# Money and Capital in an OLG Model

D. Andolfatto

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## Environment

- Time is discrete and the horizon is infinite ($t = 1, 2, ..., \infty$)
- At the beginning of time, there is an *initial old* population that "lives" (participates) for one period only
- At the beginning of every time period, a new generation of young agents is born (enter the economy)
- Let $N_t$ denote population of young born at date $t$; assume $N_{t+1} = nN_t$

- Young agents “live” (participate) for two periods; they become old and then die (exit the economy)

- An agent born at date $t = 0, 1, .., \infty$ has preferences $U_t = c_{t+1}$

- Young agents have an endowment $y$

- All agents have access to an investment technology, where $k_t$ units of investment at date $t$ delivers $f(k_t)$ at date $t + 1$

- Capital depreciates fully after it is used in production

- Assume that $f$ is strictly increasing, concave, and satisfies the Inada conditions

## Competitive equilibrium

- Let $r_{t+1}$ denote the real (gross) rate of interest on risk-free debt

- No-arbitrage-condition implies $f'(k_t) = r_{t+1}$

- The young save all their income, so $k_t = y$ and $c_{t+1} = f(y)$ for all $t \geq 1$ (the initial old consume zero)

- This implies an equilibrium real rate of interest $r^e = f'(y)$

- The CE is not Pareto efficient if $r^e < n$ (dynamic inefficiency)

## Gift-giving equilibria

- Suppose that the young are asked to make a gift of output $q \in [0, y]$ to the old

- In return, the young expect to receive $nq$ units of output when they are old

- Payoff is $c = f(y - q) + nq$

- Any $q \geq 0$ such that $f(y - q) + nq \geq f(y)$ can be supported as a Nash equilibrium with *tit-for-tat* strategies

- Note: $q > 0$ iff $f'(y) < n$ (Re: dynamic inefficiency); otherwise, only equilibrium is autarky

- Note: maximum payoff for representative young agent is achieved by setting $q$ to maximize $c = f(y - q) + nq$, so that

$$f'(y - q^*) = n \tag{1}$$

- If $f'(y) < n$, then all such gift-giving equilibria constitute Pareto improvements over competitive equilibrium allocation

- Note: initial old consume $q \geq 0$

## Supporting gift-giving equilibria with government debt

- In what follows, I make no distinction between money and bonds; i.e.,
  - a bond is like interest-bearing money
  - money is like a zero-interest bond
- Let $M_t$ denote the nominal stock of money at date $t$
- One dollar of money held at date $t$ constitutes a risk-free claim for $R_{t+1}$ dollars of money in period $t+1$
- So $R_{t+1}$ is the (gross) nominal interest rate on government money/debt

- Assume that the initial stock of money $M_0$ is distributed evenly across the initial old (so $M_0/N_0$ dollars per initial old agent)
  - note: the old (of any generation) are willing to dispose of their money holdings at any price

- The money supply grows according to $M_{t+1} = \mu_{t+1} M_t$

- There is a government budget constraint

$$(R_t - 1)M_{t-1} = (M_t - M_{t-1}) + T_t$$

$\Rightarrow$ interest on debt is financed by new money creation and/or taxes

- Lump-sum tax (transfer, if negative) $T_t$ is applied to the old

*Rearrange GBC...*

- Rewrite GBC as $R_t M_{t-1} = M_t + T_t$

- From money growth equation, $M_{t-1} = M_t/\mu_t$; plug into GBC above to get

$$T_t = \left[\frac{R_t}{\mu_t} - 1\right] M_t \tag{2}$$

- Note special cases: $(R_t, \mu_t) \in \{(1, \mu), (R, 1), (1, 1)\}$

- Note: usually, the restriction $R_t \geq 1$ is imposed (idea is that currency in circulation cannot be taxed)

*Individual budget constraints*

- Let $p_t$ denote the price level at date $t$

- Budget constraints for representative young agent

$$\begin{aligned} p_t y &= p_t k_t + m_t \\ p_{t+1} c_{t+1} &= p_{t+1} f(k_t) + R_{t+1} m_t - T_{t+1}/N_t \end{aligned}$$

- Define $q_t \equiv m_t/p_t$ (real money balances) and $\tau_{t+1} \equiv T_{t+1}/(N_t p_{t+1})$, then rewrite above constraints as

$$c_{t+1} = f(y - q_t) + (R_{t+1}/\Pi_{t+1}) q_t - \tau_{t+1} \tag{3}$$

- where $\Pi_{t+1} \equiv p_{t+1}/p_t$ is the gross rate of inflation

*Demand for money*

- The demand for real money balances $q_t^d = q(y, R_{t+1}/\Pi_{t+1})$ satisfies (at an interior)

$$f'(y - q_t^d) = \frac{R_{t+1}}{\Pi_{t+1}} \tag{4}$$

- This also implies a demand for capital investment, $k_t^d = y - q_t^d$

- Money/bond demand is increasing in $y$ and $R$, decreasing in $\Pi$

*General equilibrium*

- Money trades for goods on competitive spot market in each period; market-clearing condition implies (for all $t \geq 1$)

$$\frac{M_t}{p_t} = N_t q_t^d \tag{5}$$

- Assume $R_t = 1$, $N_t = N$, $M_t = M$ for all $t \geq 1$ and define $v_t \equiv 1/p_t$

- Combining previous two restrictions...

$$v_{t+1} = v_t f'(y - v_t M) \equiv h(v_t)$$

which is a first-order difference equation in $\{v_t\}_{t=1}^{\infty}$

*Steady-state equilibria*

- A steady-state equilibrium has the property $v_t = v_{t+1} = v$ for all $t$

- There are two steady-states, one monetary $(v > 0)$ and one nonmonetary $(v = 0)$

- Monetary steady-state $v^* > 0$ satisfies

$$1 = f'(y - v^* M)$$

- Note: the monetary equilibrium is an asset "bubble" (fiat money is priced above its fundamental value)

*Nonstationary price-level dynamics with constant fundamentals*

- $h'(v) = f'(y - vM) - vMf''(y - vM) > 0$

- $h''(v) = -2vf''(y - vM) + vMf'''(y - vM)$

- Let's assume $h''' \approx 0$ so that $h'' > 0$

- Equilibrium sequence $\{v_t\}$ must be bounded; i.e., $0 \leq v_t M \leq y$

- Then there exists a continnum of nonstationary monetary equilibria indexed by initial condition $v_1 \in (0, v^*)$ where $v_t \searrow 0$ (inflation)

**Exercise 1** *Does there exist an interest rate policy rule, say, in the form* $R_{t+1} = R(\Pi_t)$, *that will ensure long-run stability of the price-level? If so, describe the properties of this policy (or any other policy that may do the trick).*

*Stationary equilibria*

- Let $M_t = \mu M_{t-1}$ and $R_t = R$ with $N_t = nN_{t-1}$

- Then from market-clearing condition (5)

$$\frac{M_{t+1}/p_{t+1}}{M_t/p_t} = \left[\frac{N_{t+1}}{N_t}\right]\left(\frac{q^d_{t+1}}{q^d_t}\right) \Rightarrow \Pi_{t+1} = \left[\frac{\mu}{n}\right]\left(\frac{q^d_t}{q^d_{t+1}}\right)$$

- In a stationary equilibrium, $q^d_t = q^d_{t+1}$, so that $\Pi_{t+1} = \mu/n$ for all $t$

- So conditional on a policy $(R, \mu)$, with associated lump-sum tax/transfer to balance budget, the stationary monetary equilibrium is characterized as follows...

- Equilibrium inflation rate is $\hat{\Pi} = \mu/n$

- From condition (4) we have equilibrium quantity of real money balances

$$f'(y - \hat{q}) = \frac{R}{\mu} n \tag{6}$$

- Initial price-level is given by $v_1 M = N_1 \hat{q}$

- From GBC (2) we have $\hat{\tau} = [(R/\mu) - 1] \, n\hat{q}$

- From BC (3) we have $\hat{c} = f(y - \hat{q}) + n\hat{q}$

- Note $\hat{q} = q^*$ for any policy with $R = \mu$ (compare conditions 1 and 6)

## Money neutrality

- Money is said to be *neutral* if a one-time change in $M$ has no real effects (i.e., if only nominal values change)

- Money is said to be superneutral if a one-time change in $\mu$ has no real effects

- Money is clearly not superneutral in this model

- But money may or may not be neutral

- Depends on how new money is injected into the economy

- Note: this is a heterogeneous agent economy (the young and old have different consumption propensities)

- A lump-sum injection of cash to the old turns out to be neutral

  - the old compete among themselves and bid up the price of goods in proportion to their new money

- A lump-sum injection of cash to the young is neutral in the long-run, but not in the short-run

  - new money puts upward pressure on the price-level and diminishes the purchasing power of cash held by old

  - wealth transfer from old to young (and a temporary investment boom)

**Exercise 2** *Work out the economic consequences of a one-time increase in the money supply at date 1, where the new money is injected as a lump-sum transfer to the initial young.*

## Inflation as a tax and the limits to seigniorage

- Let $R = 1$ and $M_t = \mu M_{t-1}$ so that $T_t = [1/\mu - 1]\, M_t$

- Define seigniorage $S_t \equiv -T_t$ so that $S_t = [1 - 1/\mu]\, M_t$

- Convert into real terms and divide by $N_{t-1} = N_t/n$

$$s_t = [1 - 1/\mu]\, n \left( \frac{v_t M_t}{N_t} \right)$$

- So $s_t$ is real seigniorage revenue per old agent

- Looks like $s_t$ is an increasing function of $\mu$; but is this necessarily the case?

- We know that condition (6) must hold; i.e.,

$$f'(y - \hat{q}) = \frac{n}{\mu}$$

- Since $\hat{q} = v_t M_t / N_t$ in a stationary equilibrium, we have

$$\hat{s}(\mu) = [1 - 1/\mu] \, n\hat{q}(\mu)$$

- Note that the real tax base $\hat{q}(\mu)$ is a decreasing function of $\mu$

- Consequently, $\hat{s}(\mu)$ exhibits a "Laffer curve" property

## An increase in the money supply that has no price-level effect

- An increase in the money supply need not have any effect (including *nominal* effects) if the increase is temporary

- Example: a lump-sum injection of cash $\Delta M$ at some arbitrary date $t$ (to the young at date $t$) that is to be reversed at date $t+1$ (via a lump-sum tax on the old at date $t+1$)

- Such a policy has no effect on the lifetime wealth of young agents; they will simply save $\Delta M$ to pay back an anticipated tax bill $R\Delta M$ next period

- The demand for money rises one-for-one with the supply of money (Ricardian equivalence holds in this case)

**Exercise 3** *Repeat the experiment above except assume now that the lump-sum money transfer accrues to the old, instead of the young. Explain why Ricardian equivalence does or does not hold in this case.*

## Monetary policy as asset purchases

- In reality, central banks expand/contract the supply of base money by way of purchases/sales of assets

- Traditionally, the assets involved are typically short-term government debt instruments (e.g., U.S. Treasuries)

- Occasionally, other asset classes are involved; e.g., long-term government debt and mortgage-backed securities

- Collateralized lending arrangements (e.g., the discount window), sale and repurchase agreements (repo), are closely related policies

*Purchases of capital*

- Think of the Fed's massive MBS purchase program–what does our theory say?

- Let's begin in a steady-state

- Government then steps in, prints new money $\Delta M$, and uses it to purchase output (in the form of capital, $k$)

- Money supply will be permanently higher, unless the government plans to reverse the money injection

- One way to reverse $\Delta M$ is to re-sell the capital (or its output, in this case) for money in the future

- Such a policy is likely to be completely neutral here (I have not checked)

- Upshot: the inflationary consequences of an increase in the supply of base money depends critically on whether the increase is (or is expected to be) reversed in the future

- Note: the Fed has repeatedly suggested that it has an "exit strategy" in place

## Broad money aggregates and the money multiplier

- Most of an economy's money supply consists of private debt
- In reality, debt in the form of demandable liabilities (demand deposit liabilities)
- Broad money measure M1 defined roughly as cash plus demand deposits
- We can identify a similar (though not identical) object in our model
- In our model, think of $k$ as work effort devoted to producing new capital goods

- Imagine that a firm borrows banknotes from a bank to pay for $k$

- Banknotes constitute a claim against future output; worker deposits banknotes back into bank

- Young worker uses banknotes to purchase future output from firm, firm uses sales to pay back loan to bank

- Nominal value of private money is $p_t N_t k_t$

- The aggregate money supply is $M1_t = M_t + p_t N_t k_t$

- From the market-clearing condition $p_t = M_t/(N_t q_t)$

- Combining the previous two expressions...

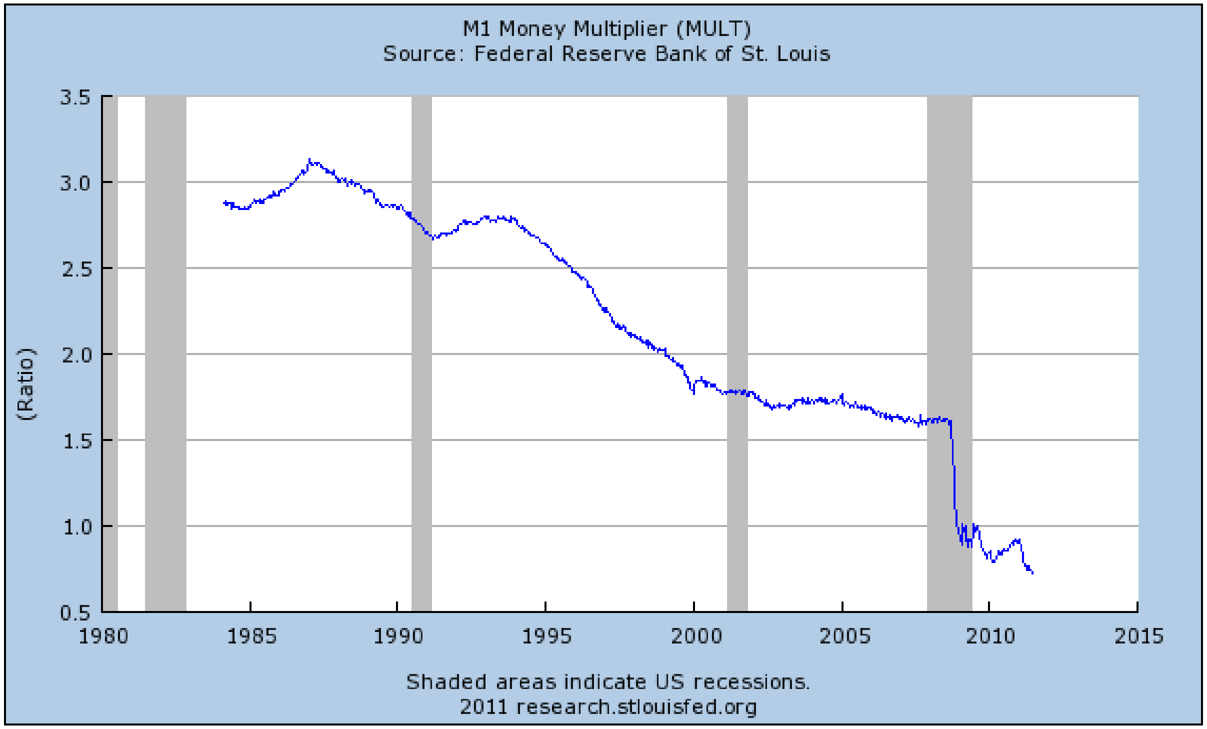
$$M1_t = \left[1 + \frac{k_t}{q_t}\right] M_t$$

- The money mulitiplier is defined as $M1_t/M_t$; i.e., the object in square brackets

- Anything that causes $k_t$ to expand (or $q_t$ to contract) will cause an expansion in the broad money supply even if $M_t$ remains constant (banks are expanding their balance sheets)

- Conversely, any shock that induces a large increase in the demand for government money (at the expense of private money) will cause a decline in the supply of broad money (banks are contracting their balance sheets)

U.S. money multiplier plunges in the great recession...



## Excess money demand?

- Many economists/commentators explain recessions as the outcome of some shock that causes an "excess demand" for money (and/or government securities)

- People get scared (irrationally?) and collectively flock into relatively safe assets (dumping risky assets)

- Now, the flight into money is particularly troubling because, *if nominal prices are sticky*, there is now not enough money to facilitate a "normal" level of transactions, so output must decline (an excess supply of output is the other side of an excess demand for money)

- Our model offers a slightly different interpretation that does not rely on sticky prices

- Modify the investment technology as follows

$$E\left[z_{t+1} f(k_t) \mid \eta_t\right]$$

  where $z$ is a parameter that indexes the productivity of capital investment, and where $\eta$ is information (or psychology) that influences conditional forecasts

- Assume that $E\left[z_{t+1} \mid \eta_t\right]$ is increasing in $\eta_t$ and that $\eta_t$ follows an exogenous stochastic process

- Our theory implies

$$E\left[z_{t+1} \mid \eta_t\right] f'(y - q_t)v_t = RE\left[v_{t+1} \mid \eta_t\right]$$

- Now, invoke the market-clearing condition $v_t = N_t q_t / M_t$; so that

$$E\left[z_{t+1} \mid \eta_t\right] f'(y - q_t)q_t = \left(\frac{Rn}{\mu}\right) E\left[q_{t+1} \mid \eta_t\right]$$

- I conjecture that $q_t = q(\eta_t)$

- To simplify, assume that $\eta_t$ is an *i.i.d.* process (might be interesting to generalize later); consequently $\bar{q} \equiv E\left[q(\eta_{t+1}) \mid \eta_t\right]$ is a constant and

$$E\left[z_{t+1} \mid \eta_t\right] f'(y - q(\eta_t))q(\eta_t) = \left(\frac{Rn}{\mu}\right) \bar{q}$$

- Note that $f'(y-q)q$ is increasing in $q$

- Consequently, $q(\eta_t)$ must be decreasing in $\eta_t$

- The implication is that "bad news" (or psychological depression) will lead to an increase in the demand for money (asset substitution away from private money)

- That is, $(k/q)$ falls $\Rightarrow$ money multiplier falls

- Moreover, this type of shock is deflationary (the price level jumps down)

# U.S. price level (CPI)

## Policy responses

- If the "flight to quality" is deemed excessive (driven perhaps by animal spirits), then an appropriate policy response is to lower the interest rate $R$

- Re: $q$ is an increasing function of $R$ in this model (and lower $R$ stimulates investment)

- If there is a constraint $R \geq 1$, then there is only so far this policy can go

- Some economists (e.g., Paul Krugman) argue that increasing $\mu$ is desirable if $R = 1$

## Unconventional policy responses

- Conventional monetary policy organized around a "Taylor rule"

$$R_t = R + a(\Pi_t - \Pi) + b(y_t - y)$$

- But constrained by $R_t \geq 1$

- So try "unconventional" measures, like quantitative easing (QE)

- Not as unconventional as people make out

- Basic idea is to print money to finance large-scale purchases of other asset classes (mortgage-backed securities and longer-dated Treasuries)

- The impact of these QE policies is hotly debated

- There is some evidence to suggest they increased inflation expectations (at a time when deflation was a worry)


- QE2 formally announced in early November 2010 (hinted earlier in summer at Jackson Hole)