Money, Capital, and Banking

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1 The Environment

Let me consider a variant of the OLG model that we studied earlier. Let $t \in T \equiv \{0, 1, 2, ..., \infty\}$ and let $y \in Y \subseteq \mathbb{R}_+$, where Y is a compact set. Let $F : Y \to [0, 1]$ be a continuous cumulative distribution function.

An agent's type is denoted by the pair $(t, y) \in T \times Y$. The interpretation here is that a type (t, y) agent has preferences

$$U_t = c_{t+1}; \tag{1}$$

and an endowment of output y. In our earlier model, we assumed that there was a labor-leisure choice, so that agents could choose how much output to generate when young. In this setup, we simply assume that agents are endowed with output when 'young.' Moreover, there is a distribution of income across young agents given by the c.d.f. $F(y)$. I also make the simplifying assumption here that agents are risk-neutral (there is no desire for insurance).

There is also a storage technology such that k units of output invested at date t yields xk units of output at date $t+1$. The capital investment k depreciates fully after it produces its dividend. The parameter $x > 1$ represents the (expected) productivity of capital investment; it is the (future) marginal product of capital. It also represents the (gross) real rate of return on investment; and so, you can think of it as the real interest rate.

To motivate the need for an intermediary, assume that no single agent himself has enough income to exploit the storage technology (i.e., a miniumum level of capital investment is needed, and this minimum level far exceeds the resources any any single agent). Investment requires the coordinated actions of a measurable group of young agents; refer to this coalition as a bank.

Assume that the bank has a commitment technology. That is, an agent making a contribution of capital to the bank at date t is guaranteed a promised return at date $t+1$. However, at date $t+1$, individual depositors of capital to the coalition at date t cannot be relied upon to report their investment truthfully. The bank, however, is also in possession of a record-keeping technology. When a deposit of capital is made at date t, an electronic account is opened on behalf of the depositor, the deposit recorded, and a PIN is assigned that matches the depositor with the account. This record-keeping technology is costly to operate; each account holder must pay a fixed cost $\alpha > 0$ to participate. For simplicity, assume that this fixed cost is incurred at date $t + 1$.

There is also a substitute record-keeping technology available in the form of durable, divisible, and non-counterfeitable tokens. Only society (the government) has the ability to issue such tokens. These tokens may conceivably be used to purchase output at any date. There is no fixed cost associated with using tokens (cash) in any exchange.

Assume that the initial old are endowed with M_0 units of cash money. Moreover, assume that the government expands the supply of cash at the constant rate $\mu \geq 1$; so that $M_t = \mu M_{t-1}$. Assume that new money is used to finance government consumption expenditures.

2 Competitive Monetary Equilibrium

Let v_t denote the value of cash at date t and assume that $v_t > 0$ for all t (this assumption will hold true in an competitive monetary equilibrium). Define $R_{t+1} \equiv v_{t+1}/v_t$; i.e., the real gross rate of return on cash (or the inverse of the gross inflation rate). In what follows, I restrict attention to stationary equilibria; i.e., assume that $R_{t+1} = R$ for all t. Moreover, as we can anticipate that $R = 1/\mu$ in equilibrium, it follows that $R < 1 < x$. In other words, capital (bank deposits) earns a higher rate of return than cash.

The initial old face a trivial choice problem; i.e.,

$$c_1 = v_1 M_0. \tag{2}$$

Consider now the choice problem facing a young agent endowed with y units of output. As the young do not value consumption, they will want to save all their income. In this model, there are two ways in which this saving may occur: they can either sell their output for cash or deposit it in the bank. The optimal choice will depend on which saving instrument yields the higher rate of return, net of any fixed costs.

Selling output y for cash at date t yields a return Ry at date $t+1$. Investing output y as a bank deposit at date t yields a return $xy - \alpha$ at date $t+1$. Hence, if $Ry > xy - \alpha$, the young agent is better off selling his output for cash; and if $Ry < xy - \alpha$, then he is better of depositing his output in the bank. In general, there will exist a reservation income level $\hat{y}$ that satisfies $R\hat{y} = x\hat{y} - \alpha$; or

$$\hat{y} = \left(\frac{\alpha}{x - R} \right). \tag{3}$$

That is, a young agent with output $\hat{y}$ is just indifferent between using cash or bank deposits as a means to acquire future consumption.

Exercise 1 *Explain (do not just describe) how $\hat{y}$ is affected by R, x and α (you can think of a decrease in α as a technological advancement in the electronic payments system).*

To derive the equilibrium R, note that the aggregate demand for real cash balances is given by

$$Q(R) \equiv \int_0^{\hat{y}(R)} y dF(y).$$

The market-clearing conditions are given by,

$$v_t M_t = Q(R) \tag{4}$$

for all $t \geq 1$. Hence, it follows that $R^* = 1/\mu$. The equilibrium value of money (the price-level) at any date t can then be determined as

$$v_t^*(\mu) = \frac{Q(\mu^{-1})}{M_t}. \tag{5}$$

Exercise 2 *Demonstrate that $Q'(R) > 0$ and provide the economic intuition for why an increase in the (expected) inflation rate reduces the aggregate demand for real money balances. What effects does an increase in μ have on the price-level? Explain.*

Exercise 3 *Imagine that the economy is in a stationary equilibrium. Now imagine that at some arbitrary date t, young agents receive information that x has increased. Remember that x represents the return on capital at date $t+1$. How does the arrival of this 'news' shock affect the price-level at date t? What effect does this shock have on the expected inflation rate? Explain.*

2.1 The Money Multiplier

There are two operational measures of the money supply that empirical macroeconomists frequently make use of. The first measure goes by many names: the monetary base, base money, or high-powered money. Empirically, this measure corresponds to the money created by a country's central bank. In the context of our model, we can think of base money as corresponding to the supply of cash M_t.

The second measure of money is broader in scope; it recognizes that there are privately-issued liabilities that are also commonly used in making payments. The most common of these constitute the demand-deposit liabilities created by chartered banks. This broader measure of money, called M1, includes both the monetary base and these demand deposit liabilities. The **money multiplier** is then defined by the ratio $(M1_t/M_t)$.

Empirically, it appears that the money mulitiplier is postively correlated with and leads the real GDP. If we let GDP_t denote the real GDP at date

t, then statistically, what we observe is $cor(M1_t, GDP_{t+1}) > 0$. In words, an increase in the current (broad) money supply is associated with an increase in the future level of real economic activity.

At the same time, it appears that increases in the (broad) money supply are positively correlated with the price-level. In terms of our notation, we can write this statistical relation as $cor(M1_t, v_t) < 0$.

A natural conclusion to draw from these statistical facts is that a monetary authority may be able to engineer a boom in (future) economic activity by increasing the money supply. Alternatively, many appear to interpret these correlations as evidence that "money supply shocks" lead to changes in real economic activity. But are these beliefs or conclusions necessarily true? Let us try to answer this question in the context of the model above.

The monetary base in our model is given by M_t. The real value of bank-deposit liabilities is given by

$$K_t \equiv \int_{\hat{y}(R)}^{\overline{y}} ydF(y).$$

Hence, M1 is given by,

$$M1_t \equiv M_t + v_t^{-1} K_t.$$

Using condition (5), this latter expression can alternatively be expressed as,

$$M1_t \equiv M_t + \left[\frac{M_t}{Q_t} \right] K_t;$$

or, by rearranging terms,

$$M1_t \equiv \left[1 + \frac{K_t}{Q_t} \right] M_t.$$

Note that the term in the square brackets is the money mulitiplier. Evidently, the money multiplier is positively realated to the **deposit-to-cash ratio** K_t/Q_t.

Let $Y_t \equiv \int_0^{\overline{y}} ydF(y)$. Then the real GDP at date t is given by,

$$GDP_t = Y_t + x_t K_{t-1}.$$

Exercise 4 *Demonstrate that an exogenous one-time increase in M_t has absolutely no effect on the real GDP (and that is serves only to cause the price-level to jump permanently an in proportion to the increase in the supply of base money).*

Exercise 5 *Demonstrate that a permanent increase in the rate of money growth μ leads to a permanent increase in the real GDP. Explain what is going on here. Hint: Tobin effect.*

Now, imagine that the monetary authority remains completely passive; i.e., it holds μ constant over time. Imagine further that x_t fluctuates over time; i.e., that the economy is subject to 'real' shocks rather than 'nominal' shocks. What does our model predict?

As of date t, x_t is already given. Since K_{t-1} was determined in the previous period, and since Y_t is exogenous, it follows that the GDP_t is fixed. But now imagine that at date t, young agents receive 'news' that the future return to capital (the real interest) is likely to be higher (i.e., an increase in x_{t+1}). How do agents respond to this information and what are the implications of their behavior?

Our model implies the following. The increase in the (expected) real rate of interest means that agents will substitute out of cash and into bank deposits. This reduces the real demand for cash and increases the real demand for deposits; so that the deposit-to-cash ratio K_t/Q_t rises. The effect of this response is to increase $M1_t$ and the money mulitplier at date t. Essentially, the money supply increases because banks are creating more money (the monetary base remains constant).

At the same time, because Q_t falls, it follows that v_t falls as well. In other words, the effect of this news shock is to increase the price-level at date t. Hence, $cor(M1_t, v_t) < 0$ just as in the data.

What is the effect on the future GDP? Consider that

$$GDP_{t+1} = Y_{t+1} + x_{t+1}K_t.$$

In this model, $Y_t = Y_{t+1} = Y$, by assumption. But note that because both x_{t+1} and K_t are higher than before, the real GDP is expected to increase at date $t+1$. Hence, $cor(M1_t, GDP_{t+1}) > 0$ just as in the data.

Our theoretical framework here teaches us an important lesson; i.e., not to confuse statistical correlation with causation. It is not true in the example considered here that an increase in M1 'causes' a future increase in GDP. In fact, the **direction of causality is reversed**: it is the expected increase in future GDP (productivity) that causes the money supply (and price-level) to increase today.

Exercise 6 *Can you think of other examples of reverse causality? Hint: Christmas shopping and Christmas day.*

Exercise 7 *Imagine that an economy (like Japan since 1990) is constantly subject to 'bad news shocks' in the sense that the forecast of x_{t+1} is revised downward for many consecutive periods in a row. What does our model predict in terms of GDP growth, inflation, and the money multiplier?*

Exercise 8 *In difficult economic times, it is frequently alleged that there is a 'flight to quality;' that is, that investors start dumping their private securities in* *exchange for safer government securities. This behavior is commonly thought to be deflationary. Explain how our model is consistent with this idea.*

3 Demandable Liabilities and Banking Panics

A distinguishing characterisitic banks (say, in relation to other financial intermediaries) is that they issue demandable liabilities. In modern economies, bank liabilities are made demandable for cash. For this reason, banks typically hold cash reserves with which they can use to honor the redemptions that occur every day (as when people visit a bank or ATM to withdraw cash).

This demandable structure of bank liabilities is frequently thought to pose a problem. The problem is that while the bank's liabilities are 'short-term' in nature, their assets are 'illiquid' and long-term in nature. That is, there appears to be a mismatch in the structure of assets and liabilities on the bank's balance sheet. Undoubtedly, this mismatch occurs for a good economic reason (we wouldn't need banks if all assets were perfectly liquid). But on the other hand, there is a feeling that such a mismatch makes a bank vulnerable to speculation. In particular, if everyone runs at once to redeem their bank liabilities for cash, the bank will be forced to liquidate its illiquid assets at firesale prices. This may lead to bank insolvency and in turn, justify the desire to run the bank before it collapses.

In the model described above, our bank does not create liabilities that are redeemable for cash (it simply creates liabilities that are redeemable for future output). Let me now describe a slightly different environment that makes up for this deficiency.

3.1 Environment

The setup is as described above, except with the following modifications. First, imagine that there are two spatially separated locations (no communication is possible across locations). The population, at any given date, is evenly divided between the two locations. The locations are symmetric in every other respect.

As before, young agents only value consumption when old. However, assume now that they are risk-averse, so that $U_t = u(c_{t+1})$; with $u'' < 0 < u'$. Moreover, assume that all young agents now have the same endowment y.

The storage technology is as described earlier (for each location). Assume that $\alpha = 0$; so that there is no cost associated with joining a bank located in the 'domestic' economy (it is impossible to open an account in the 'foreign' bank because communication is impossible across locations).

An important element of this model is as follows. At the end of period t, some fraction $0 < \pi < 1$ of the young population is exogenously relocated to the 'other' location. This 'relocation shock' is private information. Goods are not transportable across locations. Private securities are transportable; but there is no sense in doing so as foreign private securities can be costlessly counterfeited in a domestic location. The only object that is transportable (and non-counterfeitable) is government money.

Finally, in terms of timing, assume that any domestic investment must be made before people learn whether or not they are to be relocated. After investment is put in place at date t, it can be dismantled and liquidated, but at some cost. Assume that the liquidation value is some fraction $0 \leq \theta \leq 1$ of book value.

The monetary authority is completely passive. It simply creates M units of money; this is endowed to the initial old at each location. The money supply stays constant over time. Hence, the equilibrium real rate of return on cash is fixed at unity (zero inflation).

3.2 An Intermediated Arrangement

In this environment, there are two reasons why young agents (in each location) may want to form a coalition. The first is as in the earlier model; i.e., to exploit the higher return storage technology. But there is another reason here that relates to the desire for insurance. That is, *ex ante*, each young agent faces a probability of relocating to the foreign location. Money held in a local bank is not usable in the foreign location; only cash is. Hence, to insure themselves against the risk of relocation, young agents will want to pool their resources in a way that helps them smooth their consumption across contingencies.

The way that the coalition bank can organize itself is as follows. First, the bank needs to collect deposits of cash to hold in reserve. The cash reserves will be used for those agents who subsequently realize that they need to move (they will want to withdraw cash for this purpose). The bank also needs to collect resources to finance capital investment.

The most direct way for this to happen is for all young agents to deposit their income y with the bank. The bank can then sell a part of these resources for cash (from the exisiting old); it can use the rest to finance capital investment k. If we let q denote the quantity of real money balances held as cash reserves for the bank, then it faces the following constraint:

$$q + k = y.$$

Remember that cash earns a rate of return equal to 1, while capital earns a rate of return equal to $x > 1$.

In return for a deposit of y, the bank offers a state-contingent liability. The fraction π of its depositors will realize a relocation shock; or, something that might more appropriately be labeled a 'liquidity shock.' The realization of this shock is private information. This means that the bank liability must be made contingent on individual reports of their state. In short, the liability must be made demandable. The bank liability essentially promises a rate of return on deposits equal to 1 if the deposit is liquidated early; and a rate of return $x > 1$ if the deposit is held to maturity (the next period).

Let $c(s)$ denote the future consumption of an agent who realized state $s \in \{1, 2\}$, where $s = 1$ means 'stay' and $s = 2$ means 'move.' Then the bank faces the following resource constraint:

$$xk + q \geq (1 - \pi)c(1) + \pi c(2).$$

In addition to this, the bank faces a cash-constraint. In particular, aggregate withdrawals of cash cannot exceed the bank's cash reserves; i.e.,

$$q \geq \pi c(2).$$

Since cash is dominated in rate of return, and since the bank can forecast perfectly the number of young agents who will need cash (without knowing beforehand the identity of who needs the cash), it follows that it will be optimal for the bank to minimize its cash reserves so that $q = \pi c(2)$. In turn, through the resource constraint, this implies that $xk = (1 - \pi)c(1)$.

The question now turns to the issue of how the bank should divide its resources y between cash reserves and capital investment. Since the young depositors are *ex ante* identical, they have an expected utility function given by,

$$U = (1 - \pi)u(c(1)) + \pi u(c(2)).$$

Substituting the constraints into this objective, the choice problem for the bank can be written as follows:

$$\max_q (1 - \pi)u\left(\frac{x(y - q)}{1 - \pi}\right) + \pi u\left(\frac{q}{\pi}\right).$$

The associated FOC is given by,

$$xu'\left(\frac{x(y - q^*)}{1 - \pi}\right) = u'\left(\frac{q^*}{\pi}\right).$$

Once the optimal cash reserves q^* are determined in this manner, all remaining variables can be deduced from the constraints.

Exercise 9 *Demonstrate that those agents who experience a liquidity shock consume less than those who do not.*

While this model is still highly abstract, the bank that emerges in this setting resembles actual banks in many important ways. First, a bank issues demand deposits redeemable in cash; and a bank holds cash reserves to meet these expected redemptions. Second, the bank uses its liabilities to purchase capital investment. The liabilities that the bank issues are backed in part by the cash it holds and in part by the capital it holds. As well, bank liabilities generally earn interest; at least, if people keep their 'money' in the bank for a period of time.

3.3 Modeling a Bank Run

Imagine that the economy operates exactly in the manner described above for a long period of time. Then in one period, following the regularly planned investment, something completely unforseen happens. Imagine that as the young realize their liquidity shocks, that all agents suddenly 'hear a rumor' that the bank's capital investment is not sound (the expected return to capital investment falls to $x' < 1$).

At this point, those agents that experience a liquidity shock will be visiting the bank (or ATM) to make a withdrawal of cash. But those agents that do not have a pressing need for liquidity must also be thinking about withdrawing their cash early. In particular, if the expected return on the bank's capital is indeed $x' < 1$, then withdrawing cash and 'stuffing it under the mattress' will offer a better rate of return.

Imagine then that all agents–acting in their own self-interest–rush to withdraw cash. But there is a little problem here: the bank does not have enough cash to honor all such redemptions. It might try selling some of its capital in a effort to raise cash, but by assumption all the cash in this economy is at this point in time already residing in the banking system. The only thing the bank can do is to start scrapping its capital. By assumption, it can re-convert this capital into output at some cost; i.e., k^* units of investment will yield θk^* units of output. This output can then be handed over to depositors, who can then use it for consumption in the next period (at least, those depositors who do not move). We can assume that this scrapped capital continues to earn a rate of return $x > 1$. However, the effective rate of return (taking into account the scrapping costs) is given by $x' = \theta x$; which I assume here is less than one.

To conclude, it appears possible here that the simple rumor that $x' < x$ can result in behavior that makes the rumor become a self-fulfilling prophesy.