

# An Introduction to Business Cycle Theory

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Preliminary: Comments Welcome

## 1. Introduction

Business cycle theory is concerned with explaining the recurrent fluctuations in aggregate business activity that characterize modern industrialized economies. The broadest single measure of aggregate economic activity is the real gross domestic product (output), which measures the total value of final goods and services produced domestically in a given period. A casual glance of the time-series properties of this measure reveals two immediate facts. First, the rate at which output grows is on average positive; and second, the rate of output growth fluctuates significantly over short periods of time. While growth and cycles are often treated as separate phenomena, it is far from obvious that they are independent (Schumpeter, 1939).

A more formal statistical analysis of the data requires that a stand be taken on the nature of the underlying data generating process. By far the most common approach is to view output growth as fluctuating around a single 'long-run' or 'trend' rate of growth; this is the perspective adopted below.<sup>1</sup> The term 'business cycle' typically refers to the cyclical fluctuations in output growth around a given trend. Evidently, these fluctuations are viewed as being 'cyclical' in the sense that they represent 'temporary' deviations from 'long-run' behaviour. In other words, under the assumption of a unique trend, output growth can be interpreted as displaying mean-reverting tendencies. The assumption of mean-reversion is likely implicit in the mind of anyone who speaks of 'recessions' and 'booms'.

Business cycle activity is often described with adjectives like volatility, persistence and comovement. Given a set of stationary time-series data, there are

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<sup>1</sup>A competing approach views the trend rate of output growth as shifting stochastically, with stationary components around each trend; see, for example, Hamilton (1989).

well-defined statistics that can quantify such concepts. For example, a measure of volatility is given by the standard deviation; a measure of persistence is given by the autocorrelation function; a measure of comovement is given by the cross-correlation function (at different leads and lags). Statistics such as these constitute just some of the 'stylized facts' of the business cycle; a central task of business cycle theorists is to explain the signs and magnitudes of such statistics.

## 2. Some Stylized Facts

The data described in this section are for Canada over the sample period 1975:1-1995:4 (quarterly).<sup>2</sup> Figure 1 plots the quarterly growth rate (annualized) in real per capita output: over this sample, real per capita output grew at an average rate of 1.57% per annum, with a standard deviation of 3.70%. From Table 1, we see that output growth displays some persistence. In particular, positive serial correlation is evident in output growth at lags of one to three quarters. An implication of this latter fact is that the future rate of output growth is to some extent forecastable with current and past rates of growth.<sup>3</sup>

When output is decomposed into its income or expenditure components, other interesting patterns become evident. For example, consider the division of output between consumption and saving (investment plus net exports). Consumption expenditure (including government purchases) averages about 80% of total output; this ratio has remained relatively stable over long periods of time. However, at business cycle frequencies, the consumption-output ratio is countercyclical. The autocorrelation function for consumption growth exhibits less persistence (at least at one period lag), which suggests that the level of consumption is closer to a random walk. On the other hand, saving (output minus consumption) growth exhibits positive serial correlation at short horizons. This latter fact suggests that any transitory (cyclical) component in the level of output is determined predominantly by the behaviour of investment and net exports. Growth in consumption and saving are both procyclical. Wage income comprises on average about 55% of total income; this ratio displays no secular trend. At business cycle frequencies, labour's share of income is countercyclical (the share of capital income is procyclical). However, growth in both components is procyclical.

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<sup>2</sup>The data source and description is provided in Appendix I.

<sup>3</sup>Fama (1992) documents the forecastability of output growth.

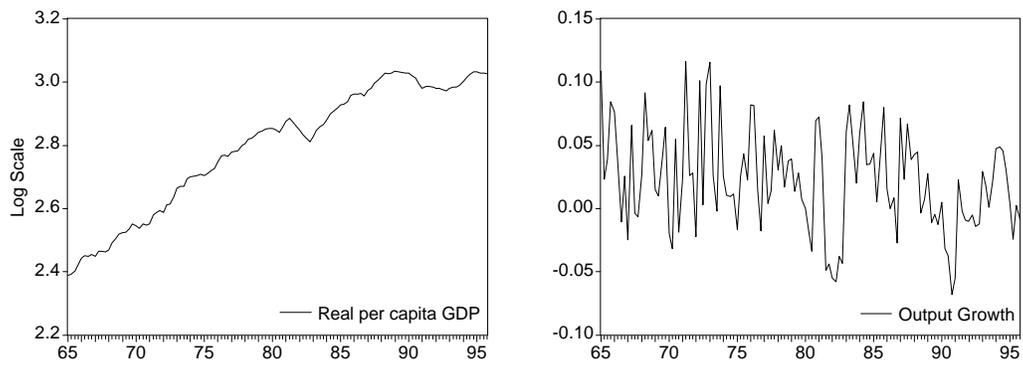


Figure 0.1:

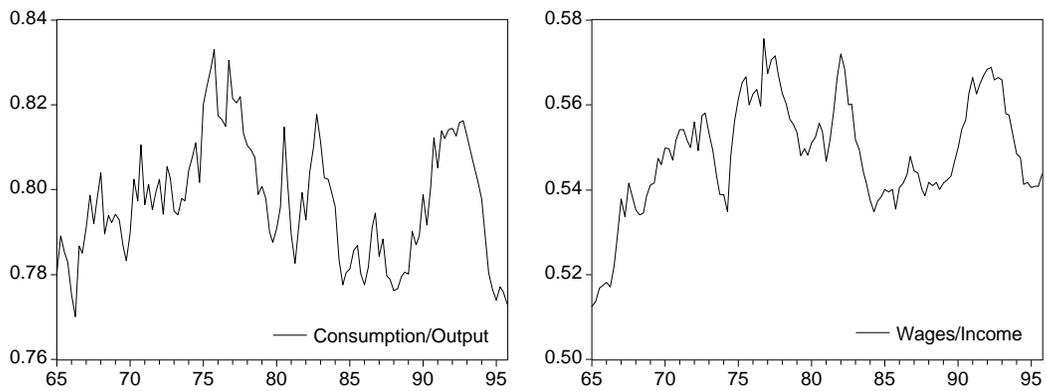


Figure 0.2:

Table 1						
	Mean	Std Dev	ACF 1{4 Lags			
$\Phi \ln(y)$	2.21	3.98	0.34	0.17	0.15	0.07
$\Phi \ln(c)$	2.11	3.50	{0.01	0.25	0.28	0.03
$\Phi \ln(wn)$	2.40	3.88	0.30	0.17	0.22	0.03
$c=y$	0.80	0.01	0.86	0.75	0.65	0.55
$(wn)=y$	0.55	0.01	0.91	0.82	0.71	0.59

Table 2									
Cross Correlogram of $\Phi \ln(y)$ with Selected Variables									
	{4	{3	{2	{1	0	+1	+2	+3	+4
$\Phi \ln(y)$	0.07	0.15	0.17	0.34	1.00	0.34	0.17	0.15	0.07
$\Phi \ln(c)$	0.16	0.21	0.23	0.25	0.53	0.26	0.17	0.22	0.04
$\Phi \ln(wn)$	0.07	0.00	0.01	0.26	0.67	0.34	0.19	0.31	0.16
$c=y$	0.14	0.16	0.18	0.11	{0.18	{0.25	{0.27	{0.26	{0.29
$(wn)=y$	0.05	{0.05	{0.13	{0.16	{0.35	{0.36	{0.35	{0.29	{0.26

### 3. Explaining Business Cycle Facts

An explanation (as distinct from description) is comprised of three basic elements: a set of phenomena to be explained; a set of given hypotheses; and a set of logical statements linking the latter to the former. This notion of explanation is equivalent to the definition of theory. Virtually all modern theories of the business cycle choose to interpret fluctuations in economic activity as emanating from some hypothesized source called the impulse mechanism, which is usually modelled as an exogenous stochastic process. These impulses or 'shocks' are somehow magnified throughout the economic system, both in space and over time; the law governing this process is called the propagation mechanism.<sup>4</sup>

One approach in 'explaining' business cycles is to view the propagation mechanism as an exogenous law, much in the way that a physicist might view the law of gravity. For example, one might hypothesize that output growth follows an autoregressive process driven by an innovation that is distributed according to some

<sup>4</sup>For an early discussion, see Frisch (1933) and Slutsky (1937).

probability law. Given time-series data on output growth, one may then employ statistical techniques designed to estimate the parameters of the stochastic process governing output growth. A more sophisticated version of this methodology has been developed by Sims (1980).<sup>5</sup> While this approach has its applications (e.g., prediction), it falls short of explanation in the sense described above since the law of motion is essentially assumed rather than derived from a more primitive set of hypotheses.

The traditional approach in explaining business cycles has been to first hypothesize the existence of a set of behavioural rules governing the economic decisions of individuals in different sectors of the economy. A classic example of such a decision rule is the 'Keynesian' consumption function, which hypothesizes a relationship between desired consumption spending and current disposable income in the household sector. Combined with other assumptions describing the operating characteristics of various markets (together with a solution concept) one may derive an 'equilibrium' stochastic process characterizing the behaviour of aggregate variables. Behavioural parameters such as the 'marginal propensity to consume' play a key role in propagating economic disturbances. A famous paper that employs this methodology fruitfully to the analysis of growth phenomena is Solow (1956). An application to business cycle theory is that of Klein and Goldberger (1955).<sup>6</sup>

Modern business cycle theory is dominated by the 'microeconomic' approach advocated by Lucas (1980) and pioneered by Kydland and Prescott (1982) together with Long and Plosser (1983). The substantive difference between the modern approach and the traditional approach is in terms of how economic behaviour is modelled. Rather than hypothesizing directly on the structure of behavioural rules, the modern approach prefers to derive behavioural rules from a more primitive set of objects that describe individual preferences, technology, endowments, information and institutional structure. In the words of Kydland and Prescott (1996), these are the parameters that describe the 'willingness and ability of individuals to substitute across different commodities'. The derived behavioural rules are optimal in the sense that they are consistent with individuals doing the best they can (according to their preferences) subject to the constraints on their behaviour. The preferred solution concept invokes consistency in the formation of expectations (rational expectations). Associated with this general

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<sup>5</sup>Cooley and Leroy (1985) provide a critique of this method.

<sup>6</sup>This methodology has been criticized by Lucas (1976).

approach is the use of a competitive market structure, representative agencies, and an impulse mechanism described by shocks to technological capabilities; such restrictions characterize what has become known as 'Real Business Cycle (RBC) Theory'. None of these latter features are necessary components of the modern approach; see for example the several recent extensions compiled by Cooley (1995).

#### 4. Intratemporal Propagation

In this section, I develop what is essentially a static general equilibrium model for the purpose of displaying, at the simplest possible level, the basic approach to RBC theory. In addition, the static framework will be useful for demonstrating how an exogenous impulse to technology can be 'magnified' into an employment and output response in a given period.

##### 4.1 Economic Structure and Decision Making

Consider a representative individual with preferences defined over current period consumption  $c_t$  and work effort  $n_t$  which are represented by  $U_t = c_t \cdot H(n_t)$ ; with  $H^0, H^{00} > 0$ : Individuals supply labour in a competitive labour market at real wage  $w_t$ . As well, individuals earn nonlabour income  $d_t$ ; which represents a dividend payment (individuals own business sector equity). There are no financial markets and capital is in fixed supply. Formally, the individual's decision problem is given by

$$V(w_t; d_t) = \max_{c_t; n_t} \{ c_t \cdot H(n_t) \mid c_t = w_t n_t + d_t \}$$

The labour supply function  $n^s(w_t)$  is characterized by the restriction

$$w_t \cdot H^0(n^s) = 0:$$

Clearly, labour supply unambiguously increases with the real wage (there is no income effect). Consumption demand is given by  $c^d(w_t; d_t) = w_t n^s(w_t) + d_t$ :

The representative firm is endowed with a unit of capital and the production technology  $y_t = F(n_t; z_t)$ ; where  $F_n, F_z, F_{nz} > 0$  and  $F_{nn} < 0$ : The parameter  $z_t$  indexes the economy-wide state of technological conditions in the business sector. The technological efficiency parameter may be interpreted narrowly as representing the effect of a new technology, or it may be interpreted broadly to include all sorts of effects, ranging from changing weather patterns to fluctuating

'animal spirits'. Formally, the decision problem faced by the firm is given by

$$j(w_t; z_t) = \max_{y_t, n_t} \{ w_t n_t - j(y_t - F(n_t; z_t)) \}$$

The labour demand function  $n^d(w_t; z_t)$  is characterized by the restriction

$$F_n(n^d; z_t) - j = 0$$

Labour demand is decreasing in  $w_t$  and increasing in  $z_t$ : The supply of output is given by  $y^s(w_t; z_t) = F(n^d(w_t; z_t); z_t)$ :

The decision problems above are stated under the assumption that individuals expect to be able to trade freely any quantity they desire at the stated price system  $w_t$ : In the aggregate, such an assumption on the part of individuals is justified only to the extent that desired trading patterns across individuals are consistent with each other. Consequently, we impose the consistency condition  $n_t^s = n_t^d$  (which implies  $c_t^d = y_t^s$ ); this restriction is usually referred to as 'market-clearing'. The restrictions on economic behaviour described above are summarized in the definition of equilibrium.

**Definition.** A competitive equilibrium for the fixed-capital economy comprises a set of decisions for households  $(n_t^h; c_t^h)$  and firms  $(n_t^f; y_t^f; d_t)$  together with a price system  $w_t$  such that:

- (i)  $n_t^h = n^s(w_t)$  and  $c_t^h = c^d(w_t; d_t)$ ;
- (ii)  $n_t^f = n^d(w_t; z_t)$ ;  $y_t^f = y^s(w_t; z_t)$  and  $d_t = j(w_t; z_t)$ ;
- (iii)  $n_t^h = n_t^f = n_t$  and  $c_t^h = y_t^f = y_t$ .

The first and second conditions restrict the outcome to be consistent with optimizing behaviour on the part of households and firms. The third condition requires that these behaviours are consistent with each other.

## 4.2 Qualitative Properties of the Equilibrium

The restrictions stated in the definition above can be used to characterize the competitive equilibrium outcome as follows.<sup>7</sup> Conditions (i) and (iii) imply

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<sup>7</sup>I am assuming existence and uniqueness.

$w_t$  and  $H^0(n_t) = 0$ , while conditions (ii) and (iii) imply  $F_n(n_t; z_t) - w_t = 0$ : Combining these latter two results yields the following restriction:

$$F_n(n_t; z_t) - w_t = 0;$$

which completely characterizes the equilibrium level of employment in this economy as a function of  $z_t$ ; i.e.,  $n_t = n(z_t)$ : Once  $n_t$  is known, it is straightforward to characterize the remaining variables of interest:  $y_t = F(n(z_t); z_t) = c_t$ ;  $w_t = H^0(n(z_t))$ ; and  $d_t = y_t - w_t n(z_t)$ :

How do economic variables in this model economy react to a sudden increase in economy-wide technological capabilities? Mechanically, the restriction that characterizes equilibrium employment tells us that

$$\frac{dn}{dz_t} = \frac{F_{nz}(n_t; z_t)}{F_{nn}(n_t; z_t) - H^{00}(n_t)} > 0;$$

That is, the equilibrium level of employment is predicted to increase. The economic intuition for this result can be obtained by studying how a change in  $z_t$  impacts directly on the decisions of individuals. From the restrictions on behaviour derived earlier, a change in  $z_t$  has no direct impact on desired labour supply, but increases the demand for labour (by raising the marginal product of labour at any given level of employment). The upward shift in labour demand puts upward pressure on real wages, a fact that can be verified by

$$\frac{dw_t}{dz_t} = H^{00}(n_t) \frac{dn}{dz_t} > 0$$

which induces an increase in the supply of labour. With wages and employment rising, total labour income rises. It is easy to verify that nonlabour income rises as well. Aggregate output rises for two reasons: first, there is a direct impact on the amount of output that can be produced with a given level of employment; second, there is an indirect impact on output via the increase in employment induced by the productivity shock, i.e.

$$\frac{dy_t}{dz_t} = F_z(n_t; z_t) + F_n(n_t; z_t) \frac{dn}{dz_t} > 0;$$

Thus, we see how an economy infected with disturbances that alter production possibilities (the ability to substitute) can display fluctuations in output, employment, wages and profit. Furthermore, these productivity shocks are seen to imply procyclical movements in employment, wages and profit | a qualitative feature that is shared with the data generated by real economies.

### 4.3 Quantitative Analysis

The first step in the quantitative analysis is to parameterize the functional forms  $F$  and  $H$ ; here we specify

$$\begin{aligned} F(n_t; z_t) &= \exp(z_t) n_t^\mu k^{1-\mu} & 0 < \mu < 1 \\ H(n_t) &= \frac{1}{1+\phi} n_t^{1+\phi} & \phi > 0; \end{aligned}$$

where  $k = 1$ : We will also need to specify the stochastic process describing the evolution of productivity shocks; here we assume a stationary AR(1) process

$$z_{t+1} = \rho z_t + \epsilon_t \quad 0 < \rho < 1$$

with  $\epsilon_t$  identically and independently distributed with mean zero and standard deviation  $\sigma > 0$ :

The next step is to assign values to the model parameters:  $\mu$ ;  $\phi$ ;  $\rho$ ;  $\sigma$ : Unfortunately, there is not at present a generally accepted method for this process of parameter selection. Below, I will describe two general procedures employed in the literature that loosely go by the labels 'calibration' and 'estimation'.

The calibration method is the favoured approach among RBC theorists; see Kydland and Prescott (1996) for an extensive discussion as well as Hansen and Heckman (1996) for a critique. In this approach, the theorist attempts to calibrate the model to aspects of reality that are considered invariant to the phenomena under study. The phenomena under study in business cycle theory are typically represented by a set of second moments (covariances), as in Table 1 above. A given parameterization will imply an equilibrium stochastic process. The lower moments (means) of this process are used to calibrate the model's parameters, while the higher moments constitute the model's predictions; the procedure is best explained by way of example.

An empirical fact is that labour's share of output has remained relatively stable over long periods of time; in a sample period of length  $T$ , its average is given by

$$\bar{x} = \frac{1}{T} \sum_{t=1}^T \frac{w_t n_t}{y_t}$$

In the model described above, there is a single parameter that governs the size of labour's income share; this parameter is  $\mu$ : Therefore, in order for the model's

equilibrium to replicate observation along this dimension, the parameter  $\mu$  must be chosen to equal  $\bar{x}$ :

The remaining parameters of the model have no bearing on model predictions concerning first moments; consequently, other information must be used to restrict their values. To calibrate the utility function of the household sector, information on certain averages across large numbers of individuals can be employed. For example, labour economists often report estimates of the labour supply elasticities based on various cross-sections of individuals. In the model above, the elasticity of labour supply with respect to the real wage is given by  $1-\sigma$ : A common estimate for this elasticity (that includes females and the extensive margin of labour supply) ranges in the neighbourhood of 0.5 and 1.5; this suggests that  $\sigma = 1.0$  constitutes a plausible value.

The parameters remaining are those that describe the technology shock process. Unfortunately, there are no direct measures of  $z_t$  available. However, given observations on output and employment, an estimate for  $z_t$  can be computed over the sample by using the hypothesized production function and the calibrated value for  $\mu$ ; in particular

$$z_t = \ln(y_t) - \mu \ln(n_t)$$

The parameter  $z_t$  is sometimes referred to as 'total factor productivity' or as the 'Solow residual'. After detrending the series  $\{z_t\}_{t=1}^T$ ; one can run a simple regression of  $z_t$  on  $z_{t-1}$  in order to estimate the parameters  $\frac{1}{2}$  and  $\frac{3}{4}$ :

With the model so calibrated, the next step is solving numerically for the model's equilibrium, which in this case comprises a set of functions of the form  $y(z_t)$ ;  $n(z_t)$ ;  $w(z_t)$  and  $d(z_t)$ : With these functions computed, one may then generate random sequences of  $\{z_t\}$  on the computer according to the estimated probability law, and record the implied equilibrium behaviour for the economic variables of interest. In particular, one may compute the average values (over hundreds of simulations) for any number of second moments and compare these values to the second moments generated by the real economy. To the extent that the predicted second moments are similar to actual second moments, one might judge the model as being of use in understanding an important economic mechanism at work in the actual economy. More often than not, the model tends to fail along several dimensions, a result that spawns further questioning and learning.

An alternative to 'calibration' for parameter selection is 'estimation'. As Hansen and Heckman (1996) point out, the distinction between these two meth-

ods appears to be super-efficient at best. An estimation strategy might proceed as follows. Let  $\theta$  denote the set of model parameters and let  $m(\theta)$  denote a set of moments predicted by the model as a function of parameters. The set of relevant moments typically includes both first and second order moments. Let  $\bar{m}$  denote the corresponding moments describing the sample generated by the real economy. Then an estimate for  $\theta$  is found by trying to make  $m(\theta)$  as close as possible to  $\bar{m}$  according to an explicit metric. For example, the value of  $\theta$  that minimizes the sum of squared errors is given by the solution to

$$\min_{\theta} \sum_i (m(\theta)_i - \bar{m}_i)^2$$

Usually, the number of moments to match is chosen to coincide with the number of parameters, so that the estimator is just-identified. One advantage of this approach is that one may compute standard errors for the parameter estimates, based on the knowledge that the minimized sum-of-squared errors statistic is distributed asymptotically according to a chi-square distribution. Except for the practice of calibrating parameter values on the basis of external data sets, it seems apparent that the calibration procedure amounts to estimation according to a method-of-moments procedure based on a loss function that only penalizes deviations from means.

One advantage of the simple model developed above is that one may compute the equilibrium in closed-form. Doing so will give us a feel for which parameters are important for propagating shocks across space.

From the restriction that characterizes the equilibrium level of employment, one may easily solve for

$$\ln n(z_t) = \mu + \sigma z_t$$

where  $\mu = (1 + \sigma \mu) > 0$  and  $\sigma = 1 + \sigma \mu > 0$ : The standard deviation in employment fluctuations (measured in percent) is given by

$$\text{std}(\ln n(z_t)) = \sigma \text{std}(z_t)$$

where  $\text{std}(z_t) = \frac{\sigma}{\sqrt{1 - \sigma^2}}$ : Thus we see that the model has the potential of generating very large employment fluctuations with only very small shocks to technology; this will be the case if  $\sigma$  is a large number. In particular, if  $\sigma$  is less than unity, then the model predicts that employment will fluctuate less (in percentage terms) than the fluctuations in technology. Thus, the magnitude of  $\sigma$  governs the way in which a technological disturbance is magnified into an employment

response (the intratemporal propagation effect on employment); not surprisingly, the magnitude of  $\psi$  is seen to depend on the willingness and ability of individuals to substitute across commodities. As the parameter  $\mu$  rises, diminishing returns to labour set in less rapidly as employment expands. Consequently, a larger  $\mu$  increases the ability to substitute from leisure into output, thereby increasing the multiplier effect  $\psi$ : As the parameter  $\sigma$  rises, the marginal disutility of work effort (marginal utility of leisure) rises more rapidly as employment expands. Consequently, a higher  $\sigma$  reduces the willingness of individuals to substitute from leisure into output, thereby reducing the multiplier  $\psi$ :

In the calibration procedure described above,  $\mu$  was chosen in order to replicate the average share of output accruing to labour; this required setting  $\mu = 0.55$ : The value for  $\sigma$  was chosen to be consistent with measurements reported by econometricians studying the labour supply behaviour of large groups of individuals; a common value reported for  $\sigma$  is around unity. For these parameter values, we see that  $\psi < 1$ , which implies that the propagation mechanism highlighted by our model is not likely to be very important quantitatively.<sup>8</sup>

Again, once the equilibrium employment function is known, the remaining variables are easy to determine:

$$\begin{aligned} \ln y_t &= \mu \ln n_t + z_t \\ \ln w_t &= \sigma \ln n_t \\ \ln d_t &= \ln(1 - \mu) + \ln y_t \end{aligned}$$

Thus, the model predicts that the standard deviation in output relative to the standard deviation in employment should be

$$\frac{\text{std}(\ln y_t)}{\text{std}(\ln n_t)} = \frac{1 + \psi\mu}{\psi} > 1;$$

that is, given our calibration, output is predicted to fluctuate more (in percentage terms) than employment. As well, with  $\sigma$  close to unity, wage fluctuations are predicted to fluctuate about as much as employment. Finally, the model predicts that profit should exhibit volatility on the same order as output. Except for the fact that empirical measures of the real wage seem to fluctuate much less

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<sup>8</sup>In contrast to the calibration exercises, the estimation procedure would attempt to let the data 'speak for itself', for example, by choosing  $\psi$  ( $\mu$  and  $\sigma$ ) to replicate the observed ratio  $\text{std}(\ln n_t)/\text{std}(z_t)$ .

than employment, the qualitative predictions of the model are consistent with observation.

One striking feature of the model developed above is in terms of its quantitative predictions for correlations across space and time. In particular, the model predicts that employment, wages and profit are all perfectly correlated with output (and the technology shock). In terms of dynamic correlations, the economic variables all inherit the assumed dynamic properties of the impulse mechanism; i.e., the model contains no endogenous dynamic propagation mechanism. In other words, assume for the moment that  $\beta = 0$  so that technology shocks are purely temporary. Then an increase in  $z_t$  results in the effects described above, but has absolutely no impact on any future variables. As will be explained below, the necessary ingredient for dynamic propagation is the existence of some form of capital, an element missing in the model above.

## 5. Intertemporal Propagation

The basic economic forces that generate dynamic propagation can be isolated in the context of a simple two-period model. For simplicity, we will abstract from a variable labour supply and uncertainty. As before, there is a representative individual, but now with preferences defined over streams of consumption  $(c_1; c_2)$  represented by the function

$$U(c_1) + \beta U(c_2);$$

with  $U'' > 0$ ;  $U''' < 0$  and where  $0 < \beta < 1$  is a time-discount factor. Note that the strict concavity of the utility function implies a consumption-smoothing motive on the part of individuals.

### 5.1 An Endowment Economy

The consumption-smoothing motive relates to the willingness of individuals to substitute consumption across time. In order to highlight the importance of the consumption-smoothing property for the dynamic propagation of transitory disturbances, assume for the moment that output is given exogenously by  $(y_1; y_2)$  and that there is a competitive financial market for risk-free private debt that earns a (gross) real interest rate equal to  $R$ : Assume that individuals begin period one with zero financial assets (or liabilities) and that liabilities cannot be passed onto generations living beyond period two. With saving defined as  $s = y_1 - c_1$ , second period consumption is then constrained by  $c_2 = y_2 + Rs$ : Substituting these relations into the objective function reveals that the individual's decision

problem essentially involves the choice of an appropriate saving policy, i.e.

$$\max_s fU(y_1 - s) + \beta U(y_2 + Rs)g :$$

The solution  $s^d(R; y_1; y_2)$  is characterized by the following restriction:

$$-U'(y_1 - s^d) + R\beta U'(y_2 + Rs^d) = 0;$$

which simply asserts that at the margin, the costs and benefits of saving must be balanced.<sup>9</sup>

In the context of this model, a purely transitory technology shock can be modelled as a change in  $y_1$ : From the restriction that characterizes the saving rule, we see that

$$\frac{ds^d}{dy_1} = \frac{U''(c_1^d)}{U''(c_1^d) + R^2\beta U''(c_2^d)} > 0 \quad (0; 1)$$

where  $c_1^d = y_1 - s^d$  and  $c_2^d = y_2 + Rs^d$ : For a given rate of interest, the temporary increase in income results in an increase in household wealth, which induces an increase in the demand for all normal goods (i.e.,  $c_1$  and  $c_2$ ). Since  $y_2$  remains fixed, the only way to finance an increase in future consumption is to increase future interest income, i.e., to increase  $(R - 1)s^d$ : The only way to accomplish this latter task is to increase saving today (i.e., refrain from consuming the entire increase in  $y_1$  at date one). In this manner, a purely transitory income shock can have implications for the level of future income through the willingness of individuals to substitute current consumption for future consumption.

The analysis above assumes a given and arbitrary interest rate  $R$ ; such an assumption would be relevant in the case of a small open economy (desired national saving  $s^d$  would then determine the country's current account position). In a general equilibrium, however, the interest rate is endogenous and will respond to the transitory productivity shock. For a closed economy with no investment opportunities, the interest rate must adjust so that desired national saving is equal to zero, i.e.,  $s^d(R; y_1; y_2) = 0$ ; resulting in consumption allocations  $c_1 = y_1$  and  $c_2 = y_2$ .<sup>10</sup> Consequently, in the aggregate there is no dynamic propagation; current period consumption absorbs the full impact of the productivity shock. To

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<sup>9</sup>Increasing saving by one unit reduces current consumption by one unit; the utility cost of this foregone consumption is given by  $-U'(c_1)$ : The one unit of saving will generate  $R$  units of future consumption; the utility benefit of this additional consumption is  $R\beta U'(c_2)$ :

<sup>10</sup>The equilibrium interest rate is given by the marginal rate of substitution between con-

understand this result, recall that the direct impact of an increase in  $y_1$  was to increase desired national saving at any given interest rate. This additional supply of loanable funds puts downward pressure on interest rates, which stimulates current period consumption demand. Interest rates must fall sufficiently to ensure that the loanable funds market clears, i.e.,  $s^d = 0$ : With net saving equal to zero, future net interest income is equal to zero, so that future aggregate income remains at  $y_2$ : Thus, while individuals have a willingness to substitute intertemporally, they do not have the ability to do so in this model.

## 5.2 Physical Capital

In the endowment model above, we demonstrated how the existence of financial capital was incapable of generating dynamic propagation (in general equilibrium). Below we demonstrate how, in addition to being insufficient, financial capital is not even necessary for dynamic propagation. Dynamic propagation hinges on the technological ability to store goods across time; i.e., on the feasibility of physical capital accumulation.

As before, assume that the labour input is fixed. Output at date  $t$  is an increasing and concave function of period  $t$  capital; i.e.,  $y_t = F(k_t)$  with  $F' > 0$ ;  $F'' < 0$ : In this setting, we may assume without loss of generality that the individual owns and operates the production technology directly. The capital stock available for production at the beginning of period  $t$  is determined solely from the previous period's capital investment  $x_{t-1}$ ; i.e.,  $k_t = x_{t-1}$  (capital is assumed to depreciate fully after use in production). Assume that there are no financial markets. Then a representative individual's intertemporal consumption pattern is constrained as follows:

$$c_1 + x_1 = y_1 \quad c_2 = F(x_1):$$

Inserting these constraints into the objective function reveals that the individual's decision problem essentially boils down to the choice of an investment (saving) policy, i.e.

$$\max_{x_1} fU(y_1 - x_1) + \beta U(F(x_1))g:$$

\_\_\_\_\_ sumption across time evaluated at the equilibrium consumption allocation, i.e.

$$R = \frac{U'(y_1)}{-U'(y_2)}:$$

The solution  $x^d(y_1)$  is characterized by the following restriction:

$$-u'(y_1 - x^d) + F'(x^d) - U'(F(x^d)) = 0:$$

The effect on current period investment from a purely transitory productivity shock is given by:

$$\frac{dx^d}{dy_1} = \frac{U''(c_1^d)}{U''(c_1^d) + [F'(x^d)]^2 - U''(c_2^d)} > 0 \quad (0; 1)$$

with  $c_1^d = y_1 - x^d$  and  $c_2^d = F(x^d)$ : The economics underlying this result is exactly the same as in the case of the endowment economy (with the future marginal product of capital  $F'(x^d)$  serving as the interest rate). As before, an increase in current period income induces an increase in desired saving. In the endowment economy, this additional desired saving had to take the form of financial paper, the net value of which must be equal to zero in the aggregate. In the economy studied here, however, this additional saving takes the form of stored goods (a technology that is ruled out in the endowment economy). These stored goods, which in reality correspond to the production/purchase of new capital goods and inventories, can be utilized to expand future production capabilities by increasing the future capital stock. In this way, a purely transitory productivity disturbance is propagated forward in time.

## 6. A Prototype Real Business Cycle Model

The economic mechanisms described above are embodied in the basic RBC model. Essentially, the environment resembles a stochastic version of a neoclassical growth model with endogenous labour supply. The formal structure of the model is as follows.<sup>11</sup>

### 6.1 Economic Structure and Decision Making

Time is discrete and denoted by  $t = 0; 1; 2; \dots; \infty$ : As before, there is a representative individual, but now with preferences defined over stochastic streams of consumption and leisure represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + H(l_t)]$$

<sup>11</sup>Hansen and Wright (1992) report the quantitative properties of a benchmark RBC model and several extensions.

where  $0 < \beta < 1$  is a time discount factor, with  $U^0; H^0 > 0$  and  $U^{00}; H^{00} < 0$ : The individual is endowed with one unit of discretionary time per period, which is allocated between work and leisure, i.e.

$$n_t + \ell_t = 1:$$

In this setting, we may assume without loss of generality that the individual owns and operates the production technology directly. The technology for producing output is represented by

$$y_t = F(k_t; n_t; z_t)$$

where  $F_k; F_n; F_z; F_{nz} > 0$  and  $F_{kk}; F_{nn} < 0$ : Capital accumulation is governed by the equation

$$k_{t+1} = (1 - \delta)k_t + x_t$$

where  $k_0 > 0$  is given and  $0 < \delta < 1$  is the (constant) rate at which capital depreciates. The technology shock is assumed to follow a first order Markov process:

$$z_{t+1} = \rho z_t + \epsilon_t$$

where  $0 < \rho < 1$  and  $\epsilon_t$  is noise with mean zero and standard deviation  $\sigma$ :

In order to characterize optimal decision making, recursive techniques can be employed (see Stokey and Lucas, 1989). To begin, note that at the beginning of any time period  $t$ ; there are two predetermined variables  $(k_t; z_t)$ ; call this pair the state of the economic system at date  $t$ : Notice that  $k_t$  is an endogenous state variable, since it has been determined by past investment choices, while  $z_t$  is an exogenous state variable, since its evolution is governed by an exogenously specified probability law. The economic relevance of the state variables is that they summarize all the information that is needed for optimal decision making at date  $t$ :

Let  $V(k_t; z_t)$  denote the indirect utility function associated with maximizing  $E_t \sum_{j=0}^{\infty} \beta^j [U(c_{t+j}) + H(\ell_{t+j})]$  subject to the constraints specified above. Then  $V$  solves the functional equation

$$V(k_t; z_t) = \max_{n_t; k_{t+1}} \beta U(F(k_t; n_t; z_t) + (1 - \delta)k_t - k_{t+1}) + H(1 - n_t) + \beta E_t V(k_{t+1}; z_{t+1})$$

with associated solution functions  $n_t = n(k_t; z_t)$  and  $k_{t+1} = g(k_t; z_t)$ : Standard restrictions guarantee the existence and uniqueness of the value function  $V$ ; with

a little extra work, one can usually establish that  $V_k > 0$  and  $V_{kk} < 0$ : The solution functions can be characterized by the following first-order conditions:

$$\begin{aligned} F_n(k_t; n(k_t; z_t); z_t)U^0(c(k_t; z_t)) - H^0(1 - n(k_t; z_t)) &= 0 \\ -U^0(c(k_t; z_t)) + -E_t V_k(g(k_t; z_t); z_{t+1}) &= 0 \end{aligned}$$

where  $c(k_t; z_t) = F(k_t; n(k_t; z_t); z_t) + (1 - \alpha)k_t - g(k_t; z_t)$ : The first condition above governs the intratemporal allocation between consumption and leisure; this restriction is similar to the one derived in Section 4 (with  $U^0(c) = \text{constant}$ ). The second condition above governs the intertemporal allocation of consumption. Using the envelope theorem, we have

$$V_k(k_t; z_t) = [F_k(k_t; n(k_t; z_t); z_t) + 1 - \alpha]U^0(c(k_t; z_t))$$

which, when combined with the second first-order condition above, yields

$$-U^0(c(k_t; z_t)) + -E_t [F_k(g(k_t; z_t); n(g(k_t; z_t); z_{t+1}); z_{t+1}) + 1 - \alpha)U^0(c(g(k_t; z_t); z_{t+1}))] = 0:$$

This latter condition resembles a similar restriction derived for the two-period economy in Section 5.

## 6.2 Qualitative Properties

Note that when  $\epsilon_t = 0$  for all  $t$ ; the RBC model reduces to a deterministic neoclassical growth model with endogenous labour supply. Under standard restrictions on preferences and technology, such a model possesses a unique (nondegenerate) steady-state, with dynamics that converge monotonically to the steady-state. For example, if the current period capital stock is below its steady-state value, then so is output, consumption, investment and leisure (employment is above its steady-state value); each of these variables converge monotonically to their long-run values.

Suppose that we begin in the neighbourhood of the steady-state and that the economy suddenly experiences a purely transitory productivity shock (i.e.,  $\epsilon_t > 0$  and  $\epsilon_{t+1} = 0$ ). What are the economic impacts and subsequent dynamics likely to be? Theoretically, the impact effect on employment is ambiguous. Current productivity (the return to work) is temporarily high so that individuals are encouraged to substitute intratemporally from leisure to consumption as well as intertemporally from current leisure into future leisure. On the other hand, wealth

is higher so that the demand for all normal goods increases, including the demand for current period leisure. Given the transitory nature of the shock, the wealth effect is likely to be relatively weak; employment is likely to respond positively to the transitory increase in productivity. With higher employment and higher productivity, current period output rises (the current period capital stock remains fixed). The consumption-smoothing motive suggests that a part of this increased output will take the form of additional new capital goods so that current period investment spending will rise (together with current period consumption).

The increase in current period investment translates into a larger capital stock for the subsequent period's production activities. With respect to this future period, the theoretical impact of this extra capital on future labour supply is ambiguous. To the extent that capital and labour are complementary inputs into production, the higher capital stock increases the marginal product of labour, which encourages intratemporal substitution from leisure into consumption. As well, with the capital stock expected to fall back to its steady-state level, the return to labour is still temporarily above its long-run value, which continues to encourage intertemporal substitution of current leisure for future leisure. Working against these substitution effects on employment is the wealth effect induced by the larger stock of capital. The net effect of these forces is likely to keep employment above its long-run value, but below the impact-period employment level; i.e., employment is expected to fall monotonically to its steady-state level. Output in the future period will be less than in the impact period because the productivity increase has passed and because employment is lower than on impact (on the other hand, capital is higher than on impact). However, the level of output is still above its expected long-run value, so that consumption and investment in the future period will remain above their average values as well. The higher than average investment continues to propagate forward the dynamic effects described above. Thus, we see how even purely transitory disturbances to productivity might manifest themselves as serially correlated movements in output, consumption, investment and employment.

We should note that the economic response to any shock will depend on how persistent it is likely to be. For example, in the case of  $\frac{1}{2} > 0$ ; an increase in  $\pi_t$  will compel individuals to revise their forecasts of future productivity levels upward; the direct impact of this is to increase the marginal benefit of current period investment since the future marginal product of capital is expected to remain high. The higher current period productivity encourages an intratemporal

substitution away from leisure to consumption, but the intertemporal substitution of current leisure for future leisure will be weaker since the higher productivity is expected to persist. Offsetting these substitution effects will be a strong wealth effect, inducing an increase in the demand for all normal goods. In particular, the wealth effect on current period consumption demand should be considerably stronger than when the shock is transitory, although higher interest rates (brought about by high investment demand) should serve to mitigate this effect. In any case, one can again see how these economic mechanisms might propagate impulses through space and time.

### 6.3 Quantitative Analysis

The model is calibrated to match the sample averages for the data generated by the real economy. To this end, the model's deterministic steady-state is characterized; in what follows, let a 'starred' variable (e.g.,  $x^*$ ) denote a steady-state level. For all practical purposes,  $x^*$  will be approximately equal to the average value of  $x$  generated over repeated stochastic simulations.

The functional forms for  $U$ ;  $H$  and  $F$  are specified as follows:

$$\begin{aligned} U(c) &= \ln(c); \\ H(\cdot) &= \frac{\bar{A}}{1-i} \cdot^{1-i}; \\ F(k; n; z) &= \exp(z)k^{1-\mu}n^\mu; \end{aligned}$$

In a deterministic steady-state,  $z = 0$ : These functional forms, together with the first-order conditions derived above, imply the following restrictions for steady-state behaviour:

$$\begin{aligned} \mu \frac{\bar{A} k^{1-\mu} n^\mu}{n^*} \frac{1}{c^*} &= \bar{A}(1-i) n^{*\mu}; \\ -4(1-i) \mu \frac{\bar{A} k^{1-\mu} n^\mu}{n^*} + 1-i &= 1; \\ c^* + x^* &= y^*; \\ y^* &= \frac{\bar{A} k^{1-\mu} n^\mu}{n^*}; \\ x^* &= \pm k^*: \end{aligned}$$

Since measurements of the steady-state values are available (using the sample averages in the data), this information together with the  $\bar{\tau}$ -ve restrictions above could be used to 'estimate' the unknown parameters ( $\mu; \bar{A}; \bar{\tau}; \pm$ ): However, most RBC researchers usually proceed along somewhat different lines (for reasons that I still do not fully understand).

In a decentralized version of the economy above, it is known that the following restrictions must also hold in the steady-state:

$$\frac{w^s n^s}{y^s} = \mu;$$

$$r^s = (1 - \mu) \frac{\bar{A} k^{\alpha} i^{\mu}}{n^s} i^{\pm};$$

where  $r^s$  is the long-run real rate of interest. This sixth restriction pins down a value for  $\mu$ ; using our Canadian data,  $\mu = 0.55$ : The seventh restriction asserts that the real rate of interest is equal to the marginal product of capital net of depreciation; combined with the Euler equation above, this implies

$$\bar{\tau} [1 + r^s] = 1:$$

The real rate of return on capital has averaged in the neighbourhood of 4% per annum over the last forty years; using quarterly data, this implies setting  $r^s = 0.01$ ; which implies  $\bar{\tau} = 0.99$ :

The next step is to combine the  $\bar{\tau}$ -rst and fourth restrictions in order to derive

$$\mu \frac{y^s}{c^s} = \bar{A} (1 - n^s) i^{\bar{\tau} n^s};$$

The parameter  $\bar{\tau}$  relates to the labour supply elasticity and is usually set to a value that is commonly reported by labour econometricians.<sup>12</sup> Measurements of the aggregate data reveal that  $(c^s = y^s) \approx 0.80$  and  $n^s \approx 0.3$  (approximately one-third of discretionary time is devoted to the labour market on average). With  $\mu$  already determined, the restriction above then implies a value for  $\bar{A}$ : The depreciation rate is usually set to  $\pm = 0.025$  (10% per annum), which evidently implies a ratio  $(k^s = y^s)$  that is close to observation (alternatively,  $\pm$  could be chosen so as to replicate the  $x^s = k^s$  ratio).

<sup>12</sup>Alternatively,  $\bar{\tau}$  is often implicitly set to unity by assuming that the function  $H$  is logarithmic.

At this stage, one might ask to what extent any over-identifying restrictions are satisfied. For example, this parameterization implies a particular value for  $(k^a = n^a)$ ; how close is this value to observation? Such questions are rarely asked in practice, possibly because the answer is expected to be embarrassing. However, what remains true is that the parameterized model economy has been calibrated to replicate a variety of key ratios observed to hold in the real economy over long periods of time; at the very least, one could argue that the parameterization is in this sense a 'plausible' one.<sup>13</sup> What remains to be seen is the extent to which the calibrated model can replicate observed business cycle phenomena when subjected to fluctuations in the technology parameter  $z_t$  that mimic the fluctuations in the Solow residual.<sup>14</sup> Here are some numbers reported by Plosser (1989) for an RBC model calibrated to annual data for the postwar U.S. economy; the moments predicted by the model are in parentheses.

Variable	Std Dev	ACF 1{3 Lags			Cor
$\Phi \ln(y)$	2.71 (2.48)	0.13 (0.30)	{0.17 (0.18)	{0.16 (0.14)	1.00 (1.00)
$\Phi \ln(c)$	1.27 (1.68)	0.39 (0.55)	0.08 (0.44)	0.05 (0.37)	0.78 (0.96)
$\Phi \ln(x)$	6.09 (4.68)	0.14 (0.14)	{0.28 (0.00)	{0.19 ({0.02)	0.92 (0.97)
$\Phi \ln(n)$	2.18 (0.89)	0.17 (0.07)	{0.32 ({0.09)	{0.24 ({0.12)	0.81 (0.87)
$\Phi \ln(w)$	1.80 (1.76)	0.44 (0.51)	{0.16 (0.40)	{0.08 (0.33)	0.59 (0.97)

## 7. Business Cycles and Labour Market Search

Much of the research in business cycle theory can be characterized as the search for empirically plausible propagation mechanisms. In this final section, I describe how a simple labour market search environment, based on a framework developed by Pissarides (1990), might propagate transitory disturbances into serially

<sup>13</sup>The extent to which any particular parameterization is deemed 'plausible' likely depends on the purpose at hand. The approach to parameter selection described in the text can be easily justified if the purpose is simply to develop a quantitative theory designed to understand various economic mechanisms or interpret economic phenomena (analogous to the qualitative theories developed and utilized by economists for centuries).

<sup>14</sup>The technology parameter  $z_t$  is measured by the Solow residual:

$$z_t = \ln(y_t) - (1 - \mu) \ln(k_t) - \mu \ln(n_t)$$

The parameters for the stochastic process  $z_{t+1} = \frac{1}{2}z_t + \epsilon_t$  are then estimated via simple regression; typical estimates are  $\frac{1}{2} = 0.95$  and  $\text{std}(\epsilon_t) = 0.007$ :

correlated movements in employment.

The economy is comprised of a fixed number of individuals with mass normalized to unity. Individuals have identical preferences defined over stochastic sequences of consumption:

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t$$

Notice that these individuals are risk-neutral and that they do not value leisure. There is no physical capital and financial markets are absent.<sup>15</sup>

Individuals are endowed with one unit of time per period. At any point in time, individuals differ in terms of their available job opportunities. In particular,  $n_t$  individuals enter period  $t$  with a job opportunity that offers them a real wage  $w_t$ ; given that leisure is not valued, these individuals will supply their time endowment inelastically at this wage. The remaining  $(1 - n_t)$  individuals use their time in the only other available activity: job search. These nonemployed workers can be interpreted as being unemployed.

Not all individuals are employed (even though, in this model, they would like to be) for two reasons: (i) jobs are subject to idiosyncratic shocks that occasionally drive the price of (demand for) their output to zero, an event which drives the worker's wage to zero, thereby inducing either a quit or a layoff; and (ii) workers who 'lose their jobs' in one period cannot instantaneously find another job, but must spend at least some time engaged in active job search.

Assume that there are many potential tasks or jobs in the economy that labour might fruitfully employ labour in production. Jobs exist in three states: active, vacant, or dormant. An active job is one that is currently matched with a suitable employee and producing output; a vacant job is one that is currently engaged in costly recruiting activities in the search for a suitable employee; a dormant job is simply a potential job vacancy. For simplicity, assume that all active jobs are identical in terms of their productivity; let  $y_t = F(z_t)$  equal the value of output produced by one worker at one job when productivity in the business sector is  $z_t$ : Assume that  $F'(z) > 0$ :

Active jobs earn flow profit  $y_t - w_t$  during period  $t$  and with probability  $(1 - \delta)$  survive into the next period; thus  $0 < \delta < 1$  represents the job-destruction rate. In the event of job destruction, the worker becomes unemployed and the job becomes

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<sup>15</sup>This latter assumption is made without loss of generality; given the assumption of risk-neutrality, individuals have no desire to purchase insurance.

either vacant or dormant (whichever yields the greater value). Let  $(J_t; Q_t)$  denote the capital value of an active job and vacant job, respectively; the value of a dormant job is normalized to zero. Given risk-neutral individuals, the equilibrium (gross) rate of interest will equal  $1 = \bar{r}$ ; consequently the relevant discount factor for future profits is  $\bar{r}$ . Anticipating that  $J_t \geq Q_t \geq 0$  in equilibrium, the capital value of an active job satisfies the following recursive relationship:

$$J_t = y_t - w_t + \bar{r} E_t [(1 - \delta)J_{t+1} + \delta Q_{t+1}]$$

Assume that wages are determined by an exogenous share rule  $0 < \alpha_t < 1$  that generates a constant wage  $w = \alpha_t y_t > 0$ : Such an outcome is more likely to occur when individuals are risk-averse and have imperfect access to insurance markets, but nothing below hinges on this result. The key thing to note is that there exists a match surplus that must be divided in some manner between the worker and firm and that here we avoid modelling the negotiation game explicitly.

A vacant job must expend  $\delta > 0$  units of output in search activities in return for the chance of recruiting a suitable employee during period  $t$ ; let  $q_t$  denote the probability of finding a suitable worker. With probability  $(1 - q_t)$  the vacant job fails in its quest and remains dormant for the remainder of the period. Thus, the capital value of a job vacancy must satisfy:

$$Q_t = -\delta + q_t J_t + (1 - q_t)(0):$$

Finally, we assume free-entry in the sense that dormant jobs are free to become vacant jobs if they so choose. Clearly, entry will occur as long as  $Q_t > 0$ : Thus, the assumption of free-entry implies that  $Q_t = 0$  for all  $t$ :

The number of new matches  $m_t$  that occur during period  $t$  depends on the search and recruiting activities of unemployed workers and vacant jobs. The number of unemployed workers is given by  $u_t = (1 - n_t)$ ; let  $v_t$  denote the number of job vacancies posted during period  $t$ : Thus, in the aggregate, there are  $u_t$  workers and  $v_t$  jobs searching for suitable matches. The outcome of this decentralized search market is the formation of a number of 'newly created jobs'  $m_t$ ; the level of which is governed by an aggregate matching technology  $m_t = m(v_t; u_t)$ ; where  $m$  is increasing, concave, and displays constant returns to scale. Under the assumption that vacant jobs are equally likely to find a suitable worker, we have  $q_t = m_t/v_t$ : Define the 'labour market tightness' variable  $\mu_t \equiv v_t/u_t$ : Given constant returns to scale in the matching technology, the matching probability  $q_t$  depends only on  $\mu_t$ : In particular,  $q_t = q(\mu_t)$  with  $q'(\mu_t) < 0$ :

In this model, as with the RBC model studied earlier, there are two state variables: an endogenous state variable  $n_t$ ; and an exogenous state variable  $z_t$ : While there is no physical capital in this model, there is a sense in which the stock of employment  $n_t$  represents a form of 'relationship capital'. In particular, because job-worker matches are costly to form and because they are durable (depreciate at the rate  $\delta$ ), such relationships carry with them the ability to transport productive capabilities across time, much in the same way that physical capital provides a technology that allows the transportation of production across time.

Assume that the idiosyncratic risk faced by individuals 'washes out' in the aggregate; in this case,  $\delta$  represents the fraction of active jobs are destroyed at the end of the period and  $p_t = m_t = u_t$  represents the fraction of unemployed workers that become employed during the period. Thus, the stock of workers employed at the beginning of period  $t + 1$  is given by:

$$n_{t+1} = (1 - \delta)[n_t + p_t(1 - n_t)];$$

where  $p_t = p(\mu_t) = m(\mu_t; 1)$  is increasing in  $\mu_t$ :

In this environment, it turns out that the equilibrium value functions depend only on the exogenous state variable  $z$ :<sup>16</sup> Thus, imposing the free-entry condition  $Q(z) = 0$  for all  $z$ ; the function determining the capital value of an active job can be written as:

$$J(z) = (1 - \delta)w(z)F(z) + \delta \int_0^z J(z^0) dG(z^0);$$

where  $G(z^0) = \Pr[z_{t+1} \leq z^0]$  and  $w(z) = w = F(z)$ : Clearly,  $J(z)$  is an increasing function of  $z$ : Similarly, from the equation that characterizes the value of a vacancy, we have:

$$q(\mu(z))J(z) = \delta v$$

In other words, with  $J(z)$  determined by the former equation, this latter condition determines the equilibrium ratio of vacancies to unemployed workers when technology is  $z$ . Clearly,  $\mu$  is an increasing function of  $z$ : Intuitively, an increase in  $z$  increases the marginal product of labour which increases the value of an active enterprise. As the capital value of production increases, entrants are drawn into the industry. Entrants must, however, first post vacancies in order to recruit workers; this recruitment activity leads to an economy-wide increase in the number of

<sup>16</sup>With risk-averse preferences, the interest rate would depend on  $n_t$  so that capital values would depend on  $n_t$  as well.

vacancies. Since the stock of unemployed workers from which recruitment takes place is fixed in any period, the ratio of vacancies to unemployment rises (the labour market tightens), which reduces the probability that any single vacancy will be successful in finding a worker (the value of  $q_t$  falls). Entry continues to occur until the congestion in the search market reduces the expected capital value of recruitment (the marginal benefit of hiring) to (the marginal cost of hiring).

Consequently, a temporary increase in productivity will lead to a contemporaneous increase in employment (new job matches); i.e.,

$$\frac{dm_t}{dz_t} = (1 - n_t)p^0(\mu_t) \frac{d\mu}{dz_t} > 0:$$

Intuitively, the increase in the number of vacancies means that unemployed workers are more likely to find jobs during their search activity ( $p_t$  rises). The magnitude of the response in new hires is seen to depend on the nature of the matching technology, on the responsiveness of businesses in their recruitment efforts, and on the current level of unemployment (notice that the impact of any shock is larger when unemployment is higher). This impact effect propagates into the future since current recruitment activities to some extent determine the subsequent period's stock of employment. In particular, from the stock-flow equation governing the evolution of  $n_t$ :

$$\frac{dn_{t+1}}{dn_t} = (1 - \delta)(1 - p(\mu_t)) \geq (0; 1):$$

Qualitatively, this model possesses several attractive features. To begin, employment fluctuates in the model despite the fact that individual labour supply is perfectly inelastic.<sup>17</sup> This particular model also displays the feature that purely transitory shocks are propagated dynamically even though individuals have no consumption-smoothing motive.<sup>18</sup> In response to a permanent shock, the forecastable movements in employment can be shown to adjust in the same direction as the forecastable movements in output and consumption.<sup>19</sup> As well, the model

<sup>17</sup>Especially among males, labour supply elasticities are estimated to be very low.

<sup>18</sup>In the RBC model, if individuals were modelled as risk-neutral in consumption, a transitory productivity shock would be fully absorbed by current consumption, leaving investment unaffected.

<sup>19</sup>Rotemberg and Woodford (1996) point out this feature of the data, together with the RBC model's failure to replicate this fact.

is capable of addressing additional stylized facts concerning the cyclical properties of job vacancies (the Beveridge curve). Finally, the model is consistent with a countercyclical wage bill. When this search framework is extended to include physical capital and an intensive labour supply margin, the model's quantitative properties display improvement along several margins relative to a standard RBC model (Andolfatto, 1996). In particular, the search mechanism appears to embody an empirically relevant dynamic propagation mechanism.

## Appendix I Data Source and Construction

### 0.1. CANSIM Label

D10373: Real GDP.

D1: Population.

D20488: Real Private Consumption (including durables).

D20490: Real Private Consumption, motor vehicles, parts and repairs.

D20491: Real Private Consumption, furniture and household appliances.

D20492: Real Private Consumption, other durables.

D20465: Real Government Purchases (includes capital goods).

D20002: Wages, Salaries and Supplemental Income.

D20556: GDP De<sup>o</sup>ator.

### 0.2. Data Construction

1. Per capita output =  $D10373/D1$ .
2. Per capita consumption =  $(D20488\{D20490\{D20491\{D20492+D20465\})/D1$ .
3. Per capita wage bill =  $D20002/(D1\text{E}D20556)$ .

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