Notes on Contracts and Money

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1 A Standard Principal-Agent Problem

A risk-neutral principal hires a risk-averse agent. The agent applies effort n; which generates output $y \in \{y_l, y_h\}$, $0 < y_l < y_h < \infty$. Let $\phi(n) = \Pr[y = y_h]$, with ϕ increasing and concave.

The agent's preferences over consumption c and effort are:

$$U(c,n) = u(c) - n.$$

The principal's utility is $y - c$ (allowed to be negative). The agent has an outside option valued at v; and all the bargaining power resides with the principal.

The first-best allocation solves:

$$\max \phi(n) [y_h - c(y_h)] + (1 - \phi(n) [y_l - c(y_l)])$$

subject to:

$$\phi(n)u(c(y_h)) + (1 - \phi(n))u(c(y_l)) - n \geq v.$$

The FONC:

$$\begin{aligned} \phi'(n) [y_h - y_l] + \lambda \left[\phi'(n)(u(c(y_h)) - u(c(y_l)) - 1\right] &= 0; \\ -1 + \lambda u'(c(y_h)) &= 0; \\ -1 + \lambda u'(c(y_l)) &= 0. \end{aligned}$$

The latter two conditions imply full consumption insurance; $c(y) = c^*$. Hence, optimal consumption and effort are characterized by:

$$\begin{aligned} \phi'(n^*) [y_h - y_l] u'(c^*) &= 1; \\ u(c^*) - n^* &= v. \end{aligned}$$

1.1 Unobservable Effort

Full consumption insurance cannot be a solution, as the agent would optimally choose $n = 0$. Since y is observable, the only way the principal can elicit effort is by choosing state-contingent consumption; and an educated guess tells us that it must be the case that $c(y_h) > c(y_l)$. Consider a given consumption allocation and study the choice problem faced by the agent:

$$\max_n \phi(n)u(c(y_h)) + (1 - \phi(n))u(c(y_l)) - n.$$

The FONC is given by:

$$\phi'(\hat{n}) [u(c(y_h) - u(c(y_l)] = 1.$$

Clearly $\hat{n} > 0$ iff $c(y_h) > c(y_l)$. The condition above now constitutes an additional constraint. Note that this "first-order approach" is not in general valid. However, there are conditions under which it is. These are: (1) agent is risk-averse with separable preferences; (2) the principal is risk-neutral; and (3) the probability of realizing higher output is higher for higher effort levels; see Prescott (1999).[1] These conditions are satisfied here.

Exercise 1 Recast this problem assuming that n represents the probability of success and where $-g(n)$ denotes the disutility of effort.

Exercise 2 Recast this problem assuming that effort is indivisible with $n \in \{0, 1\}$ and that the probability of success is ϕ when $n = 1$.

2 Nominal Contracting with Commitment

Assume now that y is not observed by either party. Instead, both parties observe a signal (nominal sales):

$$s = py;$$

where p (price-level) is a random variable. This signal is received after contracting is completed; but p is observed before consumption occurs. The idea here is that the price-level is observed only with some lag (as is the case in reality). Hence, indexed contracts are feasible. Since $y = s/p$, the contract $\hat{c}(s/p)$, which is the solution to the problem above, is also the solution here.

3 Nominal Contracting without Commitment

The problem is just as above, except now the parties cannot commit to the contract $\hat{c}(y)$. The signal s is observed first, immediately after effort is sunk. The agent knows n, but so does the principal, because he can infer it in equilibrium. At this point then, the two parties have the same beliefs over p. Sales alone

[1] Prescott, Edward S. (1999). "A Primer on Moral Hazard Models," Federal Reserve Bank of Richmond *Economic Quarterly*, Volume 85/1 Winter.

reveal neither y nor p perfectly, so that the conditional density $\pi(p \mid s)$ is nondegenerate.

Since $\hat{c}(y)$ is not a constant function, $\hat{c}(s/p)$ will be a random variable even if s is given. The question to consider here is whether, given the option to renegotiate, will both parties prefer a contract different from $\hat{c}(s/p)$? Conditional on seeing some signal s_0, the expected payoff to each party if they stick with the original contract is:

$$\int \left[\frac{s_0}{p} - \hat{c}\left(\frac{s_0}{p}\right)\right] \pi(p \mid s_0) dp$$

and

$$\int u\left(\hat{c}\left(\frac{s_0}{p}\right)\right) \pi(p \mid s_0) dp.$$

Now, define the certainty equivalent $\mathbf{c}(s_0)$ by:

$$u(\mathbf{c}(s_0)) \equiv \int u\left(\hat{c}\left(\frac{s_0}{p}\right)\right) \pi(p \mid s_0) dp.$$

Since u is strictly concave,

$$\mathbf{c}(s_0) < \int \hat{c}\left(\frac{s_0}{p}\right) \pi(p \mid s_0) dp.$$

Hence, the principal can be made strictly better off by getting the agent to accept $\mathbf{c}(s_0)$; leaving the agent no worse off.

The intuition is that at the renegotiation stage, the tension between insurance and incentives has disappeared; i.e., effort is sunk. Since the IC condition is no longer relevant, the principal should insure the agent fully. This requires that any renegotiated contract depend only on nominal sales. Such a contract is ex ante inefficient (but it is renegotiation proof).

For those interested in exploring this idea in more detail, refer to the authors' working paper: NBER 5637.