sensibly of debt, we need at a minimum three states. A standard debt contract in this context would entail a constant payout to the investor over the medium and high states; with the punishment inflicted on the manager only in the low state. Is such a contract optimal? (And, is the optimal contract unique?).

Let us consider the IC constraints for the manager. Partial provability implies that the manager will (must) tell the truth when $y = y_1$. Suppose that $y = y_2$. In this case, $\hat{y} = y_1$ is a feasible lie; but $\hat{y} = y_3$ is not. IC therefore requires:

$$\phi(y_1) + R(y_1) \geq \phi(y_2) + R(y_2). \tag{4}$$

Finally, suppose that $y = y_3$. Here, we have two IC constraints to worry about:

$$\phi(y_2) + R(y_2) \geq R(y_3); \tag{5}$$

$$\phi(y_1) + R(y_1) \geq R(y_3); \tag{6}$$

where here, I anticipate that it will be optimal to set $\phi(y_3) = 0$. Interpret the LHS of the expressions above as the cost of lying; and the RHS as the cost of truth-telling.

The allocation R must also respect the investor's PC; from the manager's perspective, it will be optimal to set:

$$\pi(y_1)R(y_1) + \pi(y_2)R(y_2) + \pi(y_3)R(y_3) = k. \tag{7}$$

To make things interesting, assume that $0 \leq y_1 < k$. Now, consider a debt contract of the following form:

$$R(y_1) = y_1; R(y_2) = R(y_3) = R^* \tag{8}$$

with R^* satisfying:

$$R^* = \left[\frac{k - \pi(y_1)y_1}{\pi(y_2) + \pi(y_3)} \right]. \tag{9}$$

This contract satisfies the investor's PC (by construction). Note: I am assuming parameters here such that $R^* \leq y_2$ (if this did not hold, then this debt contract is not feasible).

Now, rewrite the IC constraints (4)-(6) assuming the debt contract above; i.e.,

$$\begin{aligned} \phi(y_1) + y_1 &\geq \phi(y_2) + R^*; \\ \phi(y_2) + R^* &\geq R^*; \\ \phi(y_1) + y_1 &\geq R^*. \end{aligned}$$

Efficiency dictates that we minimize the cost of punishment, subject to maintaining incentive-compatibility. This implies

$$\phi(y_1) = R^* - y_1 \text{ and } \phi(y_2) = 0.$$

Hence, if $R(y_2) = R(y_3)$, there is no need to punish the manager when he reports y_2.

In this case, the manager's expected payoff is given by:

$$\begin{aligned} V^0 &= \pi(y_1)\left[y_1 - R(y_1) - \phi(y_1)\right] + \pi(y_2)\left[y_2 - R(y_2)\right] + \pi(y_3)\left[y_3 - R(y_3)\right]; \\ &= \pi(y_1)\left[y_1 - R(y_1) - \phi(y_1)\right] + \pi(y_2)\left[y_2 - R^*\right] + \pi(y_3)\left[y_3 - R^*\right]; \\ &= \pi(y_1)\left[y_1 - \phi(y_1)\right] + \pi(y_2)y_2 + \pi(y_3)y_3 - k. \end{aligned}$$

This payoff can still be postive, but is clearly lower than the first-best payoff by the expected punishment cost $\pi(y_1)\phi(y_1)$.

I have just described an allocation that respects IC and PC. The contract that supports this allocation has the form of a standard debt contract. That is, the debtor (manager) agrees to pay either a fixed payment R^* or something less than this. In most states, he will make the payment. But there is a chance that he will not. In the default state, the manager is punished (and the less that the creditor can seize in the default state, the greater must be the nonpecuniary punishment). That is, a lower y_1 implies a larger $\phi(y_1)$.

Is this debt contract efficient? The answer is clearly yes. That is, the contract delivers just enough resources to the investor to induce participation and, at the same time, minimizes the expected utility cost of default.

Is there any other contract that delivers the same expected payoff to the manager? The answer appears to be no (this is in contrast to the case in which y is verifiable). To see this, imagine that we relax the restriction $R(y_2) = R(y_3)$. There are only two possibilities to consider; either $R(y_2) < R(y_3)$ or $R(y_2) > R(y_3)$. Let us consider the first case.

Imagine that $R(y_2) < R(y_3)$. By the investor's participation constraint (3), this implies that $R(y_2) < R^* < R(y_3)$. Now, consider the IC constraints (4)-(6):

$$\begin{aligned} \phi(y_1) + y_1 &\geq \phi(y_2) + R(y_2); \\ \phi(y_2) + R(y_2) &\geq R(y_3); \\ \phi(y_1) + y_1 &\geq R(y_3). \end{aligned}$$

Setting these last two equations to equality (to miminize the punishment cost), we have

$$\begin{aligned} \hat{\phi}(y_2) &= R(y_3) - R(y_2); \\ \hat{\phi}(y_1) &= R(y_3) - y_1. \end{aligned}$$

If these two conditions hold, then the first one will as well.

Clearly, $\hat{\phi}(y_2) > \phi(y_2) = 0$ and $\hat{\phi}(y_1) > \phi(y_1) = R^* - y_1$. In other words, the degree of punishment is increased in states y_1 and y_2; but increasing the punishment in this manner does nothing to improve the allocation. Therefore, $R(y_2) < R(y_3)$ cannot be optimal. It is a simple matter to check that $R(y_2) > R(y_3)$ cannot be optimal either (do this as an exercise). Hence, the standard debt contract is optimal (and unique).

Exercise 3 *The Miller-Modigliani Theorem for corporate finance asserts that under some conditions, the method of financing (whether by debt or equity) a capital project does not affect the value of the firm. Let us measure the value of the firm as the maximum amount that the manager would be willing to pay for the opportunity of operating the capital project. Explain why this value is higher when project returns are not observable when the manager chooses to finance his project with debt rather than equity. Explain why the common notion that one should invest in firms with "clean" balance sheets (low levels of debt) might be misguided.*

2.3 General Case (Incomplete)

The analysis above can be generalized to the case in which y lies on a continuum; see Diamond (1984). Let $y \in [0, \overline{y}] \equiv Y$ be distributed according to cdf $F(y)$. A positive NPV project requires:

$$\int y dF(y) > k.$$

Anticipating that IC will induce truth-telling, the return function must satisfy the investor's PC:

$$\int R(y) dF(y) \geq k; \tag{10}$$

which will hold with equality (the feasibility restriction $R(y) \leq y$ is implicit).

Let m denote the manager's report of y. Consider a given penalty function $\phi(y)$. IC on the part of the manager requires:

$$y - R(y) - \phi(y) \geq \max_{m \in Y} \{y - R(m) - \phi(m)\};$$

or,

$$\min_{m \in Y} \{R(m) + \phi(m)\} \geq R(y) + \phi(y) \tag{11}$$

for every $y \in Y$.

The manager's problem is then to choose a penalty function $\phi(y)$ and a return function $R(y)$ that maximizes expected utility:

$$\max_{R,\phi} \int [y - R(y) - \phi(y)] \, dF(y)$$

subject to IC (11) and investor PC (10); and feasibility $0 \leq R(y) \leq y$.

This is not quite the same way Diamond (1984) formulates the problem (I do not fully understand what he is doing). Evidently, the optimal return function takes the form:

$$R(y) = \begin{cases} y & \text{if} \quad y < h; \\ h & \text{if} \quad y \geq h; \end{cases}$$

where $h \in (0, \overline{y})$ satisfies:

$$F(h) \int_0^h y dF(y) + [1 - F(h)] \, h = k.$$

That is, the optimal contract takes the form of debt; i.e., a bond with a face value of h and that promises to repay $y < h$ in the default state. The least-cost penalty function that respects IC is evidenty given by:

$$\phi(y) = \max \{h - y, 0\};$$

i.e., see Diamond (1984), Proposition 1.

Exercise 4 *Formulate this problem correctly and provide a formal proof of Diamond's proposition.*

3 Costly State Verification

The analysis above appears to draw a great deal on the early work of Townsend (1979). The key difference with what I have done above is that Townsend regards $\phi(y)$ as an **exogenous** penalty function that is imposed on the **investor** (rather than the manager).[2] In the simple case, $\phi > 0$ is a constant and is incurred by

[2] See also the appendix to chapter 5 in Hart (1995). The idea of making ϕ endogenous appears to be attributable to Diamond (1984).

the investor whenever he wants to perform an audit that reveals the true state of nature.

As with our earlier analysis, it will make sense to economize on the audit expense subject to IC. The best way to do this is to promise a fixed payment over a large region of the state space (the higher end). When the manager reports a level of output below some critical level, the investor is obliged to undertake an audit (even though the investor knows that, in equilibrium, the manager is not lying).

Let's see how this works for the simple case in which $y \in \{y_1, y_2\}$, $0 \leq y_1 < k$ and $\pi y_1 + (1-\pi)y_2 > k$. In this case, an audit will have to be performed (at least with some positive probability) when $\hat{y} = y_1$. Assume, for the moment, that the audit decision is restricted to be a zero or one choice (I will allow for randomization later). The return function will have to induce participation, so that:

$$\pi \left[R(y_1) - \phi\right] + (1-\pi)R(y_2) - k \geq 0.$$

As usual, this will be driven to equality. It will also turn out to be optimal to set $R(y_1) = y_1$. Hence, we have:

$$R(y_2) = \left[\frac{k - \pi\left(y_1 - \phi\right)}{1-\pi}\right] \equiv R^*.$$

Here, we assume that ϕ is sufficiently small such that $R^* \leq y_2$. Given that the audit actually takes place in the low state, this allocation obviously satisfies IC.[3]

Evidently, matters can be improved here by adopting a stochastic audit. Let α denote the probability of an audit. Imagine that the high state occurs. By telling the truth, the manager receives a payoff $y_2 - R(y_2)$. But if he lies, he is now only discovered with probability α. Therefore, with probability $(1-\alpha)$ he receives a payoff $y_2 - R(y_1)$ and with probability α he receives a payoff equal to zero. The IC constraint is:

$$y_2 - R(y_2) \geq (1-\alpha)\left[y_2 - y_1\right].$$

To show that a stochastic audit can improve matters, assume that $R(y_2) = R^*$ (hence, the gross payments received by the investor remain unchanged). Next, choose α^* to set the IC constraint above to equality;

$$\alpha^* = \left[\frac{R^* - y_1}{y_2 - y_1}\right] \in (0,1).$$

Hence, this stochastic contract is Pareto superior to the deterministic contract as the gross payments remain the same and expected audit costs are reduced.

[3] Note that, even though the investor bears the cost of the audit, the manager suffers as well in that the return he must promise in the good state is higher than it would otherwise be absence costly auditing.

Several writers have criticized the costly state verification model as a theory of debt because (they argue) it is difficult to interpret the optimal (stochastic audit) contract as debt; see Hart (1995, pg. 124-125). I do not entirely agree with this, but there is something to this (read Hart).

One question that arises is whether a stochastic punishment might improve efficiency in our earlier environment where the punishment is endogenous. In our earlier setup, we have $R(y_1) = y_1$ and R^* satisfying

$$R^* = \left[\frac{k - \pi y_1}{1 - \pi}\right].$$

Notice here that the return promised to the investor does not depend on $\phi(y_1)$, as the punishment is (endogenously chosen) to be inflicted on the manager. If the manager tells the truth when $y = y_2$, he receives a payoff $y_2 - R^*$. Imagine that he is only punished with probability α if he lies. Then IC requires:

$$y_2 - R^* \geq y_2 - y_1 - \alpha\phi(y_1);$$

or,

$$\alpha\phi(y_1) \geq R^* - y_1.$$

The expected payoff to the manager is given by:

$$V^0 = \pi\left[-\alpha\phi(y_1)\right] + (1 - \pi)\left[y_2 - R^*\right].$$

The efficient punishment that induces IC satisfies $\alpha\phi(y_1) = R^* - y_1$. Reducing α here (introducing a random punishment) simply serves to increase the size of the punishment $\phi(y_1)$ when it happens. The expected punishment cost remains unchanged. Hence, there is no scope for random punishment here to improve the allocation (this is kind of interesting).

4 Costly State Verification and Diversification

4.1 A Simple Model

This section presents a simplified version of the delegated monitoring analysis of Diamond (1984).

The analysis above considered one manager and one investor. Imagine now that there are $N > 1$ managers, each with his own project that requires a capital expenditure k. For simplicity, assume that each project produces one of two outcomes, $y \in \{0, x\}$. It is known beforehand that $F < N$ of these projects will fail (i.e., produce zero output). However, no one knows who is endowed with a bad project (not even the managers). Hence, from an individual project's point of view, there is a probability $\pi = F/N$ that a project will fail. Note, however, that there is no aggregate uncertainty; aggregate output is given by $(1-\pi)Nx$.

If y is known only to the manager, then the solution will entail a set of contracts (one for each manager) identical to the one described earlier (recall the two-state example). That is, whether we are dealing with one manager or many, does not change our earlier conclusion.

Imagine now that there is a costly state verification (CSV) technology available. In particular, imagine that the investor has the option to audit a project following the manager's report; this monitoring activity consumes $\mu > 0$ utils for the agent undertaking the audit.

Now, instead of imposing the deadweight punishment $\phi(0) = R(x)$ on each manager reporting zero output, imagine that the investor commits to performing an audit in the event that zero output is announced (this will induce truth-telling on the part of the manager). In this case, the return function $\hat{R}$ must satisfy:

$$\pi\,[0 - \mu] + (1-\pi)\hat{R}(x) \geq k$$

in order to induce investor PC. The manager will choose $\hat{R}$ such that the above expression holds with equality, so that:

$$\hat{R}(x) = \left[\frac{k + \pi\mu}{1-\pi}\right].$$

That is, the return promised in the good state must be increased in order to compensate the investor for the expected monitoring expense. (Matters could be improved here by allowing for a stochastic audit, but I only consider deterministic auditing here).

Now, let's take a look at the manager's expected return under the two scenarios considered here. Under the deadweight punishment scheme, the manager receives a payoff:

$$V^0 = (1-\pi)x - \left(\frac{1}{1-\pi}\right)k;$$

i.e., see above. Under the CSV scenario, the manager receives a payoff:

$$\begin{aligned}\hat{V} &= (1-\pi)\left[x - \hat{R}(x)\right] \\ &= (1-\pi)\,x - k - \pi\mu.\end{aligned}$$

Clearly, for μ sufficiently small, it is possible that $\hat{V} > V^0$ (this is independent of whether we allow for stochastic monitoring or not).

But regardless of the size of μ here, it turns out that a superior allocation can be achieved by diversifying risk and delegating a monitor. To see this, consider the following arrangement.

Imagine that there is another agent with zero personal wealth; call this agent a delegated monitor (or intermediary). Consider the following contractual arrangement. The intermediary accepts deposits Nk from the investor and promises them a fixed return $R^* = k$ per project (this is sufficient to induce participation). How can the intermediary offer a risk-free return? The key lies in the fact that full diversification is possible here; the intermediary knows that if it lends the Nk resources to the entrepreneurs, that in aggregate, this diversified portfolio will return $(1-\pi)Nx$ units of output. Since $(1-\pi)x > k$ by assumption, there will be more than enough resources to make good on this promise. Moreover, diversifying its portfolio in this manner severely restricts the intermediary's ability to lie about the return on its assets (it cannot lie when there is no aggregate risk).

The only question left to answer is whether managers have an incentive to report their project outcomes truthfully to the intermediary. Let $P(y)$ denote the payment required of the manager in state y. Assuming that managers tell the truth, an intermediary that performs α audits earns the payoff (per project) equal to:

$$(1-\pi)P(x) + \pi(0) - k - \left(\frac{\alpha}{N}\right)\mu.$$

A competitive intermediary will earn zero profits; so that:

$$P(x) = \left[\frac{k + \left(\frac{\alpha}{N}\right)\mu}{1-\pi}\right].$$

Now, the claim here is that an optimal contract will be able to implement the first-best allocation (i.e., set $\alpha = 0$). How will this possible? Once again, the key will lie in the ability to exploit the risk-free nature of the diversified portfolio.

To induce truth-telling on the part of managers, consider the following contract. The intermediary lends k units of output to each manager and asks for a payment $P(\hat{y})$. The intermediary promises (commits) to auditing any manager that reports $\hat{y} = 0$, **but only in the event that the aggregate reported output falls short of** $(1-\pi)Nx$. In other words, if even one manager lies, the intermediary will know that some manager lied (without knowing the identity of the transgressor). Since the intermediary is committed to auditing in this event, the transgressor will be discovered. With this **credible threat** of an audit, no individual manager has an incentive to lie. In equilibrium then, it will turn out that $\alpha = 0$.

The result that the first best is implementable is not a general one. Nevertheless, the example here drives home the basic point delivered in Diamond (1984); namely, that there are efficiency gains to be had by appointing a delegated monitor and by having this agent construct a diversified portfolio. Portfolio diversification is valued not because people are risk-averse; rather, it is valued for the incentives it provides to managers (and intermediaries) to allign incentives correctly.

4.2 A Generalization (Incomplete)

Let me now describe the model in a way that more closely resembles Diamond's original exposition.

As before, there are N managers, each of whom has a project that requires capital k and returns output y according to the cdf $F(y)$ on the interval $[0, \overline{y}]$. Unlike the previous formulation, assume that project returns are *i.i.d.* across managers. This is important because although diversification is still possible, it will not be possible to construct a completely risk-free portfolio (at least, for N finite).

Consider an intermediary that contracts with N managers (the benefits of diversification can be studied by varying N). Collectively, these managers generate an aggregate output equal to $Y_N = \sum_{i=1}^{N} y_i$. Define $y^N \equiv Y_N/N$. The random variable y^N will be distributed according to some cdf $Q^N(y^N)$ that has the same expected value as $F(y)$, but with smaller variance.

The intermediary must attract lenders. In order to do so, it must offer a payment function $R(y^N) \leq y^N$ that at least weakly induces participation. To prevent the intermediary from misreporting y^N (investors do not observe this), the intermediary must be assigned a penalty function $\phi(y^N)$ for precisely the same reason described in an earlier section. The optimal contract in this case is, as before, a debt contract:

$$R(y^N) = \begin{cases} y^N & \text{if} \quad y^N < h^N; \\ h^N & \text{if} \quad y^N \geq h^N; \end{cases}$$

with $h^N \in (0, \overline{y})$ satisfying:

$$Q^N(h^N) \int_0^{h^N} y^N dQ^N(y^N) + \left[1 - Q^N(h^N)\right] h^N = k.$$

As before, the penalty function that minimizes the cost of alligning incentives correctly is given by:

$$\phi(y^N) = \max\left\{h^N - y^N, 0\right\}.$$

We must now ask what motivates individual managers to report their output truthfully to the intermediary. Following Diamond (1984), let us simplify matters a little by assuming that the decision to audit must be made before anyone knows the outcome of any project. In this case, every manager is audited and that therefore, the output of every manager's project is observed by the intermediary (there is obviously no scope for lying in this case).

The intermediary expends μ units in monitoring expense per project. In the event that $y^N < h$, the intermediary is punished $\phi(y^N)$ utils per project. Hence, the combined expected cost of doing business (per project) for the intermediary is:

$$\int_0^{h^N} \phi(y^N) dQ^N(y^N) + \mu.$$

The intermediary must earn an expected payoff that compensates it for this expense.

Exercise 5 *I am unable to follow the exposition in Diamond. As an exercise, try to complete this section (find some classmates and work on this together).*

5 References

1. Diamond, Douglas (1984). "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, LI, 393–414.

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