

Notes on Jacklin (1987)

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February 2008

1 Basic Diamond-Dybvig (Review)

There are $N > 1$ agents (a finite integer). Each agent has an endowment $y > 0$. There are two periods, $t = 1, 2$ and agents have preferences for ‘current’ and ‘future’ consumption (c, f) given by:

$$U(c, f; \omega) = \begin{cases} u(c + \eta f) & \text{if } \omega = i; \\ u(c + f) & \text{if } \omega = p; \end{cases}$$

where u is increasing and concave, and $0 \leq \eta < 1$. Hence, one can interpret $\omega = i$ as someone who turns out to be ‘impatient’ and $\omega = p$ as someone who turns out to be ‘patient.’ Let $0 < I < N$ denote the number of agents who will turn out to be impatient. However, *ex ante*, no agent knows his type. Hence, there is no aggregate uncertainty, but agents do face idiosyncratic risk.¹ Define $\pi \equiv I/N$.

All agents have access to a storage technology. In particular, any output not consumed at date 1 can be invested for consumption at date 2; let $R > 1$ denote the gross return on investment.

The autarkic allocation is denoted $\{c^a(\omega), f^a(\omega)\}$. Assume that $\eta R < 1$. Then clearly, $\{c^a(i), f^a(i)\} = \{y, 0\}$ and $\{c^a(p), f^a(p)\} = \{0, Ry\}$. This will generate an *ex ante* utility payoff:

$$V^a = \pi u(y) + (1 - \pi)u(Ry).$$

The first-best allocation is denoted $\{c^*(\omega), f^*(\omega)\}$. Clearly, $c^*(p) = 0$ and $f^*(i) = 0$. Hence, the decision concerns $\{c^*(i), f^*(p)\}$. A planner (who can exploit the fact that there is no aggregate uncertainty) faces the following resource constraint:

$$\pi N c(i) + R^{-1}(1 - \pi)N f(p) \leq Ny.$$

Naturally, this constraint will bind; hence, write:

$$f(p) = (1 - \pi)^{-1}R[y - \pi c(i)]$$

and insert into the objective:

$$\max_{c(i)} \left\{ \pi u(c(i)) + (1 - \pi)u\left((1 - \pi)^{-1}R[y - \pi c(i)]\right) \right\}.$$

¹Diamond and Dybvig consider a continuum of agents with *i.i.d.* preference shocks.

The solution must satisfy:

$$u'(c^*(i)) = Ru'(f^*(p)); \tag{1}$$

with $f^*(p) = (1 - \pi)^{-1}R[y - \pi c^*(i)]$. Notice that as $R > 1$, we have:

$$y < c^*(i) < f^*(p) < Ry.$$

It should be evident that:

$$V^* = \pi u(c^*(i)) + (1 - \pi)u(f^*(p)) > V^a.$$

We have already shown in previous notes that the first-best allocation is implementable here even if types are private information.

2 Optimal Risk-Sharing Using Equity Shares

This analysis is based on Jacklin (1987). The question posed is whether the Diamond and Dybvig banking structure is the only way to implement an efficient allocation.

Consider instead a firm that raises capital (accepts deposits) Ny and issues depositors (shareholders) equity shares. Equity is designed to pay a per share dividend d_1 at date 1 to the shareholders of record (those who initially purchased the shares). The firm also promises to pay a liquidating dividend at date 2; $d_2 = R(y - d_1)$.

At date 1, shareholders receive their dividend d_1 and a market in the ex-dividend shares opens. At this stage, all agents know their types, so there are potential gains from trade. In particular, impatient agents will want to trade their shares for additional date 1 consumption. Patient agents are indifferent between current and future consumption; thus, if the price of ex-dividend shares is less than $R(y - d_1)$, they will be willing to trade their dividend income d_1 for additional ex-dividend shares (which represent claims to $R(y - d_1)$ units of future consumption).

Let us work backwards by first considering the ex-dividend market for equity, conditional on a given dividend policy d_1 . At this stage, every shareholder has income $d_1 \leq y$. Let ϕ denote the ex-dividend share price at date 1. Without loss of generality, we can normalize the total number of shares to N (each agent initially purchases one share). Let s denote the number of shares carried into the future.

At this stage, types are known. The choice problem for an impatient agent is given by:

$$\max \{u(c + \eta f) : c = d_1 + \phi(1 - s), f = sR(y - d_1)\}.$$

Let me simplify here by assuming that $\eta = 0$. In this case, the impatient agent will certainly want to dispose of all of his shares for any strictly positive share price $\phi > 0$; i.e., $s^D(i) = 0$ (I suppose that this presumes that short-selling is not allowed). In this case, $c^D(i) = d_1 + \phi$ and $f^D(i) = 0$.

Now consider the choice problem for a patient agent:

$$\max \{u(c + f) : c = d_1 + \phi(1 - s), f = sR(y - d_1)\};$$

or,

$$\max_s u(d_1 + \phi(1 - s) + sR(y - d_1)).$$

This agent will choose s by comparing ϕ to $R(y - d_1)$. He will be willing to buy additional shares if they are sufficiently cheap (i.e., offer a high enough return)—in other words, if $\phi \leq R(y - d_1)$.

Imagine then that the following restriction holds:

$$0 < \phi \leq R(y - d_1). \quad (2)$$

(We will have to check that it does in equilibrium). As $c \geq 0$, this implies that the maximum number of shares that a patient agent can purchase is limited by $d_1 + \phi(1 - s) \geq 0$ or $s \leq \phi^{-1}[d_1 + \phi]$. We can anticipate that $c^D(p) = 0$ so that $s^D(p) = 1 + (d_1/\phi)$ and $f^D(p) = s^D(p)R(y - d_1)$.

Market-clearing implies:

$$\pi s^D(i) + (1 - \pi)s^D(p) = 1;$$

so that:

$$(1 - \pi)[1 + (d_1/\phi)] = 1;$$

or,

$$\phi(d_1) = \left(\frac{1 - \pi}{\pi}\right) d_1. \quad (3)$$

Now, let's move back to the *ex ante* period, where the optimal dividend policy is formed. The firm (an economy-wide conglomerate) takes into account the pricing-function above when formulating its dividend policy for the representative shareholder. Let's perform some substitutions first.

$$\begin{aligned} c^D(i) &= d_1 + \phi(d_1) = \pi^{-1}d_1; \\ f^D(p) &= s^D(p)R(y - d_1) = (1 - \pi)^{-1}R(y - d_1). \end{aligned}$$

Hence, the *ex ante* objective is to:

$$\max_{d_1} \pi u(\pi^{-1}d_1) + (1 - \pi)u((1 - \pi)^{-1}R(y - d_1)).$$

And the optimal dividend policy \hat{d}_1 solves:

$$u'\left(\frac{\hat{d}_1}{\pi}\right) = Ru'\left(\frac{R(y - \hat{d}_1)}{1 - \pi}\right); \quad (4)$$

where,

$$\begin{aligned}\hat{c}(i) &= \pi^{-1}\hat{d}_1 \\ \hat{f}(p) &= (1 - \pi)^{-1}R(y - \hat{d}_1).\end{aligned}$$

Notice that conditions (1) and (4) correspond; so that:

$$\begin{aligned}\hat{c}(i) &= c^*(i) \\ \hat{f}(p) &= f^*(p).\end{aligned}$$

We should check to see that condition (2) holds. Using the asset-pricing formula (3),

$$\hat{\phi} = \left(\frac{1 - \pi}{\pi}\right)\hat{d}_1;$$

where $\hat{d}_1 = \pi\hat{c}(i) = \pi c^*(i)$. Hence,

$$\hat{\phi} = (1 - \pi)c^*(i) > 0.$$

Now, is it true that

$$\hat{\phi} \leq R(y - \hat{d}_1) = (1 - \pi)\hat{f}(p).$$

Clearly, this requires

$$(1 - \pi)c^*(i) \leq (1 - \pi)f^*(p);$$

which clearly holds as $c^*(i) < f^*(p)$.

In other words, it appears that we can implement the first-best allocation here by allowing a firm to issue equity with an appropriate dividend policy. This result hinges critically on the ability of agents to exchange equity shares in a stock market subsequent to the realization of their preference shocks. The market for ex-dividend equity essentially allows impatient agents to liquidate their assets; an act that corresponds to impatient agents withdrawing their money from the bank early in the Diamond-Dybvig bank. Moreover, one advantage of this “market mechanism” is that agents have no incentive to misreport their types.

3 Banking and Ex Post Trading Opportunities

The Diamond-Dybig banking arrangement implicitly assumes that, subsequent to the realization of their preference shocks, the agents do not engage in any ex post trades with each other. The question I want to pursue here is whether this assumption is restrictive (if it is, the banking arrangement will be dominated by the equity arrangement above, where people do trade ex post).

Imagine then that agents agree ex ante to the allocation $(c^*(i), f^*(p))$. They then realize their types and report themselves truthfully. The impatient agents

receive income $c^*(i)$ and the patient agents receive the **promise** of future income $f^*(p)$. I assume that the bank can commit to this promise.

However, imagine now that agents have an opportunity to trade on a competitive loan market at some interest rate R_0 . Then an impatient agent has the following choice problem:

$$\max \{u(c(i)) : c(i) + R_0^{-1}f(i) \leq c^*(i)\};$$

and a patient agent has the following choice problem:

$$\max \{u(c(p) + f(p)) : c(p) + R_0^{-1}f(p) \leq R_0^{-1}f^*(p)\};$$

or,

$$\max u(R_0^{-1}f^*(p) + (1 - R_0^{-1})f(p)).$$

The solution for the impatient agent is clearly $c^d(i) = c^*(i)$, independent of the interest rate. The solution for the patient agent is given by:

$$f^d(p) = \begin{cases} 0 & \text{if } R_0 < 1; \\ \text{indeterminate} & \text{if } R_0 = 1; \\ R_0 f^*(p) & \text{if } R_0 > 1. \end{cases}$$

Market-clearing requires:

$$\begin{aligned} \pi c^d(i) + (1 - \pi)c^d(p) &= \pi c^*(i); \\ \pi f^d(i) + (1 - \pi)f^d(p) &= (1 - \pi)f^*(p). \end{aligned}$$

Of course, since $c^d(i) = c^*(i)$ and $f^d(i) = 0$, it must be the case that the interest rate adjusts to a point where $f^d(i) = 0$ and $f^d(p) = f^*(p)$. The equilibrium interest rate consistent with this outcome is given by $R_0^* = 1$.

Hence, we conclude that the no-trade assumption was not restrictive in this case. Is this a robust conclusion, or does it depend on the special structure of preferences considered here? We will examine this question next.

4 Smooth Preferences

Retain the basic structure above, except that preferences are now given by:

$$U(c, f; \omega) = \begin{cases} u(c) + \beta u(f) & \text{if } \omega = i; \\ u(c) + u(f) & \text{if } \omega = p; \end{cases}$$

where $0 < \beta < 1$. Moreover, assume $R\beta = 1$.

As usual, let us begin by characterizing the first-best allocation. The planner's problem is:

$$\max \{\pi [u(c(i)) + \beta u(f(i))] + (1 - \pi) [u(c(p)) + u(f(p))]\}$$

subject to:

$$y \geq \pi [c(i) + R^{-1}f(i)] + (1 - \pi) [c(p) + R^{-1}f(p)].$$

The FONC are given by:

$$\begin{aligned} u'(c(i)) &= \lambda; \\ R\beta u'(f(i)) &= \lambda; \\ u'(c(p)) &= \lambda; \\ Ru'(f(p)) &= \lambda. \end{aligned}$$

Recall that $R\beta = 1$. Then, the first-best allocation is characterized by:

$$u'(c^*(i)) = u'(f^*(i)) = u'(c^*(p)) = Ru'(f^*(p));$$

together with the resource constraint.

Here, we see that the first-best dictates that both impatient and patient consume the same level of current consumption; and that the impatient consume this same level of consumption in the future; i.e.,

$$x^* = c^*(i) = f^*(i) = c^*(p).$$

Define $z \equiv f(p)$. Then, from the resource constraint:

$$y = \pi [x^* + R^{-1}x^*] + (1 - \pi) [x^* + R^{-1}z^*].$$

Hence, the first-best is completely characterized by the following two restrictions:

$$\begin{aligned} u'(x^*) &= Ru'(z^*); \\ Ry &= (\pi + R)x^* + (1 - \pi)z^*. \end{aligned}$$

Clearly, $x^* < z^*$. The first-best (ex ante) level of welfare is:

$$V^* = \pi [u(x^*) + \beta u(x^*)] + (1 - \pi) [u(x^*) + u(z^*)]$$

Is the first-best allocation incentive-compatible? That is, does it satisfy:

$$\begin{aligned} u(x^*) + \beta u(x^*) &\geq u(x^*) + \beta u(z^*); \\ u(x^*) + u(z^*) &\geq u(x^*) + u(x^*). \end{aligned}$$

The answer is clearly **no**. In particular, the impatient agent will want to misreport himself as a patient agent. The IC constraints clearly require that the allocation satisfy:

$$\begin{aligned} x &\geq z \\ z &\geq x. \end{aligned}$$

Hence, and IC allocation will have to satisfy the restriction $x = z$. Such an allocation is called **second-best**.

Let x^0 denote the second-best allocation; this is the solution to:

$$\max_x \{ [2 - \pi(1 - \beta)] u(x) : y \geq (1 + \beta)x \}.$$

Clearly,

$$x^0 = (1 + \beta)^{-1}y.$$

The second-best (ex ante) level of welfare is:

$$V^0 = \pi [u(x^0) + \beta u(x^0)] + (1 - \pi) [u(x^0) + u(x^0)]$$

Exercise 1 *Prove that $x^* < x^0 < z^*$ and $V^0 < V^*$.*

The second-best allocation is certainly implementable if agents are restricted from ex post trading. What if they are not restricted from ex post trading? Is this constraint binding? To find out, let us imagine that agents can trade ex post at interest rate R_0 . Then impatient agents face the following choice problem:

$$\max \{ u(c(i)) + \beta u(f(i)) : (1 + \beta)x^0 \geq c(i) + R_0 f(i) \};$$

with FONC:

$$u'(c(i)) = R_0 \beta u'(f(i)).$$

Patient agents face the choice problem:

$$\max \{ u(c(p)) + u(f(p)) : (1 + \beta)x^0 \geq c(p) + R_0 f(p) \};$$

with FONC:

$$u'(c(p)) = R_0 u'(f(p)).$$

Observe that for impatient agents to desire a constant consumption stream, we must have $R_0 = 1/\beta$. On the other hand, for patient agents to desire a constant consumption stream, we must have $R_0 = 1$. This is impossible. The ex post equilibrium interest rate will lie somewhere $R_0^* \in (0, \beta^{-1})$ and ex post trading **will** occur (the impatient will borrow and the patient will save). Hence, the second-best allocation is not “renegotiation proof.”

Hence, absent any restrictions on ex post trading, the only equilibrium will be one in which there are no ex ante trades; but where trading will occur subsequent to the realization of preference shocks. In other words, the insurance market will break down and the resulting allocation will be **third-best**. Haubrich (1988) derives a similar result.

Exercise 2 *Characterize the third-best equilibrium.*

5 Literature Review on Optimal Trading Restrictions

This is a classic example where relaxing a restriction in a second-best world will not necessarily make agents better off. I believe that Hart (1975) was probably the first one to make this point explicitly (perhaps others preceded him). The result is similar to one in the taxation literature, which shows that whenever the optimal taxation policy is nonlinear, it is vulnerable to resale; see Hammond (1979). Along with Jacklin (1987) and Haubrich (1988), others have argued that a bank-based financial system may be able to prevent ex post trading and thus be able to provide a better form of insurance against liquidity shocks than mutual funds holding shares in publicly traded firms and operating in the secondary market; see Diamond (1997) and Allen and Gale (2000). And finally, the general point appears to arise in one of my own papers as well; see Andolfatto (2002).

5.1 Postscript

Actually, now that I think about it, the general conclusions in the literature cited above are reminiscent of Cole and Kocherlakota (2001), who consider an environment with hidden income (instead of hidden preferences) and hidden storage. The idea that savings behavior can be hidden is analogous to the idea that ex post trading on a financial market cannot be prevented. The effect of this friction is to shut down insurance markets. Allen (1985) makes this point too in the context of a simple two-period model.

6 References

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Appendix: Old Notes

Retain the structure above, except that preferences are now given by:

$$U(c, f; \omega) = \begin{cases} u(c) + \beta u(f) & \text{if } \omega = i; \\ \beta u(c) + u(f) & \text{if } \omega = p; \end{cases}$$

where $0 < \beta < 1$. Hence,

$$\frac{u'(c)}{\beta u'(f)} > \frac{\beta u'(c)}{u'(f)}$$

for all values of (c, f) .

As usual, let us begin by characterizing the first-best allocation. The planner's problem is:

$$\max \{ \pi [u(c(i)) + \beta u(f(i))] + (1 - \pi) [\beta u(c(p)) + u(f(p))] \}$$

subject to:

$$y \geq \pi [c(i) + R^{-1}f(i)] + (1 - \pi) [c(p) + R^{-1}f(p)].$$

The FONC are given by:

$$\begin{aligned} u'(c(i)) &= \lambda; \\ R\beta u'(f(i)) &= \lambda; \\ \beta u'(c(p)) &= \lambda; \\ Ru'(f(p)) &= \lambda. \end{aligned}$$

For simplicity, assume $R\beta = 1$. Then, the first-best allocation is characterized by:

$$u'(c^*(i)) = u'(f^*(i)) = R^{-1}u'(c^*(p)) = Ru'(f^*(p));$$

Together with the resource constraint.

Hence, the impatient agent receives a constant level of consumption, $x^* \equiv c^*(i) = f^*(i)$; and the patient agent receives $c^*(p) < x^* < f^*(p)$. The allocation is completely characterized by:

$$\begin{aligned} u'(x^*) &= Ru'(f^*(p)); \\ u'(c^*(p)) &= R^2u'(f^*(p)); \\ (1 - \pi) [Rc^*(p) + f^*(p)] &= Ry - \pi (R + 1) x^*. \end{aligned}$$

The first-best allocation is IC iff:

$$\begin{aligned} u(x^*) + \beta u(x^*) &\geq u(c^*(p)) + \beta u(f^*(p)); \\ \beta u(c^*(p)) + u(f^*(p)) &\geq \beta u(x^*) + u(x^*). \end{aligned}$$

Let us assume that IC is satisfied. Then, one can think of this as a demand deposit contract where the bank offers all depositors a minimum guaranteed

short-term return $c^*(p)$ plus the option to make an additional short-term withdrawal equal to $(x^* - c^*(p))$. Those that exercise the option receive an additional x^* return in the future; and those that do not exercise the option receive the higher future return $f^*(p)$.

Now, imagine that this contract is in place, that types are then realized, and that a competitive loan market opens with interest rate R_0 . The choice problem facing the impatient agent is:

$$\max \{u(c(i)) + \beta u(f(i)) : c(i) + R_0^{-1}f(i) \leq c^*(i) + R_0^{-1}f^*(i)\}.$$

The FONC is:

$$u'(c^d(i)) = R_0 \beta u'(f^d(i));$$

together with the budget constraint.

The choice problem facing the patient agent is:

$$\max \{\beta u(c(p)) + u(f(p)) : c(p) + R_0^{-1}f(p) \leq c^*(p) + R_0^{-1}f^*(p)\}.$$

The FONC is:

$$u'(c^d(p)) = R R_0 u'(f^d(p));$$

together with the budget constraint. Clearly, $R_0^* = R$ is an equilibrium (verify this). There is no problem here.

However, assume now that even one of the IC constraints is violated for the first-best allocation. An educated guess tells me that this is likely to be the case for the impatient agent. Assume that demand deposits cannot be traded. Letting ψ denote the Lagrange multiplier for the impatient agents' IC constraint, the constrained-efficient allocation is characterized by:

$$\begin{aligned} \pi u'(c(i)) - \pi \lambda + \psi u'(c(i)) &= 0; \\ \pi \beta u'(f(i)) - \pi R^{-1} \lambda + \psi \beta u'(f(i)) &= 0; \\ (1 - \pi) \beta u'(c(p)) - (1 - \pi) \lambda - \psi u'(c(p)) &= 0; \\ (1 - \pi) u'(f(p)) - (1 - \pi) R^{-1} \lambda - \psi \beta u'(f(p)) &= 0. \end{aligned}$$

Rearranging these restrictions:

$$\begin{aligned} (\pi + \psi) u'(c(i)) &= \pi \lambda; \\ (\pi + \psi) u'(f(i)) &= \pi \lambda; \\ (1 - \pi) \beta u'(c(p)) - \psi u'(c(p)) &= (1 - \pi) \lambda; \\ (1 - \pi) R u'(f(p)) - \psi u'(f(p)) &= (1 - \pi) \lambda. \end{aligned}$$

The first two restrictions imply that an IC allocation will feature $x = c(i) = f(i)$. Note that as $\psi > 0$, this implies that $x > x^*$. The IC level of x must satisfy:

$$u(x) + \beta u(x) = u(c(p)) + \beta u(f(p)).$$

Note that the IC constraint above is satisfied for $x = c(p) = f(p)$. But it is likely that the constrained-efficient allocation can do better than assign patient agents a constant consumption stream. The last two FONCs above can be combined to form:

$$(1 - \pi)\beta u'(c(p)) - \psi u'(c(p)) = (1 - \pi)R u'(f(p)) - \psi u'(f(p));$$

or,

$$(1 - \pi)u'(c(p)) - (1 - \pi)R^2 u'(f(p)) = \psi R [u'(c(p)) - u'(f(p))].$$

An educated guess suggests that $c(p) < c^*(p)$ and $f(p) > f^*(p)$.

Hence, the impatient agent receives a constant level of consumption, $x^* \equiv c^*(i) = f^*(i)$; and the patient agent receives $c^*(p) < x^* < f^*(p)$. The allocation is completely characterized by:

$$\begin{aligned} u'(x^*) &= R u'(f^*(p)); \\ u'(c^*(p)) &= R^2 u'(f^*(p)); \\ (1 - \pi) [R c^*(p) + f^*(p)] &= R y - \pi (R + 1) x^*. \end{aligned}$$

Here is the intuition. The impatient agent would like to mimic the patient agent at the first-best. The way to persuade him not to misreport is to assign him more current consumption (which he values highly); hence $x > x^*$. At the same time, misreporting can be made more painful by lowering the current consumption of patient agents; hence $c(p) < c^*(p)$. The patient agent can be compensated for this to some extent by allocating more future consumption (which he values highly); hence $f(p) > f^*(p)$.

Exercise 3 *Verify what I have said here either theoretically, or computationally.*

Now, the issue here is whether the constrained-efficient allocation above is “renegotiation proof.” That is, will agents have an incentive to trade ex post? The arguments laid out in Jacklin (1987) and Haubrich (1988) suggest that the answer is yes. But if the answer is yes, then the constrained-efficient allocation cannot be an equilibrium. The only equilibrium will be the renegotiation proof equilibrium; and this can be attained simply by allowing agents to trade their endowment ex post. In an ex ante sense, agents are made worse off. Or, another way to put this, agents would be made better off in an ex ante sense if they were somehow not permitted to trade ex post.

Exercise 4 *Verify what I have said here either theoretically, or computationally.*