Econ 807: Midterm 2
09 March 2001

David Andolfatto Name_____

INSTRUCTIONS: **Please write neatly and label all diagrams clearly; if I cannot read your answer, then it is wrong. Limit your answer to the space provided below each answer (do not write on the back of the exam paper and do not hand in material that is on other paper). Note: in deriving expressions, you do not have to show me all your rough work; just write down the key steps in your derivation. The exam is worth 25 marks. Time allotted: 90 minutes.**

1. [10 Marks] Consider an individual with preferences for time-dated consumption given by: $U = \ln c_1 + \beta \ln c_2$. The individual is endowed with a known earnings profile (y_1, y_2) and can borrow or lend at a risk-free (gross) real rate of interest $R > 0$.

   (a) Derive the first-period consumption demand and show that it takes the form of a 'Keynesian' consumption function: $c_1 = a + by_1$, where $a > 0$ and $1 > b > 0$.

   (b) Derive expressions for the marginal propensity to consume out of current income, first under the assumption that the income change is temporary $(\Delta y_1 > 0 = \Delta y_2)$ and second, under the assumption that the income change is permanent $(\Delta y_1 = \Delta y_2 = \Delta y > 0)$. Explain any differences.

2. [10 Marks] A person has preferences for current and future consumption given by $U = \ln c_1 + \beta c_2$, where $\beta > 0$. As well, the person owns and operates a production technology $y = A_t k_t$, where $A_t > 0$ and k_t represents capital in place at the beginning of period t. (Assume that k_1 is given and that capital depreciates fully after one period). The productivity parameter A_t follows a two-state Markov process; i.e., let $\Pr[A_{t+1} = A^j \mid A_t = A^j] = \pi > 1/2$, where $A^j \in \{A^L, A^H\}$, $A^L < A^H$.

   (a) Let $R(A^j)$ denote the (gross) expected return to investment when $A_t = A^j$. Show that $R(A^H) > R(A^L)$ and explain.

   (b) Solve for the optimal investment choice and explain how it depends on the current state of technology $A_t = A^j$ (there are two effects to consider: a wealth effect and an expectations effect).

3. [5 Marks] *The 'stimulative' effects of government spending in a 'Keynesian' model.*

Now, sit back and relax on this one. This is a question that is designed to teach you the difference between *description* (i.e., measurement) and *explanation* (i.e., theory). Now, we know from our basic income accounting exercises that one way to measure GDP is to add up the expenditure components; i.e., $Y \equiv C + I + G$ (let's forget about net exports). This expression is *true by definition*. Students sometimes mistakenly reason that, since this expression is true, it must be the case that an increase in G (government purchases) leads to an increase in GDP. Certainly, we were are all exposed to models (i.e., theories) that generated such an implication. However, what is important to realize here is that such a prediction cannot be made solely on the basis of the identity above. In order to make a prediction, one needs a theory and the identity above is just a way of measuring things–it is not a theory. In order to demonstrate this point, let me develop a model in which the identity above holds, but nevertheless predicts that an increase in G results in a decline in Y. Your job is to explain the economic intuition behind this result.

The model is based on the IS-LM-FE analysis that you've likely encountered as an undergrad. Let us consider the labour market first. Labour supply is given by a function $N^s(w)$ and labour demand is given by $N^d(w, R)$. Here, w represents the real wage and R represents the real interest rate. Assume that labour supply is increasing in w and that labour demand is decreasing in w. We will also make the assumption that the demand for labour depends negatively on R (for example, this might be reasonable if the business sector must borrow funds to finance the period's wage bill). For a given interest rate R, we can solve for the market-clearing wage rate by setting $N^s(w) = N^d(w, R)$. We can denote by $N(R)$, the equilibrium level of employment conditional on a given interest rate R. Likewise, we can derive an 'aggregate supply' function by inserting $N(R)$ into the business sector's production function; i.e., $Y^s(R) = F(N(R))$, where F is increasing in the labour input. Convince yourself that $Y^s(R)$ is a decreasing function of R.

Now, let us consider the aggregate demand for output. Consumer demand is given by $C^d = C_0$ (a parameter). Investment demand is given by the function $I^d(R)$, which is decreasing in R. The demand for output from the government sector is given by $G^d = G_0$ (a parameter). Therefore, the aggregate demand for output is given by $Y^d(R, C_0, G_0) = C_0 + I^d(R) + G_0$. Clearly, Y^d is a decreasing function of R and an increasing function of C_0 and G_0. The equilibrium interest rate is given by the R that satisfies the goods-market clearing condition: $Y^s(R) = Y^d(R, C_0, G_0)$. It will be helpful for you to draw each of these functions on a diagram, with Y on the vertical axis and R on the horizontal axis. Assume that the Y^s curve is 'flatter' than the Y^d curve. Show that, for this model, we have $Y = C + I + G$ and $dY/dG < 0$. What is going on here?