A Model of Inside and Outside Money

Reading: Champ and Freeman, 2nd Ed., Chapter 9

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1 Money and Capital

Consider an economy of 2-period lived overlapping generations with preferences given by:

$$U = \ln c_1 + \beta \left[\ln c_2 - \lambda e \right],$$

where $e \in \{0, 1\}$ is an 'effort' variable and $\lambda \geq 0$ is some parameter indexing the utility cost of effort. Let us think of people as differing in terms of this parameter; e.g., assume that λ is distributed across the population according to the cumulative distribution function $F(\lambda)$.

Let N_t denote the population of young people and assume that population grows at the (gross) rate $n > 0$. The young have an endowment $(y, 0)$ and have access to a capital project that takes current output k and yields future output xk, where $x > n$.

There is a government that prints money at (gross) rate $z \geq 1$ and uses new money to purchase output, which it consumes.

Individuals have access to two savings instruments: fiat money or capital. In a stationary equilibrium, fiat money earns a rate of return equal to (n/z), while capital investment yields a rate of return $x > (n/z)$.[1] However, in order to secure the return from capital, individuals must exert some effort $e = 1$. Depending on the context, this effort can have many interpretations. Here, we can interpret it as the effort required to run the capital project in its period of maturity (with people differing in their ability to do so, indexed by the parameter λ).

Let R denote some generic rate of return on saving. Then, each young person faces the budget constraint:

$$c_1 + \frac{c_2}{R} \leq y.$$

[1] Assume that capital depreciates fully after yielding its return.

Utility maximization implies the saving function: $s^D = \theta y$, where $\theta \equiv \beta/(1+\beta)$. This saving function is independent of which savings instrument is chosen. Consequently, we can form the following 'indirect utility function' (by plugging the saving function into the utility function):

$$V = \kappa + \beta \left[\ln R - \lambda e \right],$$

where $\kappa = \beta \ln \theta - \ln(1+\beta) + (1+\beta) \ln y$ is a constant.

At this stage, an individual must determine how to save: should the θy units of output be used to purchase fiat money or capital goods? The rate of return on money is low $R = n/z$, but requires no effort; i.e., $e = 0$. The rate of return on capital is high $R = x$, but requires effort to operate; i.e., $e = 1$. Each option yields the following utility payoff:

$$\begin{aligned} V^m &= \kappa + \beta \ln(n/z); \\ V^k(\lambda) &= \kappa + \beta \left[\ln x - \lambda \right]. \end{aligned}$$

If $V^m > V^k$, then the person should save with the monetary instrument; if $V^m < V^k$, then the capital instrument is preferred.

Notice that we can identify some individual with a $\hat{\lambda}$ such that $V^k(\hat{\lambda}) = V^m$; this $\hat{\lambda}$ is given by:

$$\hat{\lambda} = \ln x - \ln(n/z), \tag{1}$$

which is roughly the percentage difference in the rates of return between capital and money. As the rate of return differential rises, so does $\hat{\lambda}$. Everyone with $\lambda > \hat{\lambda}$ chooses to purchase fiat money (from the old) and all those with $\lambda \leq \hat{\lambda}$ choose to purchase capital goods (from themselves).

In this economy, the demand for fiat is given by $Q_t = \left[1 - F(\hat{\lambda})\right] N_t \theta y$; i.e., this is the amount of output seeking to purchase the outstanding stock M_t of money. Thus, the equilibrium price-level is given by:

$$p_t^* = \frac{M_t}{Q_t}.$$

The remaining amount of saving is invested in capital $K_t = F(\hat{\lambda}) N_t \theta y$.

Let us now run through the experiment considered in the textbook: i.e., an exogenous increase in x. This shock can be interpreted as a positive productivity shock, for example, a technological advancement that increases the expected return to capital.

We see from (1) that $\hat{\lambda}$ is increasing in x so that the fraction of individuals choosing to save via capital investment $F(\hat{\lambda})$ increases. In other words, the real demand for fiat falls, leading to an increase in the price-level. This period's increased capital investment leads to a higher level of future GDP; i.e.,

$$GDP_t = N_t y + N_{t-1} x K_{t-1}.$$

Notice something interesting here: even though real output rises from period $t+1$ onward, this has absolutely no effect on the future level of prices (in contrast to the standard *Quantity Theory of Money*). That is, if the increase in x is permanent, then this results in a permanent increase in the price-level (reflecting the permanent decline in the demand for real money balances). Even though future output rises, this extra output accrues to individuals who do not value fiat money (i.e., the old individuals with maturing capital projects).

Now, how does any of this relate to inside money, the money multiplier, or reverse causality? Basically, the textbook wants to interpret K_t as the (real) supply of inside money (private debt). This will make more sense if we think of the economy as working in the following way.

2 Inside and Outside Money

Suppose that the investment technology is such that it needs a very large minimum investment in order to succeed; i.e.,

$$k \Rightarrow \begin{cases} xk & \text{if} \quad k \geq k_{\min} \\ 0 & \phantom{\text{if}} \quad k < k_{\min} \end{cases},$$

where $k_{\min} > \theta y$.

Given these scale economies, it makes sense for large firms to somehow pool household savings and operate the investment technology. One way that firms could raise the necessary capital would be to issue shares or corporate debt instruments that entitle the security holder to some fraction of future returns. Let R denote the real interest rate offered on such a debt instrument. Given the nature of the investment technology, free entry into this sector is going to ensure that $R^* = x$.

We are now free to interpret an individual's purchase of k as a purchase of a corporate bond that promises to pay back (principal and interest) xk units of future output. In a sense, the individual is purchasing a piece of paper, carrying through time, and then using this paper to buy output (with a redemption expense or monitoring effort equal to λ). As such, we might think of this private debt instrument as constituting some form of money (inside money).

But maybe you do not like to think of corporate debt as money. Well then, let's suppose that firms cannot issue debt (or that it is too costly to market). Let us introduce another agent, called a 'bank' that has the ability to issue debt instruments that are widely accepted in exchange.[2] Assume that there is free entry into the banking business and that there are no costs to operating a bank (none of our main conclusions are sensitive to these simplifying assumptions).

So now we have a large firm that needs to acquire capital (from the young). The firm takes its plan to the bank and asks for a 'money' loan. The bank creates a bunch of bank money (you can think of these as either physical banknotes or simply accounting entries) and lends it to the firm (i.e., the firm sells a corporate debt instrument to the bank), requiring that the firm repay the money loan in the form of bank money. Because the units of measurement are arbitrary here, assume that each banknote is issued at par with government fiat.[3] For simplicity, suppose that households have accounts at this same bank.

The firm can now approach households and purchase k using bank money. One way this could be done is by writing a cheque. A cheque is an instruction for the bank to transfer bank money credits from the firm's account to the household's account. Or, we can think of the firm paying households in banknotes. In either case, households must bear a utility cost λ (perhaps a time cost of setting up an account, or travelling to the bank, or inspecting

[2] We are thinking of a situation where it is difficult for households to evaluate the credit worthiness of investment firms, but not of banks. Banks are agencies that can identify credit worthy investment projects and monitor the behaviour of management. Consequently, banks are in a position to issue liabilities that will be held by households. These liabilities are backed by the bank's ability to enforce the contractual terms of the corporate debt it purchases from investment firms.

[3] Note that this does not necessarily mean that bank money is redeemable for fiat or that bank money and fiat exchange with each other.

the banknote, etc.). Whether banknotes are held physically or whether they remain 'deposited' at the bank, it will be in the interest of a profit-maximizing bank to pay interest on its bank money. Again, free-entry and zero costs imply that the equilibrium interest rate on bank money (inside money) is going to equal $R^* = x$. So now we see that k can also be interpreted as the (real) value of bank money held by an agent.

When the capital project matures, the firm has output to sell. It will want to sell it for bank money, since it is obliged to collect enough bank money to pay off its bank loan. Agents with banknotes or bank accounts can now collect their interest and use their bank money to purchase the output they desire.

Roughly speaking, the definition of the $M1$ is base money plus bank money. The nominal value of the monetary base in our economy is M_t. The nominal value of bank money (assuming that it is issued and remains at par with government money) is $p_t K_t$. Consequently,

$$M1_t = M_t + p_t K_t.$$

Since $p_t = M_t / Q_t$, we can rewrite this as:

$$M1_t = \left[1 + \frac{K_t}{Q_t} \right] M_t. \tag{2}$$

The term in the square brackets is called the *money multiplier*; i.e., it is that number by which one multiplies base money in order to calculate the total money supply.

Now, let us return to the experiment considered in the text; i.e., an exogenous increase in x. As mentioned above, the effect of such a shock is to increase the real return to capital spending, which in turn leads to an increase in the demand for capital spending. In order to acquire new capital, firms must first acquire the money necessary to purchase it. They do so by approaching the banking sector for additional (bank) money loans (backed by the prospect of the higher returns to capital spending). Competition among banks ensures that this new bank money pays a higher real rate of interest, reflecting the higher productivity x. As the real return on bank money rises, individuals substitute out of fiat money (Q_t falls) and into bank money (K_t rises). The reduction in the real demand for fiat money causes the price level to rise. In addition, from (2), we see that this behaviour leads to an immediate increase in both the money multiplier and money supply, even though the stock of base money remains (in period t) fixed at M_t. In subsequent periods, the level of real GDP rises, reflecting the higher productivity and higher level of real capital spending.

A casual observer is apt to interpret the economic events above in an incorrect manner. In particular, one might be inclined to interpret the increase money supply ($M1$) as exogenous (perhaps induced in some manner by the monetary authority). The sudden increase in liquidity is interpreted as putting upward pressure on the price level. Subsequently, this increase in money (somehow) stimulates the economy, leading to a future increase in real GDP. In the context of the model developed above, we see that this line of reasoning is completely wrong. One cannot tell simply by observing the correlation:

$$Cor(M1_t, GDP_{t+1}) > 0$$

that 'increases in money supply *cause* increases in future real GDP.' In the context of the present model, the direction of causality is in fact reversed: the money supply is increased today in response to the private sector's expectation of higher future GDP. Basic lesson here: be careful when interpreting correlations.