Liquidity and Finanical Intermediation

David Andolfatto

1. Long-term Capital Investment and Private Debt

Consider a constant population of 3-period lived overlapping generations with preferences given by $U = u(c_2) + \delta u(c_3)$ and an endowment stream given by $(y, 0, 0)$. Each young person is also endowed with an investment project that takes k units of current output and returns $f(k)$ units of output in the third period of life. Individuals can also borrow or lend in a market for private risk-free debt at (gross) interest rate R. The sequence of expenditure relations is given by:

$$
\begin{aligned}
a + k &= y; \\
c_2 + a' &= Ra; \\
c_3 &\leq f(k) + Ra',
\end{aligned}
$$

where a denotes net bond-holdings when young and a' denotes net bond-holdings when old. (Note that we have imposed $a'' = 0$, so that people cannot die in debt). These relations can be combined to form the lifetime budget constraint:

$$
R^{-1}c_2 + R^{-2}c_3 \leq y - k + R^{-2}f(k).
$$

Desired capital spending k^D is characterized by the condition $f'(k^D) = R^2$. With k^D so determined, we can plug it back into the budget constraint to derive lifetime wealth $W = y - k^D + R^{-2}f(k^D)$ as a function of R. Maximizing utility subject to the wealth constraint entails the usual Euler equation: $u'(c_2) = R\delta u'(c_3)$. This restriction together with the wealth constraint characterize the desired consumption plan (c_2^D, c_3^D).

Now, the next step is to impose the market-clearing condition:

$$
k^D + c_2^D + c_3^D = y + f(k^D).
$$

This is one equation in the one unknown; i.e., the equilibium interest rate R^*. Note that it is quite often the case in this class of models that the equation above possesses multiple solutions (generally, R^* is not unique).

In the equilibrium of this economy, money takes the form of private bonds backed by the returns to capital investment. Both bonds and capital investment (at the margin) earn the same rate of return $R^* = (f'(k^D(R^*)))^{1/2}$.

2. Illiquid Capital and Fiat Money

Suppose now (following Champ and Freeman, Chapter 7) that the return to capital investment $f(k)$ is inalienable (noncollateralizable). This restriction implies a complete breakdown in the market for private debt. In particular, young agents would face the following borrowing constraints: $a, a' \geq 0$. And since no debt would actually be repaid in equilibrium, no one would want to save using debt instruments; i.e., $a = a' = 0$. Consequently, the equilibrium allocation here is given by:

$$(k, c_2, c_3) = (y, 0, f(y)).$$

Clearly, there is a role for fiat money here. For example, suppose that the government injects M dollars into the economy (distributed to the middle-aged individuals) and then holds the money stock fixed over time. Let π denote the inflation rate so that $(1 + \pi)^{-1}$ equals the (gross) rate of return on fiat money. Then the typical young person faces the following sequence of expenditure relations:

$$
\begin{aligned}
q + k &= y; \\
c_2 + q' &= (1 + \pi)^{-1} q; \\
c_3 &\leq f(k) + (1 + \pi)^{-1} q'.
\end{aligned}
$$

Since agents cannot print fiat money, they are also faced with the restrictions $q, q' \geq 0$.

Clearly, the constraint $q \geq 0$ is not going to bind here, since people must hold fiat in order to acquire c_2. The constraint $q' \geq 0$ may or may not bind. Suppose that it does not bind. Then agents face the following wealth constraint:

$$R^{-1} c_2 + R^{-2} c_3 \leq y - k + R^{-2} f(k),$$

where R is now defined to be the rate of return on fiat; i.e., $R = (1 + \pi)^{-1}$. Optimal decision making is characterized by:

$$
\begin{aligned}
f'(k) &= R^2; \\
u'(c_2) &= R\delta u'(c_2).
\end{aligned}
$$

2

The money market clearing condition is given by:

$$q + q' = v_t M,$$

for all t (where v_t is the value of fiat measured in units of output). In a stationary equilibrium, $(v_{t+1}/v_t) = 1.0$, so that $\pi = 0$. In this monetary equilibrium, fiat money earns the same rate of return as capital.

Suppose, however, that the constraint $q' \geq 0$ binds (individuals would like to borrow, if they could). Let λ denote the Lagrange multiplier associated with the constraint $q' \geq 0$. The optimality conditions can be written as follows:

$$
\begin{aligned}
Ru'(Rq - q') &= f'(y - q)\delta u'(f(y - q) + Rq'); \\
u'(Rq - q') &= R\delta u'(f(y - q) + Rq') + \lambda.
\end{aligned}
$$

Of course, if $q' = 0$ then $\lambda > 0$ and we have:

$$
\begin{aligned}
Ru'(Rq) &= f'(y - q)\delta u'(f(y - q)); \\
u'(Rq) &= R\delta u'(f(y - q)) + \lambda.
\end{aligned}
$$

The money market clearing condition is now $q = v_t M$ for all t, which again implies $R = 1$ ($\pi = 0$), so that the general equilibrium q is characterized by:

$$u'(q) = f'(y - q)\delta u'(f(y - q)).$$

We also know:

$$\lambda = u'(q) - \delta u'(f(y - q)) > 0,$$

so that it must be the case that $f'(y - q) > 1$; i.e., capital dominates fiat money in rate of return.

That capital dominates fiat in rate of return can be made obvious by assuming (the way Champ and Freeman do) that $f(k) = Xk$, where $X > 1$. In this case, we know that $q' = 0$ and that q is determined by:

$$u'(q) = X\delta u'(f(y - q)).$$

3. Champ and Freeman on Liquidity and Financial Intermediation

We're now in a position to better understand Chapter 7 (2nd Edition) of Champ and Freeman. The authors begin the chapter with the analysis undertaken in Section 2 above and under the assumption that the constraint $q' = 0$ binds. Here we have a situation where individuals acquire a low rate of return (but liquid) asset to finance short-term consumption expenditures (c_2) and acquire a high rate of return (but illiquid) asset to finance long-term consumption expenditures (c_3).

What makes capital illiquid? Essentially, it is the inability (or unwillingness) of the capitalist to commit to repaying debt (or equity) out of the future returns to capital investment. If individuals can commit to honouring debt, then the analysis of Section 1 applies and capital is perfectly liquid in the sense that claims to capital can be easily used to finance both short-term and long-term consumption.

Where does financial intermediation fit into this picture? Champ and Freeman introduce a set of individuals that for some reason can commit to honouring debt contracts. They call these agents 'banks' (for obvious reasons). The arbitrage plan worked out by the authors is not quite correct (but the essence of their argument is still valid). I think that the best way to understand financial intermediation in the present context is as follows.

Let us begin with the scenario outlined in Section 2 and assume that the allocation in Section 1 involves $a > 0 > a'$. Let us also suppose, for simplicity only, that $f(k) = Xk$, with $X > 1$. Now, suppose that there is one agent that has commitment power. How can this agent exploit this ability?

Let me denote the allocation in Section 1 with a 'star' superscript and the allocation in Section 2 with an 'M' superscript. A proper analysis would require that we begin with the equilibrium in Section 2 and then ask how the monopoly bank might behave in order to maximize its profits. This approach, however, would require that we take into account transition dynamics–something that I would like to avoid here. So, let us try to imagine this monopoly bank has operated for some time–enough time for the economy to have settled into a new steady state. What might this steady state look like?

A monopoly bank would be in a position to implement a Pareto improvement and to extract the entire surplus so as to make all other individuals

just indifferent between living in the new steady state versus the one prevailing in Section 2. Suppose, for example, that the bank offers young people a one-period bond paying an interest rate R and a two-period bond paying an interest rate X. Young people would willing hand over their entire endowment to the bank as long as $R \geq 1$. In fact, why don't we just assume that the bank promises to deliver the allocation (c_2^M, c_3^M) to young agents in exchange for their endowment y and rights to their investment projects.

In a steady state, the bank will delivering $c_2^M + c_3^M$ units of output and acquiring $y + Xk$ units of output per period. The period consumption for the monopoly bank is $c_t = y + Xk_t - k_{t-2} - c_2^M - c_3^M$. Assume that the monopolist has preferences:

$$\sum_{t=0}^{\infty} \delta^t u(c_t).$$

The Euler equation is given by:

$$u'(c_t) =$$

Actually, just impose steady-state to begin.