

Money and Credit in Quasi-linear Environments*

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Abstract

I consider a version of the quasi-linear environment suggested by Lagos and Wright (2005). In the first part of these notes, I lay out the basic environment and characterize the Pareto optimal allocation.

In the second part, I view the resource allocation problem from the perspective of a mechanism (or planner) with various tools at his disposal and subject to various constraints on available information and commitment. I assume throughout this part that the mechanism has a costless record-keeping technology, in which case one can show that there is no role for fiat money. I show how optimal (and constrained-optimal) allocations require the use of a well-designed (history-dependent) system of punishments and rewards.

In the third part of these notes, I assume away the record-keeping technology. Evidently, this is equivalent to assuming that individual trading histories can be costlessly falsified (an extreme form of identity theft). Here, I study the relationship between the demand for money and the punishment/reward system that a mechanism would have adopted in a similar environment with perfect record-keeping.

Part I

Basics

1 Preferences and Technologies

Time is discrete and the horizon is infinite; $t = 0, 1, \dots, \infty$. Each period is divided into two subperiods; stage 1 and stage 2 (day and night). There

*I would like to thank Randy Wright for several helpful comments.

is a unit mass of infinitely-lived *ex ante* identical agents $i \in [0, 1]$, with preferences defined over stochastic sequences:

$$\{e_t(i), c_t(i), n_t(i) : t \geq 0\},$$

where $e_t(i)$ denotes effort during stage 1; $c_t(i)$ denotes consumption during stage 2; and $n_t(i)$ denotes effort during stage 2. These preferences are represented by an expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t [-\alpha e_t(i) + U_t(c_t(i), n_t(i))],$$

with $0 < \beta < 1$, $\alpha > 0$, and where,

$$U_t(c_t(i), n_t(i)) = \begin{cases} u(c_t(i)) & \text{w.p. } \delta_1 & \text{(consumer);} \\ -g(n_t(i)) & \text{w.p. } \delta_2 & \text{(producer);} \\ 0 & \text{w.p. } 1 - \delta_1 - \delta_2 & \text{(neither).} \end{cases} \quad (1)$$

Assume that $u'' < 0 < u'$ and $0 < g', g''$ with $g(0) = 0$ and $u(0) = u_0 \geq -\infty$. In what follows, I restrict $\delta_1 = \delta_2 = 1/2$. This is done primarily for ease of exposition; nothing critical hinges on this restriction.

The available production technologies are as follows. First, assume that $e_t(i) \in \mathcal{R}$, so that first-stage effort is unbounded from above and below. Negative effort in this case can be interpreted as a positive level of consumption (and vice-versa).¹ Second stage output is produced according to $y_t(i) = n_t(i)$. Assume that $n_t(i) \geq 0$ and that output is nonstorable.

2 Pareto Optimal Allocation

Consider the choice problem of a planner who maximizes the expected utility of a representative agent (recall that agents are *ex ante* identical). Since there is no storage (capital), and since the preference/technology shocks in (1) are *i.i.d.* (across individuals and time), we can restrict attention to a stationary allocation (so we can drop the time subscripts). The planner is

¹Allowing for negative consumption here is not restrictive. Most of what I have to say here will continue to hold if I instead assume that first-stage utility is given by $u(c) - \alpha e$ or $c - g(e)$, with $c, e \geq 0$. The key assumption here is the quasi-linearity of preferences; which renders utility transferable.

subject to the following two resource constraints:

$$\begin{aligned}\int e(j)dj &\geq 0; \\ \int n(j)dj &\geq \int c(j)dj.\end{aligned}$$

Let $\lambda_1(i)$ and $\lambda_2(i)$ denote the Lagrange multipliers associated with each of these constraints, respectively. Then the FONCs (at an interior) are given by:

$$\begin{aligned}\alpha &= \lambda_1(i); \\ \frac{1}{2}u'(c(i)) &= \lambda_2(i); \\ \frac{1}{2}g'(n(i)) &= \lambda_2(i);\end{aligned}\tag{2}$$

together with the resource constraints (holding as equalities).

The last two restrictions in (2) imply that $c(i) = n(i) = y(i) \forall i$, allowing us to characterize the optimal consumption/production pattern in stage 2 as the y^* that satisfies:

$$u'(y^*) = g'(y^*).\tag{3}$$

That is, with probability (1/2), the agent is a producer and is asked to produce $y^* = n^*$ units of output; and with complementary probability, the agent is a consumer and is entitled to $c^* = n^*$ units of output. With some law of large numbers holding, the planner is in essence transferring output from the producing half of the population to the consuming half. *Ex post*, producers would like to renege on this obligation. But *ex ante*, all would agree to commit to such a program, since over the course of time they are equally likely to be consumers.

Let us now consider the determination of $e(i)$. From the first equation in (2), a one-unit increase in $e(i)$ costs the agent α utils; this cost must be offset by the marginal utility benefit $\lambda_1(i)$, which measures the marginal value of relaxing the first resource constraint. Note that the planner sets $\lambda_1(i) = \alpha \forall i$, which implies that $e(i) = e^*$. The only e^* that satisfies the first resource constraint with equality is therefore given by $e^* = 0$.

Note that the proper interpretation of e^* is that of an *expected* value (since preferences are linear in $e(i)$, certainty equivalence holds). In other words, as part of the optimal allocation, the planner is free to choose any

lottery over $e(i)$, subject to the restriction that the expected value of this lottery be equal to $e^* = 0$.² For example, the allocation (lottery):

$$e(i) = \begin{cases} \sigma & \text{w.p. } 1/2; \\ -\sigma & \text{w.p. } 1/2, \end{cases} \quad (4)$$

for any $\sigma \geq 0$ is also consistent with Pareto optimality. In any case, the welfare payoff associated with the Pareto optimal allocation is given by:

$$W^* = (1 - \beta)^{-1} \frac{1}{2} [u(y^*) - g(y^*)]. \quad (5)$$

This is all very nice. But exactly how is such an allocation supposed to be implemented? Imagine that there is a mechanism (or planner) charged with the problem of maximizing the *ex ante* welfare of individuals. To implement the PO allocation $(e, y) = (0, y^*)$, the mechanism must be endowed with a certain amount of power. This power will be the product of what is assumed in the way of available technologies describing what is feasible in terms of: (a) collecting and recording information; (b) enforcement; and (c) commitment.

In the present context, we need to assume that the mechanism knows the parameters u, g, α, β , as well as the idiosyncratic probability structure governing the preference shocks; assuming this much is standard. But the mechanism must also be able to observe an individual's *type*; i.e., whether an individual is a producer or consumer in the stage 2 subperiod. As well, the mechanism must be able to force or otherwise compel stage 2 producers to deliver the required y^* units of output (which is then distributed to the stage 2 consumers). Alternatively, one must assume that the individuals themselves are endowed with a commitment technology. Of course, in the absence of any private information and commitment problems, one could implement the PO allocation with any one of a number of decentralized mechanisms (e.g., an Arrow-Debreu securities market).

In what follows, I consider what type of allocations can be implemented when the mechanism's powers are limited along various dimensions. The first scenario considers the case in which individual type is private information. The second scenario I consider is one that features limited commitment/enforcement. In both of these cases, I assume that the mechanism can identify individuals and keep track of their histories.

²In fact, the planner is even free to choose any deterministic and nonstationary allocation $\{e_t(i) : t \geq 0\}$ such that the discounted value of $\{e_t(i)\}$ is equal to zero. As well, the planner may also choose any lottery over $\{e_t(i) : t \geq 0\}$, as long as the expected discounted value of the lottery is again equal to zero.

Part II

Credit Economies

3 Private Information and Lack of Commitment

In the second stage of each period, an idiosyncratic shock determines whether an individual is a producer or consumer. Imagine now that the realization of this shock constitutes private information. The question here is whether individuals have an incentive to alter their behavior for personal gain. In particular, will stage 2 producers have an incentive to follow through on their obligation to produce y^* units of output? While such a commitment makes sense from an *ex ante* perspective, what if the requisite commitment technology is absent? In this case, we must restrict attention to allocations that satisfy *sequential rationality* (i.e., actions that are *individually rational* at each point in time).

The environment considered here constitutes a game of incomplete information (and a lack of commitment). For such an environment, we can make use of the so-called *Revelation Principle*, which asserts that (under some conditions, at least) any Bayes-Nash equilibrium payoff of a game of incomplete information can be achieved as a Nash equilibrium of a simple *revelation game* where individuals report their types truthfully.

To see how this works, consider the following *direct revelation mechanism*. That is, consider a mechanism that asks individuals to make a *report* of their type (when the shock is realized at the beginning of each stage 2 subperiod). The mechanism then conditions an allocation on the basis of the information received in these reports. Given the allocation, individuals then play a game in which they make sequential reports of their type (taking as given the reports they expect to be made by other individuals). To ensure that these reports are truthful, the mechanism must ensure that the allocation satisfies incentive-compatibility (IC). In other words, it must be in the interest of individuals to adopt strategies that constitute truthful revelations. To ensure that individuals actually have an incentive to carry through the actions prescribed by the allocation, the allocation must satisfy individually rationality (IR).

In general, characterizing the optimal allocation in a dynamic game of incomplete information can be difficult. The reason for this is because op-

tinality will in general require the mechanism to condition an allocation on the entire set of individual histories (describing individual reports and actions)—a large dimensional object.

Fortunately, the structure of the Lagos-Wright environment admits a particularly simple recursive representation of the optimal contract. In particular, the problem falls into the class of quasi-linear mechanisms (with strict budget balance). That is, in this quasi-linear environment, the relevant history collapses into a small dimensional object. In particular, all the mechanism needs to know is the set of reports and actions that each individual made in the previous period. What this means is that the dynamic economy can be studied as a sequence of two-period economies.³ Intuitively, the mechanism will be able to use history-dependent values for e to serve as a punishment/reward system. Since e enters linearly into preferences, the ‘full’ punishment/reward can be dispensed with in ‘one shot’ (as opposed to being spread over time, as would be optimal in nonlinear environments). With strict budget balance, any random variation in e entails absolutely no (*ex ante*) welfare cost (hence, it will be optimal to restrict punishments to variations in e , rather than y). At the same time, some variation in e can help correct incentives that have been distorted by the existence of private information and the lack of commitment.

Let $\omega \in \{1, 2\}$ denote an individual’s true type at the beginning of any stage 2 subperiod, where $\omega = 1$ denotes ‘producer’ and $\omega = 2$ denotes ‘consumer.’ We seek to implement a (stationary) allocation $\{e(\omega), c(\omega), n(\omega)\}$ of the following form:

$$\begin{aligned} (c(1), n(1)) &= (0, y); \\ (c(2), n(2)) &= (y, 0); \end{aligned} \tag{6}$$

and

$$\begin{aligned} e(1) = e &\quad \text{if } (c_{-1}(1), n_{-1}(1)) = (0, y); \\ e(2) = -e &\quad \text{if } (c_{-1}(2), n_{-1}(2)) = (y, 0); \end{aligned} \tag{7}$$

with $e(\omega) = 0$ at the initial date. This allocation anticipates that it will be necessary to reward those who reveal themselves a stage 2 producers with a promise of future utility. This future utility must be financed by punishing those who revealed themselves as consumers during the stage 2 market. Note that this punishment/reward system entails no *ex ante* welfare cost; i.e., see the discussion surrounding equation (4).

³This statement remains true only to the extent that the upper and lower bounds for e are sufficiently large (which they are in this case, since $e \in \mathcal{R}$).

Individual behavior can be represented in recursive fashion as follows. Let W^p, W^c denote the value of an individual at the beginning of stage 1, who was (respectively) a producer (p) or consumer (c) in the previous period's stage 2 market. Let V^p, V^c denote the value of an individual who is (respectively) a producer or consumer in the stage 2 market. Conditional on some allocation (e, y) , these value functions must satisfy the following recursive relationships:

$$W^p(e, y) = \alpha e + \frac{1}{2} [V^p(e, y) + V^c(e, y)]; \quad (8)$$

$$W^c(e, y) = -\alpha e + \frac{1}{2} [V^p(e, y) + V^c(e, y)]; \quad (9)$$

$$V^p(e, y) = -g(y) + \beta W^p(e, y); \quad (10)$$

$$V^c(e, y) = u(y) + \beta W^c(e, y). \quad (11)$$

The mechanism's objective function is given by:

$$\begin{aligned} W(y) &= \frac{1}{2} W^p(e, y) + \frac{1}{2} W^c(e, y); \\ &= (1 - \beta)^{-1} \frac{1}{2} (u(y) - g(y)); \end{aligned} \quad (12)$$

where this last derivation is obtained by combining the relationships in (8)-(11). Note that this welfare function does not depend on e and that the unconstrained maximum is attained at $y = y^*$.

Now, from (10) and (11), we have:

$$V^p(e, y) + V^c(e, y) = u(y) - g(y) + \beta [W^p(e, y) + W^c(e, y)];$$

which implies that:

$$\begin{aligned} \frac{1}{2} [V^p(e, y) + V^c(e, y)] &= \frac{1}{2} [u(y) - g(y)] + \frac{1}{2} \beta [W^p(e, y) + W^c(e, y)]; \\ &= \frac{1}{2} [u(y) - g(y)] + \beta W(y); \\ &= W(y). \end{aligned}$$

Substituting this result into (8)-(11), we have:

$$\begin{aligned} W^p(e, y) &= \alpha e + W(y); \\ W^c(e, y) &= -\alpha e + W(y); \\ V^p(e, y) &= -g(y) + \beta [\alpha e + W(y)]; \\ V^c(e, y) &= u(y) + \beta [-\alpha e + W(y)]. \end{aligned}$$

4 Incentive-Compatibility and Individual-Rationality

The mechanism's problem is to choose the pair (e, y) to maximize (12) subject to incentive-compatibility (IC) and individual-rationality (IR) (notice that 12 already embeds feasibility). Since reports are made at the beginning of each stage 2 subperiod (when types are revealed), the relevant IC conditions pertain to stage 2 individuals. For a stage 2 producer, the payoff to telling the truth $V^p(e, y)$ must weakly dominate the payoff associated with pretending to be a consumer; i.e.,

$$-g(y) + \beta[\alpha e + W(y)] \geq -g(0) + \beta[-\alpha e + W(y)];$$

or,

$$2\alpha\beta e \geq g(y). \tag{13}$$

Likewise, a stage 2 consumer must weakly prefer to tell the truth. As it turns out, however, it appears that a consumer cannot feasibly tell a lie. That is, a producer is called upon to deliver y units of output; and a consumer has (by construction) no technology for producing goods. Hence, a consumer will trivially prefer to tell the truth.

Let us now turn to individual-rationality. Conditional on the reports made in stage 2 of a given period, the planner makes a recommendation $\{e_{+1}(\omega), y(\omega), n(\omega)\}$ according to (6) and (7). If (13) holds, then individual type-reports will be truthful. But we still have to ask whether it makes sense for individuals to carry through with any obligations associated with the mechanism's recommendation. In other words, even if it is *collectively* rational to fulfil one's obligations; it may not be *individually* rational to do so at every point in time.

Associated with any recommendation made at stage 2 are two obligations. First, a stage 2 producer is obliged to produce y units of output immediately. Second, a stage 2 consumer is obliged to deliver e units of output (utility) at the beginning of the *future* period (in the stage 1 subperiod). What happens if an individual chooses not fulfil his obligation? Note that if individuals could default on their obligations with impunity, then the only equilibrium is autarky; which yields the utility payoff:

$$W^A = (1 - \beta)^{-1} \frac{1}{2} u_0; \tag{14}$$

where, recall that $u_0 = u(0) \geq -\infty$.

There are a number of different ways to handle the question of non-compliance. The simplest (and crudest) approach is to simply assume the existence of a commitment technology.⁴ Alternatively, the mechanism may have at its disposal an ability to punish noncompliance in some credible manner. Naturally, this latter approach presumes that the mechanism can commit to some potentially horrible punishments. One such punishment, familiar from game theory, is to assume that mechanism can banish individuals to an autarkic state in the event of noncompliance. Essentially, the mechanism threatens to ‘send people to hell’ if they break any covenants. Of course, hell may not be such a bad place—whether it is or not depends on the parameter u_0 . If hell is sufficiently painful (i.e., as $u_0 \rightarrow -\infty$), then the threat of punishment effectively returns us to the case of commitment.

In the present context (assuming that the punishment for noncompliance is autarky), the IR constraints (also called participation constraints) can be written as:

$$\begin{aligned} W^p(e, y) &= \alpha e + W(y) \geq W^A; \\ W^c(e, y) &= -\alpha e + W(y) \geq W^A; \\ V^p(e, y) &= -g(y) + \beta [\alpha e + W(y)] \geq -g(0) + \beta W^A; \\ V^c(e, y) &= u(y) + \beta [-\alpha e + W(y)] \geq u(0) + \beta W^A. \end{aligned}$$

Clearly, the first constraint will not bind for any positive (e, y) . The second constraint may or may not bind. If it does not bind, then neither will the fourth constraint. If it does bind, then the fourth constraint can be written as $u(y) + \beta W^A \geq u_0 + \beta W^A$; or just $u(y) \geq u_0$. Thus, the fourth constraint will not bind for any $y > 0$. Hence, we are left with the second and third constraints; i.e.,

$$-\alpha e + W(y) \geq W^A; \tag{15}$$

$$-g(y) + \beta [\alpha e + W(y)] \geq \beta W^A. \tag{16}$$

5 Implementation: Private Information Only

Let us imagine, for the moment, that $u_0 \rightarrow -\infty$, so that the IR constraints (15) and (16) do not bind. In this case, the only relevant constraint is given by the IC condition (13). Let $Y = [0, \bar{y}] \subset \mathcal{R}_+$, with $0 < y^* \equiv$

⁴That is, assume that individuals can commit to anything other than telling the truth (about their type).

$\arg \max W(y) < \bar{y} < \infty$. Let $E = [e_L, e_H] \subset \mathcal{R}$, with $-\infty < e_L < 0 < e_H < \infty$. Note that the lower bound can be made arbitrarily small, and the upper bound arbitrarily large. Define the constraint set:

$$A \equiv \{(e, y) \in E \times Y : 2\alpha\beta e - g(y) \geq 0\}$$

Clearly, A is convex, compact, and non-empty; i.e., since W is concave, we are dealing with a concave program.

Now, define the function:

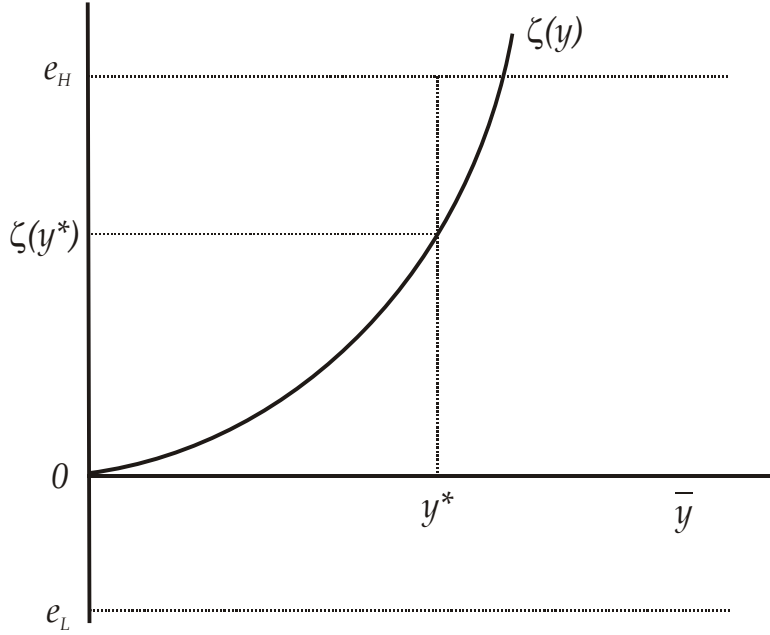
$$\zeta(y) \equiv \frac{g(y)}{2\alpha\beta}. \tag{17}$$

The function $\zeta(y)$ gives us the e that, for any given level of y , makes a stage 2 producer just indifferent between telling the truth and telling a lie. Clearly then, any $e \geq \zeta(y)$ will induce a truthful report. Observe that $\zeta(0) = 0$ and $\zeta''(y) < 0 < \zeta'(y)$. We can therefore establish the following result:

Proposition 1: *In an economy where the IR constraints (15) and (16) do not bind, y^* is implementable for any $e \geq \zeta(y^*) > 0$.*

The intuition here is simple. The mechanism is able to elicit a truthful report from the stage 2 producer simply by promising him a sufficiently large reward at the beginning of the next period's stage 1 subperiod. Of course, this reward is financed by those individuals in the stage 1 subperiod who were consumers in the last period's stage 2 subperiod. By assumption, it is individually-rational for these latter agents to make the necessary transfer. Figure 1 depicts the solution to this problem diagrammatically.

Figure 1
Implementation: Private Information



6 Implementation: Lack of Commitment Only

In this section, imagine that the mechanism can costlessly observe whether individuals are producers or consumers in the stage 2 subperiod. In this case, we can ignore the IC condition (13) and focus on the IR conditions (15) and (16). Naturally, for this to be interesting, we have to assume that hell is not particularly painful.

To begin, define the sets B and C as follows:

$$B \equiv \{(e, y) \in E \times Y : -\alpha e + W(y) \geq W^A\};$$

$$C \equiv \{(e, y) \in E \times Y : -g(y) + \beta[\alpha e + W(y)] \geq \beta W^A\}.$$

Clearly, both B and C are convex compact sets; hence, the constraint set $B \cap C$ is convex, compact, and non-empty. Once again, we are dealing with a concave programming problem here.

Let us now examine the participation constraint (15). Here, we can solve explicitly for $e = \psi(y)$, where $\psi(y)$ denotes the e that renders (15) an equality, given some value for y ; i.e.,

$$\psi(y) = \alpha^{-1} [W(y) - W^A]. \quad (18)$$

One can establish the following properties for ψ :

- [P1]: $\psi(0) = 0$;
- [P2]: $\psi'(0) = \alpha^{-1}W'(0) > 0$;
- [P3]: $\psi'(y^*) = 0$;
- [P4]: $\psi''(y) = \alpha^{-1}W''(y) < 0$.

Note that ψ essentially inherits the properties of W . Observe that we can rewrite the constraint set B as $B = \{(e, y) \in E \times Y : e \leq \psi(y)\}$.

Now, let's take a look at the participation constraint (16). We can solve explicitly for $e = \phi(y)$, where $\phi(y)$ denotes the e that renders (16) an equality, given some value for y ; i.e.,

$$\phi(y) = (\alpha\beta)^{-1} (g(y) + \beta [W^A - W(y)]). \quad (19)$$

One can establish the following properties for ϕ :

- [P5]: $\phi(0) = \phi(y_0) = 0$ for some $0 < y_0 < \bar{y}$;
- [P6]: $\phi'(0) = -(\alpha\beta)^{-1}\beta W'(0) < 0$;
- [P7]: $\phi'(y^*) = (\alpha\beta)^{-1}g'(y^*) > 0$;
- [P8]: $\phi''(y) = (\alpha\beta)^{-1} [g''(y) - \beta W''(y)] > 0$.

Note that ϕ achieves a unique minimum at y_M , implicitly defined as the solution to $g'(y_M) = \beta W'(y_M)$, where $0 < y_M < y_0 < y^*$. Thus, $\phi'(y) < 0$ for all $y \in [0, y_M)$ and $\phi'(y) > 0$ for all $y \in (y_M, \bar{y})$. Clearly, ϕ achieves a maximum at $y = \bar{y}$. Notice that the constraint set C can alternatively be written as $C = \{(e, y) \in E \times Y : e \geq \phi(y)\}$.

6.1 A Special Case: Kehoe and Levine (1993)

Imagine for the moment that $\alpha = 0$. This simple restriction essentially renders the model a version of the one studied by Kehoe and Levine (1993). In this case, the IR condition (15) holds trivially, so we need only consider (16). There are two cases to consider: either the constraint binds, or it does not. If it does not, then $y = y^*$ is implementable. If the constraint does bind, then $y = y_0$, where y_0 is defined implicitly by:

$$g(y_0) = \beta [W(y_0) - W^A], \quad (20)$$

where $0 < y_0 < y^*$. The LHS of this equation represents the immediate utility hit that a producer takes by actually producing goods. The RHS represents the (discounted) capital gain associated with producing as required.

One interpretation of y_0 is that it represents the maximum amount of borrowing (on the part of consumers) that is consistent with maximizing the *ex ante* gains from intertemporal trade without precipitating a default (when they become producers and are in a position to repay their debt). Since y_0 is endogenous, this model generates a so-called endogenous debt-constraint. Krueger and Perri (2005) and Andolfatto and Gervais (2006), among others, demonstrate how exogenous changes to the structure of fiscal policy can alter endogenous borrowing limits.

6.2 The General Case

We now consider the general case in which $\alpha > 0$. Consider, first, the following lemma:

Lemma 1: *There is a unique solution y_0 to $\phi(y_0) = 0$. If $y_0 > y^*$, then y^* is implementable, with $e^* = 0$.*

The proof of this result follows immediately by recognizing that $\phi(y_0) = 0$ implies condition (20). In this scenario, neither IR constraint binds. In what follows then, let us restrict attention to the more interesting case where $y_0 < y^*$.

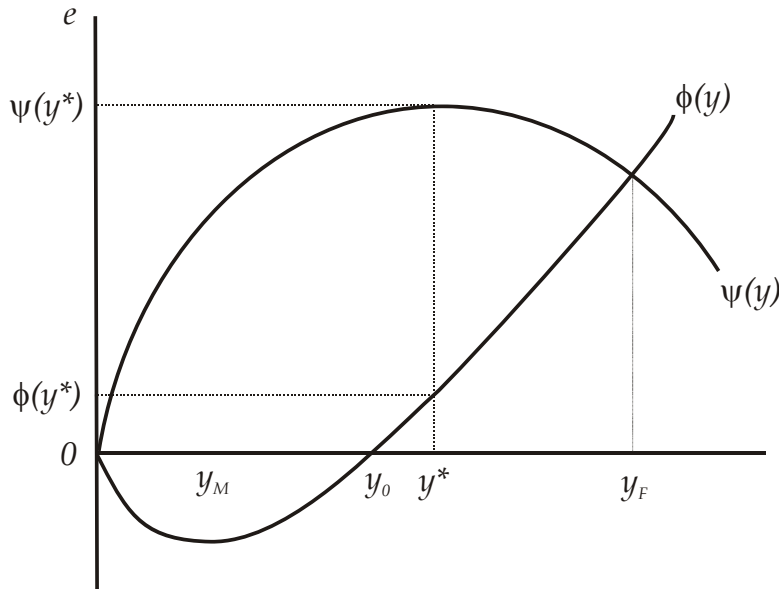
Lemma 2: *Define the function $\Delta(y) \equiv \phi(y) - \psi(y)$. Then $\Delta(y) = 0$ has two solutions: $y = 0$ and $y = y_F$, where $0 < y_0 < y_F < \bar{y}$.*

That $\Delta(0) = 0$ follows immediately from **[P1]** and **[P5]** above. The fact that there is another solution y_F follows from: **[P2]** and **[P6]** ($\Delta'(0) < 0$); **[P4]** and **[P8]** ($\Delta''(y) > 0$); together with the fact that Δ achieves a unique minimum.

Proposition 2: *Assume that $y_0 < y^*$. If $y_F > y^*$, then $0 < \phi(y^*) < \psi(y^*) < \infty$, and y^* is implementable for any $e \in [\phi(y^*), \psi(y^*)]$.*

Here, I will resort to proof by diagram; i.e., see Figure 2.

Figure 2
Implementation: Lack of Commitment



One way to interpret this result is as follows. Imagine that the mechanism is (for some reason) constrained to set $e = 0$. Then the maximum debt limit is given by y_0 . That is, for any $y > y_0$, we have $\phi(y) > e = 0$, which would violate the IR constraint (16). In other words, asking a stage 2 producer to deliver any more output than y_0 would elicit a default, since the one-time utility gain from defaulting would outweigh any benefit from continued program participation. The mechanism could, however, try to ‘bribe’

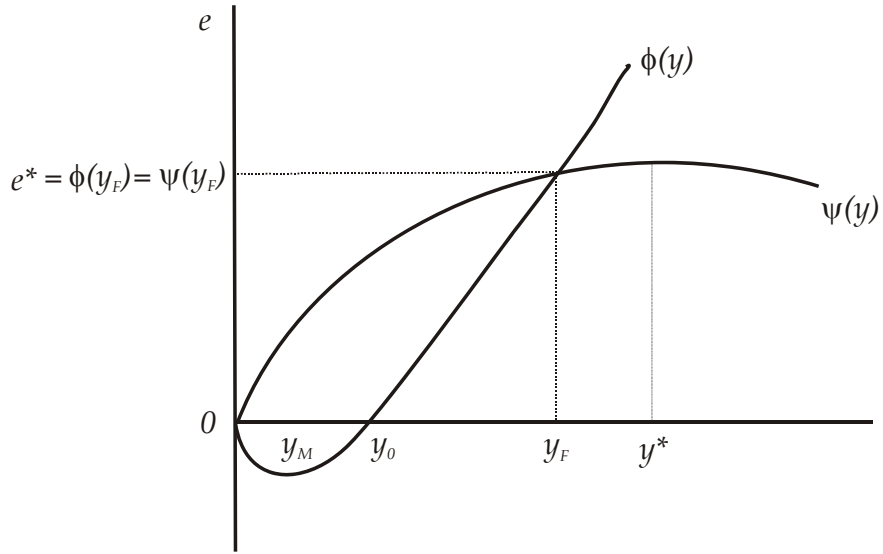
the producer with the promise of a future reward $e > 0$ large enough to compensate for the added pain of producing beyond y_0 . This promised reward must be at least as large as $\phi(y^*)$. But because any reward also constitutes a punishment (to those that must finance it), the mechanism must careful not to set e too high. In particular, any $e > \psi(y^*)$ would elicit a default on the part of stage 1 producers (the previous period's stage 2 consumers). Consequently, to satisfy both (15) and (16) simultaneously, the mechanism must choose an e such that $\phi(y^*) \leq e \leq \psi(y^*)$.

Proposition 2 describes the circumstances under which the optimal allocation can be implemented despite a lack of commitment. Unfortunately, this is not a general result as it depends on $y_F > y^*$; which, in turn, depends on functional forms. The other case to consider is one in which $y_F < y^*$ (which automatically implies $y_0 < y^*$).

Proposition 3: *If $y_F \leq y^*$, then the constrained-optimal allocation is given by y_F together with an $e = e^*$ satisfying $e^* = \phi(y_F) = \psi(y_F) > 0$.*

Figure 3 depicts the situation described in the proposition above.

Figure 3
 Constrained-Efficient Implementation:
 Lack of Commitment



7 Implementation: The General Case

The general case in which the mechanism must deal with both IC and IR is now easily investigated. In constrained-efficient cases, it will always be the case that the IR condition (15) binds and that (generally) *either*: the IR condition (16) binds; *or* the IC condition (13) binds. The proof of this is left as an exercise.

Part III

Monetary Economies

In the environments studied above, a medium of exchange is not *essential* (in the sense that the introduction of any medium would not improve the allocation that could be implemented with an appropriately designed mechanism); see Ostroy (1973), Townsend (1987), Kocherlakota (1998) and Wallace (2001). In other words, a well-designed credit system achieves the constrained-efficient allocation. The reason for this lies squarely on the assumption that the mechanism can *identify* individuals by observing each person's actions (including reports, in the case of private information) and associating with each individual a personalized *history* of such actions. Adding 'frictions' like private information and a lack of commitment simply alters what is potentially implementable; but these frictions in themselves do not make a medium of exchange essential.

Imagine then that individuals are *not* identifiable; that is to say, imagine that individuals are *anonymous*. Since individual identity *is* a personalized history, the assumption of anonymity implies the absence of a record-keeping technology.⁵ In the absence of a record-keeping technology, private debt obligations cannot be collateralized with promises of future acts of redemption (i.e., an anonymous individual can always claim not to have made the promise).⁶ Any trade, if it is to occur at all, must be made on a *quid-pro-quo* basis. Unfortunately, the lack of double coincidence of wants in our environment precludes the *quid-pro-quo* exchange of goods (with anonymity precluding the exchange of claims). Enter then, a potential role for money as a medium of exchange.

8 Equilibrium: Competitive Spot Markets

In this section, I assume away the existence of a coordination mechanism. Instead, I assume that exchange is organized on a sequence of competitive

⁵An alternative way of thinking about this is that while there is a record-keeping technology, individual records can be forged with impunity. In other words, there is a technology that allows for identity theft.

⁶There is a possibility, however, that private debt instruments can be collateralized with physical capital. But as we abstract from capital, this possibility does not exist in the current environment.

spot markets.⁷ Since agents are anonymous, the only feasible trades will involve *quid-pro-quo* exchanges of goods for some other object that does not correspond to any private obligation. This object will be fiat money (an intrinsically useless token representing a claim against nothing of intrinsic value). I assume that there exists M units of money; these tokens are durable, perfectly-divisible, non-counterfeitable, and held in fixed supply. Assume further that each individual is initially endowed with an equal share of the money supply (so that each person initially holds M units of money).

The question that concerns us here is whether this fiat money object can be valued in equilibrium. In other words, can it make sense for individuals to sacrifice goods of intrinsic value in exchange for an intrinsically useless token? The answer to this question is not obvious. What is clear is that in the absence of any money, the only competitive equilibrium entails autarky. The question here is whether the introduction of money might help facilitate trade in any way. If it does, then money has value and is essential for trade. Does there exist a competitive monetary equilibrium and, if so, what are its properties?

To begin the analysis, assume that a competitive spot market exists in each stage of each period. Let p_i denote the price of stage i output ($i = 1, 2$) measured in units of money. These prices need not be stationary and, in what follows, I adopt the convention of denoting future (one-period-ahead) variables with a ‘prime.’ I will, however, restrict attention to stationary allocations (so that p_i/p'_i will be stationary).⁸ For money to have value in equilibrium, it will have to be the case that $p_i < \infty$ at all dates (so that $p_i/p'_i > 0$).

8.1 Stage 1 Decision-Making

Let x denote an individual’s money holdings at the beginning of the stage 1 market and let m denote the amount of money this person takes into the stage 2 market. Individuals are free to purchase or sell output for money

⁷This is similar to Lagos and Wright (2005), except that these authors assume a competitive stage 1 spot market, with trade in the stage 2 market organized as a decentralized search process. Replacing the stage 2 search market as I do here (see also Rocheteau and Wright, 2005 and Berensten, Camera and Waller, 2005) allows me to abstract from bilateral bargaining considerations and does not alter the main conclusions, which hinge on anonymity and not search *per se*.

⁸Lagos and Wright (2003) demonstrate that a rich set of dynamics are possible in this type of environment.

in the stage 1 market at the exchange rate p_1 . If $e > 0$, then the individual accumulates money so that $m > x$. If $e < 0$, then the individual sells money so that $m < x$.

Note that individuals face a constraint here in that $m \geq 0$ (i.e., individuals cannot accumulate negative money balances—or, equivalently, they cannot print their own money or go into debt). Thus, the individual's first-stage budget constraint may be written as:

$$p_1 e = (m - x), \quad (21)$$

along with the CIA constraint $m \geq 0$ (which will turn out not to bind and hence will be ignored in what follows).

Let $W(x)$ denote the utility value associated with beginning stage 1 with x units of money; and let $V(m)$ denote the utility value associated with beginning stage 2 with m units of money. Note that V denotes the value *before* knowing whether one is to be a producer or consumer in the stage 2 market. Of course, both W and V presume optimal behavior on the part of individuals. These value functions must satisfy the following recursive relationship:

$$W(x) = \max_{m \geq 0} \{-\alpha p_1^{-1}(m - x) + V(m)\}, \quad (22)$$

where use has been made of (21).

Assume for the moment that $V'(m) > 0$, $V''(m) < 0$ (we will have to verify later that this is the case) and that $0 < p_1 < \infty$, the stage 1 demand for money m^d is characterized by:

$$\alpha = p_1 V'(m^d). \quad (23)$$

Observation 1: *Note that the desired level of money carried into the stage 2 market (m) is independent of how much money is brought into the stage 1 market (x). The decoupling of optimal behavior from the distribution of money is the product of the quasi-linear preferences.*

From (22), we also see that:

$$W'(x) = \alpha p_1^{-1} > 0. \quad (24)$$

8.2 Stage 2 Decision-Making

Consider a person who brings m units of money into the stage 2 market. At the beginning of this stage, the individual becomes a producer or consumer with equal probability. Let $P(m)$ and $C(m)$ denote the utility value associated with being a producer and consumer, respectively, given that one has m units of money. The *ex ante* value of entering the stage 2 market V is therefore given by:

$$V(m) = \frac{1}{2}P(m) + \frac{1}{2}C(m). \quad (25)$$

8.2.1 Producers

A producer with m units of money can produce y units of output, thereby augmenting his money balances by p_2y dollars. He then begins the next period (in the stage 1 market) with $x'_p = m + p_2y$ dollars. The choice problem can be stated as follows:

$$P(m) = \max_{y \geq 0} \{-g(y) + \beta W(m + p_2y)\}. \quad (26)$$

Assuming that $0 < p_2 < \infty$, the desired supply of stage 2 output y^s is characterized by:

$$g'(y^s) = (p'_1)^{-1}p_2\alpha\beta, \quad (27)$$

where here, use has been made of (24). From (26), we also see that:

$$P'(m) = (p'_1)^{-1}\alpha\beta, \quad (28)$$

where, again, we have made use of (24). Note that by utilizing (27), this latter expression may alternatively be expressed as:

$$P'(m) = p_2^{-1}g'(y^s). \quad (29)$$

The interesting thing to note here is that, if money is valued, then producers actually have an incentive to reveal themselves as producers and undertake costly production. Note that there is no mechanism here asking producers to reveal themselves with the promise of future output e . The ‘promise’ of future output here is embedded in the expectation that money will be valued in the future; i.e., that $p'_1 < \infty$. In this sense, a competitive spot market can be thought of as a clever mechanism that gets producers (with private information) to reveal themselves willingly (i.e., it is only by the display of output for sale that they can accumulate money).

8.2.2 Consumers

A consumer with m units of money can purchase up to $p_2^{-1}m$ units of output. This person begins the next period (in the stage 1 market) with $x'_c = m - p_2y$ dollars, where $x'_c \geq 0$. The choice problem here can be expressed as follows:

$$C(m) = \max_{y \geq 0} \{u(y) + \beta W(m - p_2y) + \lambda[m - p_2y]\}, \quad (30)$$

where $\lambda \geq 0$ represents a Lagrange multiplier associated with the stage 2 CIA constraint. If this constraint binds, then the demand for stage 2 output is given by:

$$y^d = \frac{m}{p_2}; \quad (31)$$

with an associated multiplier:

$$\lambda^d = p_2^{-1} \left[u'(y^d) - (p'_1)^{-1} p_2 \alpha \beta \right] > 0. \quad (32)$$

If the constraint is slack ($\lambda^d = 0$), then y^d is the solution to:

$$u'(y^d) = (p'_1)^{-1} p_2 \alpha \beta. \quad (33)$$

From (30), we also have:

$$\begin{aligned} C'(m) &= (p'_1)^{-1} \alpha \beta + \lambda^d; \\ &= p_2 g'(y^s) + \lambda^d, \end{aligned} \quad (34)$$

where, once again, use has been made of (24), along with (28).

8.3 Equilibrium

Consider the stage 1 market first. In equilibrium, the demand for money must correspond with its supply; i.e.,

$$m^d = M. \quad (35)$$

Hence, from (23), we have:

$$\alpha = p_1 V'(M). \quad (36)$$

Anticipating that the stage 2 debt-constraint $x'_c \geq 0$ will bind (need to verify), we have $y^d = m/p_2$, with $m = m^d$ determined in the earlier stage 1

market. Combining this with (35), we have $y^d = M/p_2$. Market-clearing in the stage 2 market requires:

$$y^s = y^d = \hat{y}; \quad (37)$$

which implies:

$$\hat{y} = \frac{M}{p_2}. \quad (38)$$

To proceed, we need to have information about $V'(m)$. From (25), we know:

$$V'(m) = \frac{1}{2}P'(m) + \frac{1}{2}C'(m).$$

From (28) and (34), we can write this as:

$$V'(m) = \frac{1}{2}(p'_1)^{-1}\alpha\beta + \frac{1}{2}\left[(p'_1)^{-1}\alpha\beta + \lambda^d\right].$$

Utilizing our expression for the multiplier (32), we then have:

$$\begin{aligned} V'(m) &= (p'_1)^{-1}\alpha\beta + \frac{1}{2}p_2^{-1}\left[u'(y^d) - (p'_1)^{-1}p_2\alpha\beta\right]; \\ &= \frac{1}{2}\left[p_2^{-1}u'(y^d) + (p'_1)^{-1}\alpha\beta\right]. \end{aligned}$$

At this stage, we are in a position to confirm whether an earlier conjecture holds. In particular, recall that we have assumed that the solution to (22) is at an interior. In other words, we assumed that $m^d > 0$ solves (23); or given what we now know about V' ,

$$\alpha = p_1 \frac{1}{2} \left[p_2^{-1} u'(y^d) + (p'_1)^{-1} \alpha \beta \right],$$

where (according to 31), $y^d = m^d/p_2$. Assuming strictly positive (and bounded) price-levels, a sufficient condition to guarantee that $m^d > 0$ is the imposition of an Inada condition; i.e., that $\lim_{c \rightarrow 0} u'(c) = \infty$. The RHS of the equation above also shows that V inherits the properties of u ; i.e, that our initial conjecture $V'' < 0$ was correct.

Notice that (28) and (29) imply that:

$$(p'_1)^{-1}\alpha\beta = p_2^{-1}g'(y^s). \quad (39)$$

If we substitute this into the expression above, we now have:

$$V'(m) = \frac{1}{2}p_2^{-1}\left[u'(y^d) + g'(y^s)\right].$$

Invoking the equilibrium conditions $m = M$, $y^d = y^s = \hat{y} = M/p_2$, we deduce:

$$V'(M) = \frac{1}{2} \left(\frac{\hat{y}}{M} \right) [u'(\hat{y}) + g'(\hat{y})]. \quad (40)$$

Now, combine (40) with (36), so that:

$$2\alpha p_1^{-1} = \left(\frac{\hat{y}}{M} \right) [u'(\hat{y}) + g'(\hat{y})]. \quad (41)$$

Since this equation must hold at every date, the following must be true as well:

$$2\alpha(p_1')^{-1} = \left(\frac{\hat{y}'}{M} \right) [u'(\hat{y}') + g'(\hat{y}')]. \quad (42)$$

Multiplying both sides of (42) by β gives us:

$$2\alpha\beta(p_1')^{-1} = \beta \left(\frac{\hat{y}'}{M} \right) [u'(\hat{y}') + g'(\hat{y}')].$$

By again making use of (39), we have:

$$2g'(\hat{y}) \left(\frac{\hat{y}}{M} \right) = \beta \left(\frac{\hat{y}'}{M} \right) [u'(\hat{y}') + g'(\hat{y}')]. \quad (43)$$

Equation (43) constitutes a first-order difference equation in \hat{y} . As we are restricting attention to a steady states ($\hat{y} = \hat{y}'$), this equation reduces to:

$$\beta u'(\hat{y}) = (2 - \beta)g'(\hat{y}). \quad (44)$$

Equation (44) characterizes the (stationary) equilibrium level of stage 2 output.⁹ Observe that since $\beta < 1$, we have $u'(\hat{y}) > g'(\hat{y})$. Hence, the equilibrium stage 2 debt constraint binds; i.e., from (32),

$$\begin{aligned} \hat{\lambda} &= p_2^{-1}u'(\hat{y}) - \alpha\beta(p_1')^{-1}; \\ &= p_2^{-1} [u'(\hat{y}) - g'(\hat{y})]; \\ &= \left(\frac{\hat{y}}{M} \right) [u'(\hat{y}) - g'(\hat{y})] > 0. \end{aligned}$$

Note that the equilibrium restriction (44) also implies:

⁹Of course, there is another steady-state equilibrium in which money is not valued.

Proposition 4: (*Lagos and Wright, 2005*) *The (stationary) competitive monetary equilibrium level of stage 2 output satisfies $0 < \hat{y} < y^*$, with $\hat{y} \rightarrow y^*$ as $\beta \rightarrow 1$.*

The fact that $\hat{y} > 0$ implies that fiat money improves the allocation away from autarky. However, the fact that $\hat{y} < y^*$ implies that the simple presence of fiat money alone is not enough to guarantee that the resulting equilibrium allocation is ‘first-best.’ Whether a superior allocation can be attained via an appropriately designed mechanism (or monetary policy), remains to be seen. What is clear, however, is that money will be essential in attaining any allocation away from autarky. This is because money is necessary as a substitute for the missing record-keeping technology.

Before moving on, let us characterize the remaining equilibrium variables. Since we know \hat{y} , the equilibrium stage 2 price-level is given by:

$$\hat{p}_2 = \frac{M}{\hat{y}}. \quad (45)$$

Note that this implies that the (gross) real rate of return on money (holding money from stage 2 at date t to stage 2 at date $t+1$) is given by $\hat{p}_2/\hat{p}'_2 = 1$. In other words, the (net) inflation rate is zero. The equilibrium stage 1 price-level can be calculated by exploiting the relationship (27); i.e.,

$$\hat{p}_1 = \hat{p}_2 \frac{\alpha\beta}{g'(\hat{y})}. \quad (46)$$

Again, note that this implies $\hat{p}_1/\hat{p}'_1 = 1$.

The distribution of money holdings at the beginning of stage 1 can be derived as follows. First, we have producers in the stage 2 market, who enter with M dollars and leave with $x'_p = M + \hat{p}_2\hat{y} = 2M$ dollars. Second, we have consumers in the stage 2 market, who enter with M dollars and leave with none (since the CIA constraint binds); i.e., $x'_c = 0$. Thus, at the beginning of stage 1, we have half the population holding all of the money. That is, let $F(x) = \Pr[x' \leq x]$. The equilibrium distribution of money at the beginning of stage 1 is given by:

$$F(x) = \begin{cases} 1/2 & 0 \leq x < 2M; \\ 1.0 & x = 2M. \end{cases} \quad (47)$$

Note, once again, that it is the quasi-linearity of preferences in the LW model that renders this distribution so tractable.

From (22), we see that everyone leaves the stage 1 market with the same amount of money; i.e., $m = M$. Therefore, the stage 1 effort choices are given by:

$$\hat{e}(x) = (M - x)\hat{p}_1^{-1} \text{ for } x \in \{0, 2M\}. \quad (48)$$

In other words, if I was a consumer last period, I need to accumulate cash now (since I may need it again in the future stage 2 market). So, I exert $e(0) = \hat{p}_1^{-1}M$ units of effort, which is consumed by who ever has the cash to pay for my services. The people with the cash to pay for these services are those who produced in the previous stage 2 market. These individuals ‘consume’ (or exert negative effort); i.e., $e(2M) = -\hat{p}_1^{-1}M$. Observe that $0.5e(0) + 0.5e(2M) = 0$ (corresponds to strong budget balance discussed earlier).

8.4 Discussion

As Kocherlakota (1998) and others have emphasized, fiat money is a form of societal memory; i.e., its main purpose is to substitute for a missing (prohibitively expensive) record-keeping technology in environments characterized by private information and a lack of commitment (and a lack of double coincidence). In environments characterized by such ‘frictions,’ a well-designed institution (mechanism) entails the conferment of ‘rewards’ on those who behave ‘well’ (producing a good valued by society when such an opportunity presents itself). Owing to the lack of double coincidence, such rewards must constitute ‘promises’ of future utility. As there is no free lunch, this reward-system must be financed by a corresponding system of ‘punishments’ (conferred on those who, in the past, have benefited from the good acts of others). For this punishment/reward system to be operational, some form of record-keeping is required. In the absence of a sophisticated (and publicly accessible) accounting system, fiat money is essential.

According to this interpretation then, when you go to work for money, you are either fulfilling an implicit obligation to society or accumulating implicit claims against the rest of society. When you spend your money to purchase goods and services from other members of society, society is implicitly rewarding your past good behavior (the work you performed to accumulate the money in the first place).¹⁰ In this sense then, the demand for money is intimately related to the concept of a social punishment/reward

¹⁰Naturally, this interpretation presumes that one cannot easily counterfeit or steal money (and that it is costless to steal or counterfeit histories).

system designed to correct incentives that would otherwise be misaligned owing to frictions like private information and the lack of commitment.

As we have seen above, however, the mere presence of a fiat money instrument together with a system of competitive spot markets does not result in an efficient allocation (even if it does improve the allocation away from autarky). Evidently, money is not a perfect substitute for the missing record-keeping technology. Why not?

The problem here appears not to be the record-keeping ability of money *per se*, but rather the intertemporal price-system induced by the competitive spot markets. The availability of these spot markets improve the allocation away from autarky. But this is still an environment in which some markets are missing. In particular, the relevant missing market here is a private debt market. If such a market was available, consumers in the stage 2 market could issue a risk-free private debt instrument to pay for their goods. This debt instrument would become due at the beginning of the stage 1 market. The equilibrium (gross) real rate of interest on such a debt instrument would be:

$$\frac{p_2}{p_1} = \frac{g'(y^*)}{\alpha\beta} = \frac{u'(y^*)}{\alpha\beta}.$$

In this equilibrium, money would not be essential. Of course, what prevents such a debt market from operating is anonymity and the lack of commitment.

In our monetary equilibrium, the (gross) real rate of return on money held from (stage 2) period t to (stage 1) period $t + 1$ is:

$$\frac{\hat{p}_2}{\hat{p}_1} = \frac{g'(\hat{y})}{\alpha\beta} < \frac{u'(\hat{y})}{\alpha\beta}.$$

In other words, the rate of return on money is ‘too low’ in a social sense. This low rate of return on money implies that the demand for real money balances $\hat{y} < y^*$ (or $\hat{e} < e^*$) is too low, so that the equilibrium punishment/reward structure is less effective than it might otherwise be. Note that this rate of return differential (between money and bonds) cannot be arbitrated away, as the bond market is missing.

This missing bond market becomes less important as $\beta \rightarrow 1$; i.e., see Proposition 4. But as $\beta < 1$, it is still of interest to ask what (if anything) might be done to improve efficiency in this economy.

9 Optimal Monetary Mechanisms

Let us think about how a mechanism might operate in this environment. By assumption, there is no record-keeping technology. It follows that current allocations cannot be conditioned directly on any history. In particular, a stage 1 choice of e cannot be conditioned directly on any action taken in the previous period's stage 2 market. On the other hand, to the extent that money can substitute for this missing memory, it is conceivable that the stage 1 punishment/reward be conditioned on money holdings. Of course, this presumes that individual money-holdings at the beginning of stage 1 are observable. Let us assume that this is the case.

Alright, suppose that the mechanism can observe money holdings at the beginning of stage 1. I suppose that there is no point in allowing the mechanism to observe anything in the stage 2 market as, by assumption, such information is nonstorable (apart from the money balances people bring with them to the next period). Still, trade must be organized in some manner in the stage 2 market. It appears that we have a number of options before us. Each of these options boils down to specifying the nature of the extensive form of the 'exchange game' to be played by producers and consumers in the stage 2 subperiod.

One option is to imagine that exchange in the stage 2 subperiod is organized as a competitive spot market. Now, the willingness of producers to accumulate cash will depend on what they expect to receive as a future reward. Since the mechanism can observe cash balances x at the beginning of stage 1, we can construct a punishment/reward function $e(x)$. From what we have learned earlier, it will be necessary to reward those with above average cash balances (and punish those with below average cash balances). As we are free to make $e(x)$ differentiable, let $e'(x) < 0$. In fact, let us be more specific and assume that:

$$e(x) = (M - x)\xi, \tag{49}$$

where ξ is a parameter to be chosen by the mechanism. Note the resemblance of this function with (48). Here, ξ reflects the purchasing power of money (in stage 1).

To rebalance money-holdings at the end of stage 1, the mechanism will also have to specify a 'money-transfer' function $\tau(x)$. In this way, the money that agents take into the stage 2 market is given by:

$$m = x + \tau(x).$$

Note that while $\tau(x)$ cannot be made conditional on any observed history (beyond what can be inferred by x), it can be made conditional on *contemporaneous* observations; in particular, on the stage 1 choice of effort $e(x)$. To guarantee that $m = M$, we need:

$$\tau(x) = M - x.$$

The value of beginning stage 1 with x units of money must satisfy:

$$W(x) = -\alpha e(x) + V(M),$$

so that:

$$W'(x) = \alpha \xi > 0.$$

Stage 2 decision-making is similar to what we have described earlier; i.e.,

$$\begin{aligned} g'(y^s) &= p_2 \beta W'(M + p_2 y^s); \\ u'(y^d) &= p_2 \beta W'(M - p_2 y^d) + \lambda^d; \end{aligned}$$

or, substituting in for W' ,

$$\begin{aligned} g'(y^s) &= p_2 \alpha \beta \xi; \\ u'(y^d) &= p_2 \alpha \beta \xi + \lambda^d; \end{aligned}$$

Now, define $p_2^* = M/y^*$, where y^* is the solution to $u'(y^*) = g'(y^*)$. Given this, supply is determined by:

$$g'(y^s) = p_2^* \alpha \beta \xi = \frac{M}{y^*} \alpha \beta \xi.$$

Note that the mechanism can elicit an efficient supply response by choosing:

$$\xi^* = \frac{g'(y^*)y^*}{\alpha \beta M}. \quad (50)$$

Consider now the consumer. If the mechanism can make $y^d = y^*$, then the CIA constraint will be slack (so that $\lambda^d = 0$). As it turns out, setting ξ according to (50) accomplishes just this; i.e.,

$$u'(y^*) = p_2^* \alpha \beta \xi^* = g'(y^*).$$

Thus, with $\xi = \xi^*$, we have $y^s = y^d = y^*$ and $p_2^* = M/y^*$.

It follows then that stage 2 producers leave the market with $2M$ dollars, while stage 2 consumers leave with 0 dollars. According to (49) and (50) then,

$$\begin{aligned} e(0) &= \frac{u'(y^*)y^*}{\alpha\beta} = \frac{g'(y^*)y^*}{\alpha\beta} > 0; \\ e(2M) &= -e(0). \end{aligned}$$

Thus, the mechanism promises asks past consumers to make a contribution of effort equal to $e(0)$. In return for making this effort, the mechanism promises a monetary transfer $\tau(0) = M$. This implies a stage 1 exchange rate equal to:

$$p_1^* = \frac{M}{e(0)} = \frac{M}{y^*} \frac{\alpha\beta}{g'(y^*)} = p_2^* \frac{\alpha\beta}{g'(y^*)}.$$

At the same time, the mechanism promises past producers a stage 1 payoff equal to $e(2M)$. In return for this payoff, the mechanism asks for a monetary contribution (makes a negative transfer) equal to $\tau(2M) = -M$. The two questions we have to answer here are: [1] Do past consumers have an incentive to give up $e(0)$ for M ; and [2] Do past producers have an incentive to give up M for $e(2M)$?

To answer these questions, imagine first that all agents are following the mechanism's recommendation $(e(0), y^*)$. We then ask whether one agent has an incentive to deviate from this recommendation. As agents are of measure zero, any individual deviation will have no affect on aggregates.

Next, we have to ask what sort of punishment may be inflicted on deviants by the mechanism. Individual trading histories are unobservable. However, the mechanism can observe money balances and stage 1 actions. Thus, if the mechanism observes money-holdings equal to zero, but no effort $e(0)$, it gives no money transfer to the agent. If the mechanism observes no monetary contribution from those agents who have $2M$ dollars, they receive no goods transfer. If the mechanism observes any stage 1 money-holdings different from either 0 or $2M$, the mechanism can 'shut out' the agent from any stage 1 trade. However, note that the mechanism cannot prevent these latter agents from trading in the stage 2 market.

Rather than prove things formally (something that should be done), let me provide the following intuitive argument. First, consider an agent with $x = 0$ money balances; this person is asked to provide effort $e(0)$ in exchange for M dollars. What would be the motivation for not following

this recommendation? Well, he forgoes the immediate utility hit of $-\alpha e(0)$. On the other hand, he enters the stage 2 market with no money balances. If $u_0 \rightarrow -\infty$, then this person will never want to subject himself to the risk of being a consumer without money. Assuming that preferences are restricted in this manner, following the mechanism's recommendation is individually rational.

Next, consider an agent with $x = 2M$ money balances; this person is asked to provide M dollars in exchange for $e(2M)$ units of goods. What would be the motivation for not following this recommendation? Well, he gets to keep the M dollars, so that he enters the stage 2 market with $2M$ dollars. On the other hand, he foregoes the immediate utility benefit $\alpha e(2M)$. In the stage 2 market, the person turns out to be either a consumer or a producer. Let us consider each of these cases in turn.

Suppose that the person turns out to be a producer. He now has $2M$ dollars and has the opportunity to accumulate even more money. Let us ask: what is the point of collecting money in the stage 2 market in the first place? The reason: to be rewarded in the future stage 1 market. But by accumulating balances beyond $2M$, the producer cannot be expected to be rewarded in such a manner (the mechanism will shut him out). And so, the best the producer can do is to produce nothing (which saves him the utility cost of producing). So, a part of the benefit associated with foregoing utility $\alpha e(2M)$ is that (with probability one-half), the agent avoids the utility cost $-g(y^*)$.

Suppose instead that the person turns out to be a consumer. He now has $2M$ dollars to spend on output (while everyone else only has M dollars). So, one of the benefits here is that the consumer may potentially consume more than y^* . But is this potential benefit worth the cost? In particular, note that $u'' < 0$ so that there are diminishing returns to consuming more. Is the marginal utility gain associated with this extra consumption worth foregoing the initial utility hit of $-\alpha e(0)$? I'm not sure, but my intuition tells me 'no.' Of course, my intuition is not a substitute for a proof.

10 The Friedman Rule

In the previous section, I assumed that the stage 1 swaps of money for goods was coordinated by a mechanism. Alternatively, we might suppose that exchange is governed by a competitive spot market. However, we already

know that left on their own, competitive spot markets will not implement an efficient allocation. The basic problem was that money is not valued as highly as it should be, so that the demand for real money balances was suboptimal. One way to potentially increase the demand for money is to increase its rate of return. If money does not pay interest, the only way to do this is by contracting the supply of money. Contracting the supply of money would require a mechanism to collect money from agents at the beginning of each stage 1 market and destroying it (removing it from circulation). Ideally, one would like to design a system whereby individuals made such contributions voluntarily. The far more common approach in the literature is to assume that the mechanism can forcibly confiscate (i.e., tax) money-holdings. In this case, the mechanism is called a government. But one may alternatively assume that individuals can commit to make such transfers. In this case, we might call the mechanism a bank (that lends individuals M dollars and asks them to repay the money loan in a sequence of decreasing installments). Either approach is designed to avoid the problem of worrying about the sequential IR constraints. In what follows, let's proceed in this manner.

To proceed then, assume that the mechanism 'lends' each individual M dollars at the beginning of time. In each subsequent (stage 1) period, the money supply is contracted at (gross) rate z ; i.e.,

$$M' = zM.$$

This implicitly defines the lump-sum contribution (tax) required of each person at the beginning of each (stage 1) period; i.e.,

$$\tau = (z - 1)M.$$

Note that if $z > 1$, then the mechanism is injecting money by way of lump-sum transfers, rather than destroying money by way of lump-sum taxes. We anticipate, however, that optimality will require $z < 1$. In either case, the stage 1 budget constraint (21) is modified as follows:

$$p_1 e = (m + \tau - x).$$

Everything that we have done earlier concerning individual decision-making remains the same. The only change comes in the specification of equation (42); i.e.,

$$2\alpha(p'_1)^{-1} = \left(\frac{\hat{y}'}{M'} \right) [u'(\hat{y}') + g'(\hat{y}')],$$

where M is now replaced by M' . This implies that condition (43) becomes:

$$2g'(\hat{y}) \left(\frac{\hat{y}}{M} \right) = \beta \left(\frac{\hat{y}'}{zM} \right) [u'(\hat{y}') + g'(\hat{y}')].$$

Imposing stationarity $\hat{y} = \hat{y}'$ and simplifying, yields:

$$\beta u'(\hat{y}) = (2z - \beta)g'(\hat{y}).$$

Note that $\hat{y} = y^*$ if $z^* = \beta < 1$. In other words, the mechanism can implement the optimal allocation by contracting the money supply at the discount rate; this is the celebrated Friedman rule.¹¹

¹¹Another policy that would work here is to pay interest on money, with interest payments financed by way of taxes (instead of printing new money).

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