

# A Basic Neoclassical Business Cycle Model with Heterogeneous Agents

David Andolfatto

January 18, 2006

## 1 Introduction

Chapter 2 in the textbook models an economy consisting of many individuals who are literally assumed to be identical; this is the so-called *representative agent* (RA) assumption.

The natural reaction to such an assumption is that it is ludicrous—people are obviously very different. How can we learn anything about the way an economy functions by assuming that people are all identical? The answer to this question is that all models (or theories) involve some element of abstraction (otherwise, they would not be models). Whether a particular assumption is a good one to make or not depends on the question that is being addressed. For some questions, the RA assumption turns out not to be a bad approximation. For other questions (for example, the issue of income redistribution), the assumption of a RA is clearly inappropriate. At the end of the day, the usefulness of a model must be judged on its predictive power and its ability to deliver a plausible interpretation of the basic economic forces at work (and not on the nature of the assumptions that are made). The common road map, for example, is an abstract representation of the country-side. A road map typically abstracts from an infinite number of real-world elements, like terrain, atmospheric conditions, people, etc. Nevertheless, we find road maps useful for navigation purposes. We criticize a road map not for its assumptions, but for its inaccuracies along particular dimensions (for example, by failing to denote the presence of a cliff that separates a road connecting point A to point B).

## 2 The Basic Model

Chapter 2 begins with an economy populated by individuals who have preferences defined over broad-based consumption; i.e., market consumption ( $c$ ) and home-production ( $l$ ). We assumed that all individuals have identical preferences

over these two objects, and that these preferences could be represented by a utility function  $u(c, l)$ . The basic idea here is that while people may obviously differ in their preferences (which are not directly observable, by the way), there are many things that people also have in common (for example, the desire for higher material living standards as measured by  $c$  and  $l$ ). The way to think of the RA assumption is that our model is highlighting these similarities, while downplaying any differences. Whether this is a useful assumption to make depends on the model's predictions/interpretations.

For simplicity, we assumed that production takes place according to a linear production function; i.e.,  $y = zn$ , where  $y$  denotes production of the market good (consumer goods and services),  $n$  denotes time spent in the labor market, and  $z$  denotes an exogenous productivity parameter. In this world, competition among firms dictates that the equilibrium real wage  $w$  will adjust to a point where  $w^* = z$  (so that firms earn zero profits).

We also assumed, for simplicity, that agents could divide their time (normalized to one unit) between two competing activities: work (in the market sector) or leisure (work in the home sector). This implies the following time constraint:  $n + l = 1$ . Since there is no saving in this model, all wage income must be consumed; this implies the budget constraint:  $c = zn$ . If we combine the time and budget constraints, we get the restriction:  $c = z - zl$ . Thus, we can represent the agent's choice problem mathematically as:

$$\max_{c,l} u(c, l) \text{ subject to: } c = z - zl.$$

Alternatively, one can write:

$$\max_n u(zn, 1 - n).$$

In words, these expressions simply assert that the agent does the best he/she can (according to his/her preferences) subject to his/her constraints.

This model predicts that the equilibrium level of employment (hours allocated to the market sector) should be a function of  $z$ , which measures the return to labor. Under some mild restrictions on preferences, the model predicts that employment (and GDP) should be an increasing function of productivity  $z$ . In fact, this is what we observe in the data. The intuition is straightforward and is discussed in the text.

What I want to focus on here is the fact that the model predicts that everyone is employed, so that variations in the labor input occur only to the extent that individuals vary the number of hours they choose to work at the prevailing wage. In the data, it turns out that most of the variation in the labor input is attributable to changes in the number of people who are employed; not in the hours of work by those who are employed. It turns out that we can extend the basic model to accommodate this fact.

### 3 Heterogeneous Agents

Here, I make two modifications to the basic model above. First, let me assume that time is indivisible, so that people may *either* work or leisure; i.e.,  $n = 1$  or  $n = 0$ . Second, let me assume that preferences are given by:

$$u(c + vl),$$

where  $v$  is a parameter that measures how much a person values leisure. Assume that people are different in how they value leisure. We can represent this heterogeneity by assuming that  $v$  is distributed across the population according to a cumulative distribution function:

$$F(x) = \Pr[v \leq x].$$

That is,  $F(x)$  represents the fraction of people with a leisure-value  $v$  less than or equal to  $x$ . Obviously,  $F$  is an increasing function of  $x$ .

The choice problem facing individuals is simple: should they work ( $n = 1$ ) or not ( $n = 0$ )? If they choose to work, they receive a utility payoff equal to  $u(z)$ . If they choose to leisure, they receive a utility payoff equal to  $u(v)$ . Thus, everyone with  $v \leq z$  should become employed; and everyone with  $v > z$  should remain at home (nonemployed). It follows that the equilibrium level of employment is given by:

$$N^*(z) = F(z).$$

Since  $F$  is an increasing function of  $z$ , it follows that  $N^*$  is too. In other words, a positive productivity shock leads to an expansion in employment (and GDP). As we can see, this basic prediction (an increase in the labor input) is exactly the same as in the basic model. Furthermore, the intuition is exactly the same too. In other words, this basic prediction is not an artifact of the RA assumption (it is a basic economic force that is present in a large class of models).