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A Joint Symbol Detection Algorithm Efficient at Low SNR for a Multi-Device STBC-MIMO System

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Motivation

- The Optimal detector (maximum likelihood (ML) detector) for Multi-device STBC-MIMO is computationally expensive.
- The Sphere decoding algorithms operation at a low SNR requires inordinately high computation.
- There is need of low complexity algorithms for Multi-device STBC-MIMO systems.

Problem Formulation

- K = Number of Mobile devices/users
- N_B = Total number of transmitted symbols
- N_R = Number of receive antennas
- N_T = Number of receive antennas
- X_k = Symbols transmitted from the *k*th device
- Y = received signal
- M = Size of symbol constellation
- b = $\log_2(M)$
- m = Problem size

Problem Formulation

- Consider the case of single mobile device, *K*=1.
- For T time slots in the space-time code block,the relation between the input and output signal is

$$\tilde{Y} = S \cdot H + \tilde{Z},\ H \in \mathbb{C}^{N_T \times N_R}, S \in \mathbb{C}^{T \times N_T}, \tilde{Y} \in \mathbb{C}^{T \times N_R},$$

where $$S = \sum_{q=1}^{Q} \left[\left(\alpha_q + j\beta_q \right) C_q + \left(\alpha_q - j\beta_q \right) D_q \right]$$

Q is the number of symbols in a space time code block.
Q is complex.

- An alternative representation is

$$Y = X\Omega + Z,\ \Omega \in \mathbb{R}^{2Q \times 2TN_R}$$

Problem Formulation

- For multiple mobile devices

$$\tilde{Y} = \sum_{k=1}^{K} S_k \cdot H_k + \tilde{Z}, S_k \in \mathbb{C}^{T \times N_T}, H_k \in \mathbb{C}^{N_T \times N_R}$$

- An alternative representation is

$$Y = \begin{bmatrix} X_1 & X_2 & \cdots & X_K \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \vdots \\ \Omega_K \end{bmatrix} + Z$$

Optimum Detector

Maximum Likelihood (ML) Detection

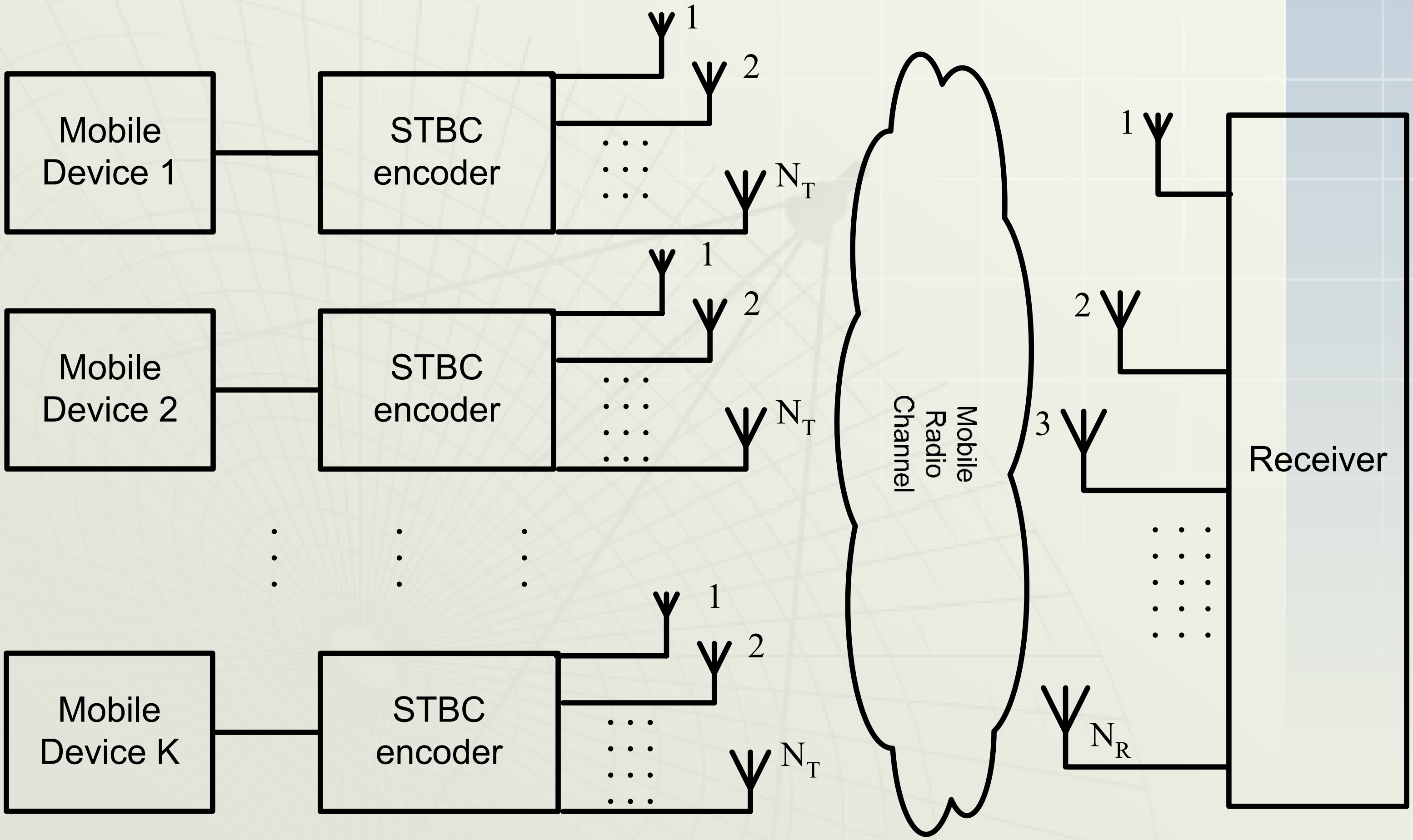
$$\arg\min \left\| Y - \sum_{k=1}^{K} X_k \Omega_k \right\|$$

ML Complexity

- Exponential Complexity (NP-Hard Problem i.e. Can not solve in polynomial time)

- Exhaustive Search with Search Space exponent

$$M^{N_B} = 2^{bN_B}$$


Mobile Device 1
STBC encoder
1
2
N_T
Mobile Device 2
STBC encoder
1
2
N_T
Mobile Device K
STBC encoder
1
2
N_T
Mobile Radio Channel
1
2
3
N_R
Receiver

Cross Entropy Optimization

- The Cross-Entropy optimization is a generalized Monte Carlo technique to solve combinatorial optimization problems
- In general the CEO first associates the target combinatorial optimization problem with a rare-event estimation problem, and then solves the problem iteratively in two phases.

- In first phase the CEO generates a sample of random data according to a specified random mechanism.
- In the second phase it updates the parameters of the random mechanism, typically parameters of probability distribution, on the basis of the data, to produce a better sample in the next iteration.

Cross Entropy Optimization

- Consider a maximization problem: $\underset{x \in \mathcal{X}}{\text{maximize}} \quad F(x)$
- Let us denote a maximum by *x** and the maximal function value by *γ**.
- $f(x;u), x \in \mathcal{X}$ is an arbitrary probability mass function used in at a stage in iterations.
- Hypothetically, if pmf

$$g(x;\gamma,u) = \frac{I_{\{F(x)\geq\gamma\}} f(x;u)}{\sum_{x\in\mathcal{X}} I_{\{F(x)\geq\gamma\}} f(x;u)} \equiv \frac{I_{\{F(x)\geq\gamma\}} f(x;u)}{l(u,\gamma)}$$

is used as the pmf at the next iteration, then every sample generated from this distribution will be a high-quality candidate.

- The CEO algorithm uses in place of $g(x;\gamma,u)$ the pmf that is closest to $g(x;\gamma,u)$ in terms of Kullback-Leibler (KL) distance (cross entropy) . That is, the pmf v that minimizes

$$D\Big(g(x;\gamma,u)\Big\|f(x;v)\Big)=\sum_{x\in\mathcal{X}}g(x;\gamma,u)\ln\frac{g(x;\gamma,u)}{f(x;v)}$$

$$=\sum_{x\in\mathcal{X}}g(x;\gamma,u)\ln g(x;\gamma,u)-\sum_{x\in\mathcal{X}}g(x;\gamma,u)\ln f(x;v)$$

- Minimizing this KL-distance by choosing pmf *v* is equivalent to maximizing

$$\sum_{x\in\mathcal{X}}g(x;\gamma,u)\ln f(x;v),$$

- This is also equivalent to maximizing

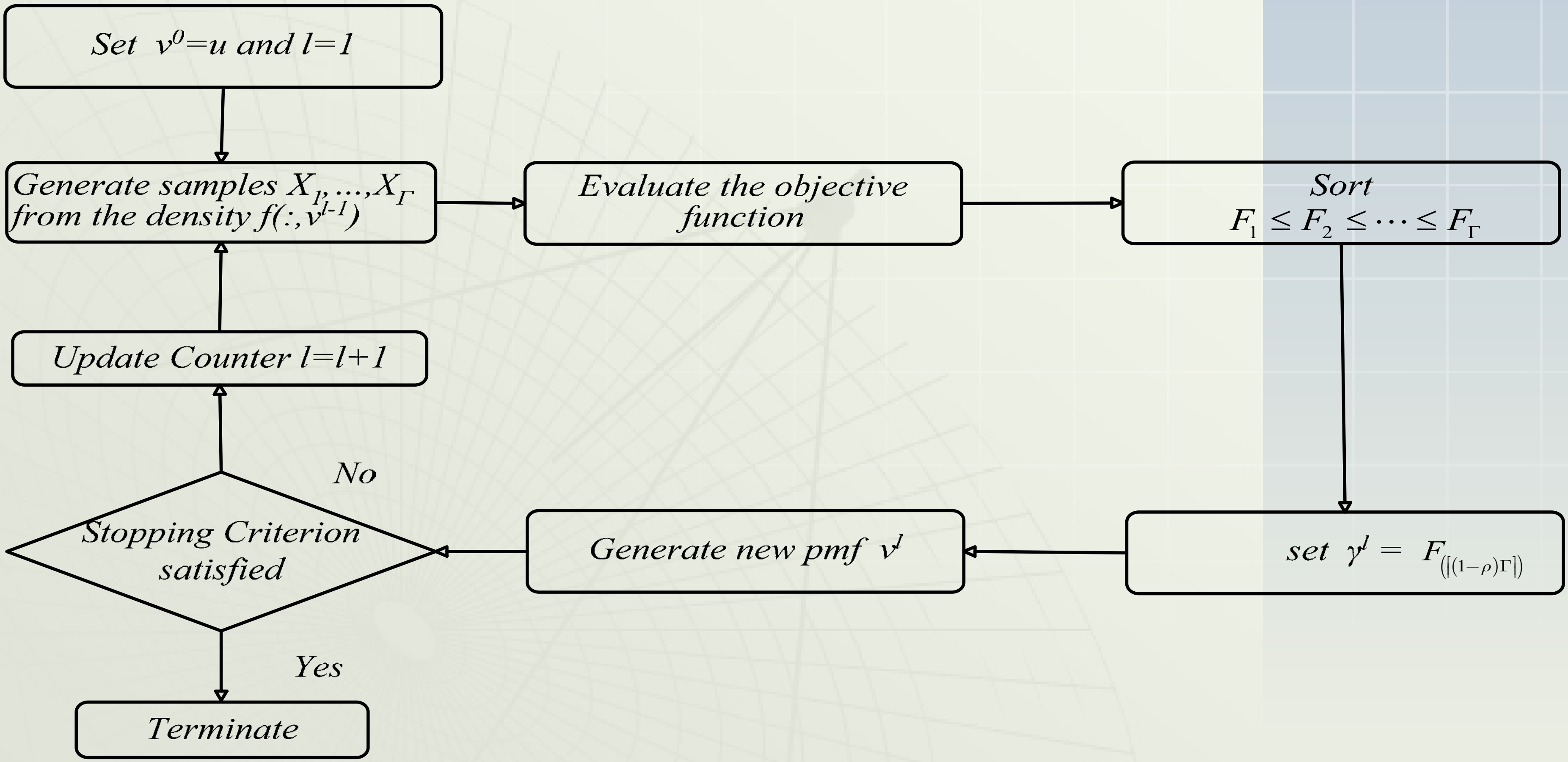
$$\sum_{x\in\mathcal{X}}I_{\{F(x)\ge\gamma\}}f(x;u)\ln f(x;v)=E_u\left[I_{\{F(X)\ge\gamma\}}\ln f(X;v)\right],$$

- In order to avoid computational complexity, a CE algorithm finds in the family of pmfs, a pmf *v* that results in the largest

$$\frac{1}{\Gamma}\sum_{i=1}^{\Gamma} I_{\{F(X_i)\geq\gamma\}} \ln f\left(X_i;v\right),$$

- In general, CE algorithm proceeds as:

1. *Define $v^0=u$, Set $l=1$ (Iteration counter)*
2. *Generate samples $X_1,\ldots,X_\Gamma$ from the density $f(\cdot;v^{l-1})$*
3. *Evaluate the objective function and order them from the smallest to largest: $F_1 \le F_2 \le \cdots \le F_\Gamma$ Then set the (1-ρ)-quantile γ^l as γ^l = $F_{(\lceil(1-\rho)\Gamma\rceil)}$*
4. *Use the same samples $X_1,\ldots, X_\Gamma$ to obtain a new pmf that results in largest (10). Denote this pmf by index v^l.*
5. *If stopping criterion satisfied then terminate otherwise set $l=l+1$ and reiterate from step 2.*


Set $v^0=u$ and $l=1$
Generate samples $X_1,\dots,X_\Gamma$ from the density $f(\cdot;v^{l-1})$
Evaluate the objective function
Sort $F_1 \le F_2 \le \cdots \le F_\Gamma$
set $\gamma^l = F_{(\lceil(1-\rho)\Gamma\rceil)}$
Generate new pmf v^l
Stopping Criterion satisfied
No
Update Counter $l=l+1$
Yes
Terminate

Cross Entropy Optimization For STBC-MIMO

We define the following parameters

1. F denotes the fitness or objective function, which is the target function to be optimized.
2. Γ is the sample size, or population.
3. Parameter m is the dimension of each sample or individual.
4. P is the parameter vector, which indicates the pmf used to generate the random samples.
5. ρ is the rarity parameter. In each iteration, a threshold γ is set to select $\rho\Gamma$ best samples among Γ samples to update P.
6. α is the smoothing parameter.
7. ε is the tolerance. The iteration is stopped when F is within $\pm\varepsilon$ of the target value.
8. I is the maximum number of iterations. The iteration is stopped when I is reached.

Pseudo code of CEO For Sensor Selection

Phase 1:

Initialize ($P^{l=0,}$ ρ, α ε). *Generate initial* Γ *data samples from Bernoulli probabilities using parameter vector* $P^{l=0}$.

Phase 2:

while (tolerance < ε or $l < I$)

Evaluate fitness function F for these Γ *data samples.*

Get the current best value.

If (*best fitness value* < *current best value*)

best fitness value = *current best value*

end

Sort the Γ *data samples according to their fitness value.*

Set $\gamma^{\ell} = F_{(\lceil 1-\rho \rceil \Gamma)}$.

Select the samples whose fitness values are greater than γ^{ℓ}

Generate new parameter vector P^{ℓ} *from selected data samples.*

Update parameter vector using the relation

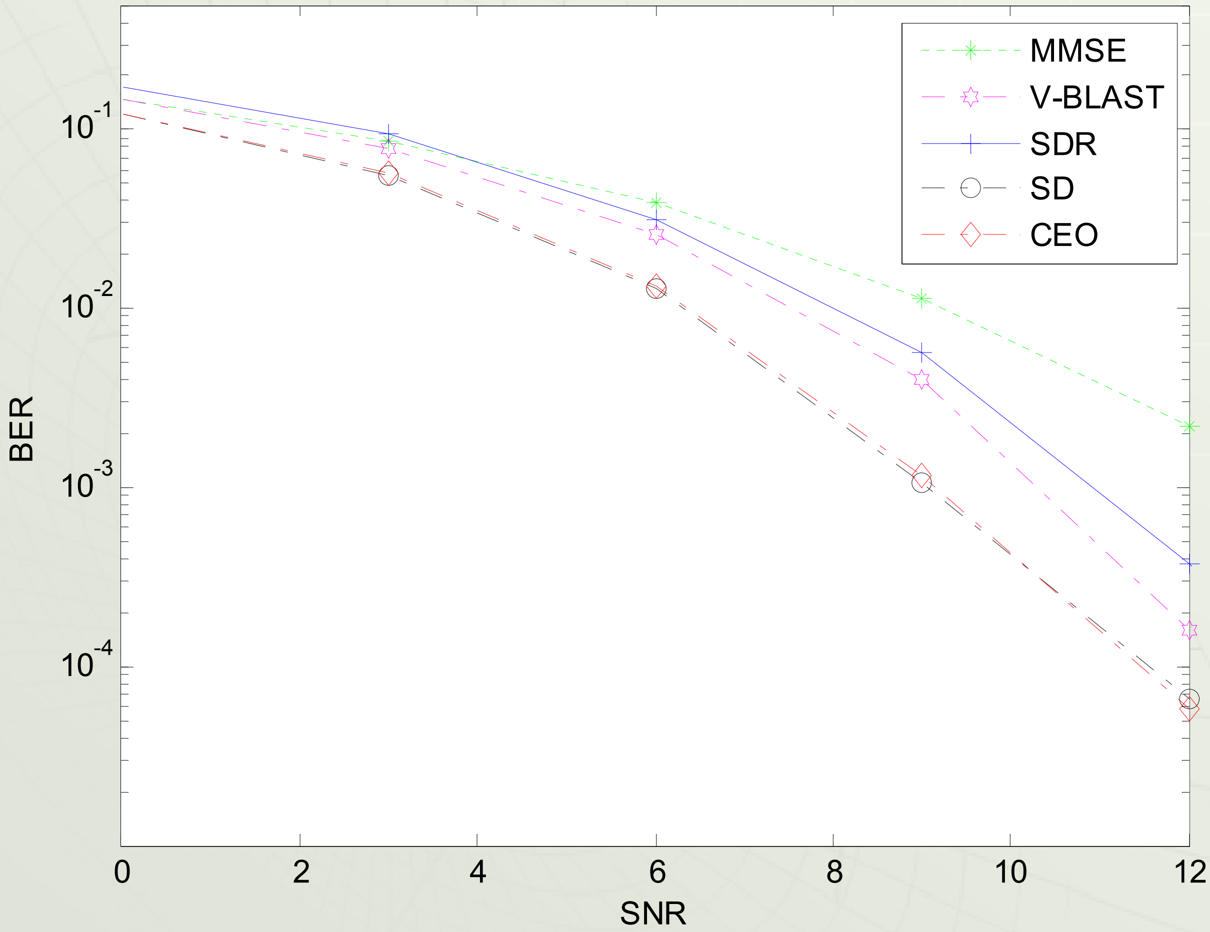
$$P^{\ell} = (1-\alpha) P^{\ell-1} + \alpha P^{\ell}$$

Generate new (1-ρ)Γ samples on the basis of this updated parameter vector P.

End % end while loop

Simulation Results

Bit Error Rate (BER) Performance



Parameters

$N_T=4$

$K=3$

4-QAM

$N_R=4$

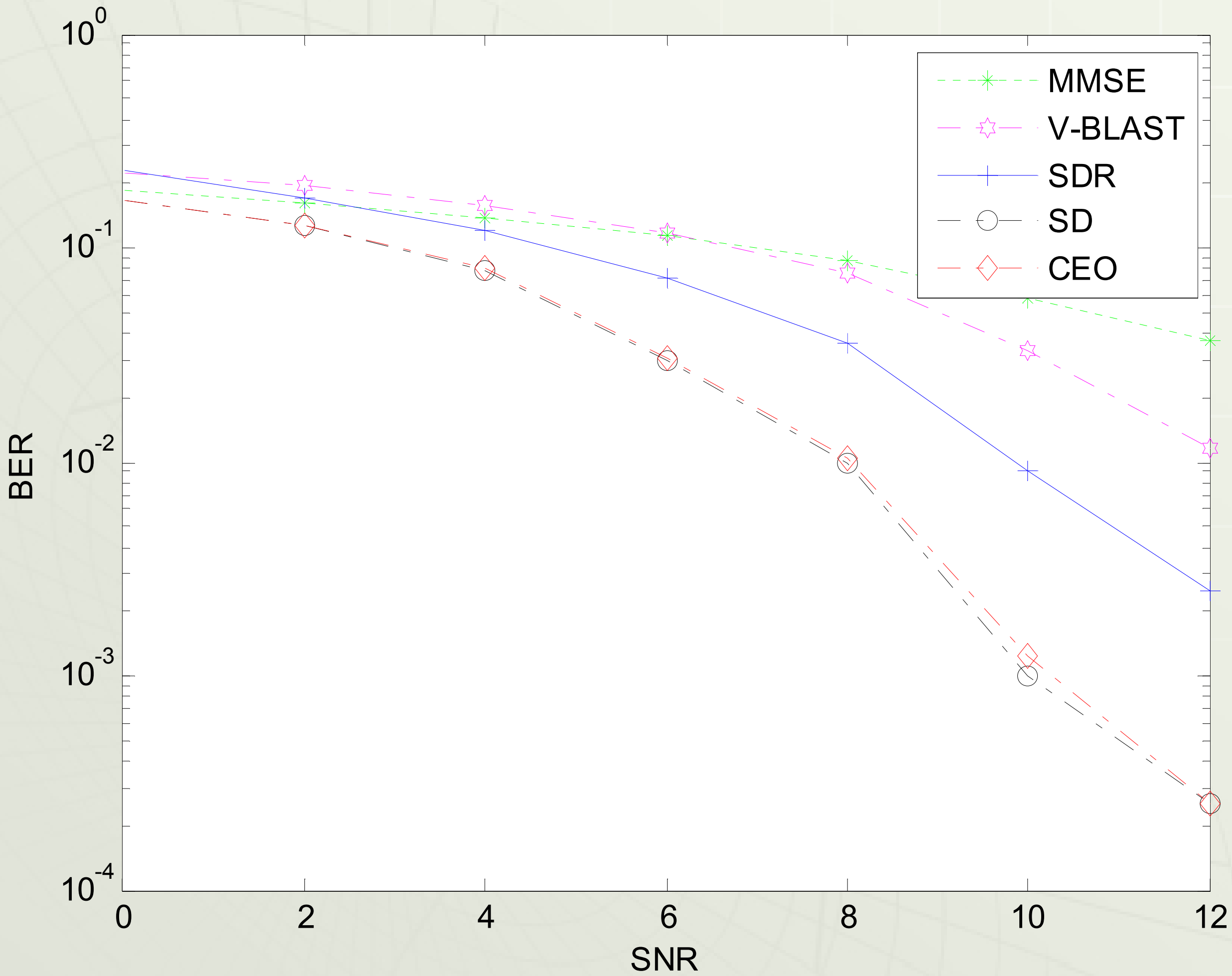
$\Gamma = 100$

$\alpha = 0.7$

$\rho = 0.3$

$m = 24$

Bit Error Rate (BER) Performance



Parameters

N_T=2

K=5

4-QAM

N_R=5

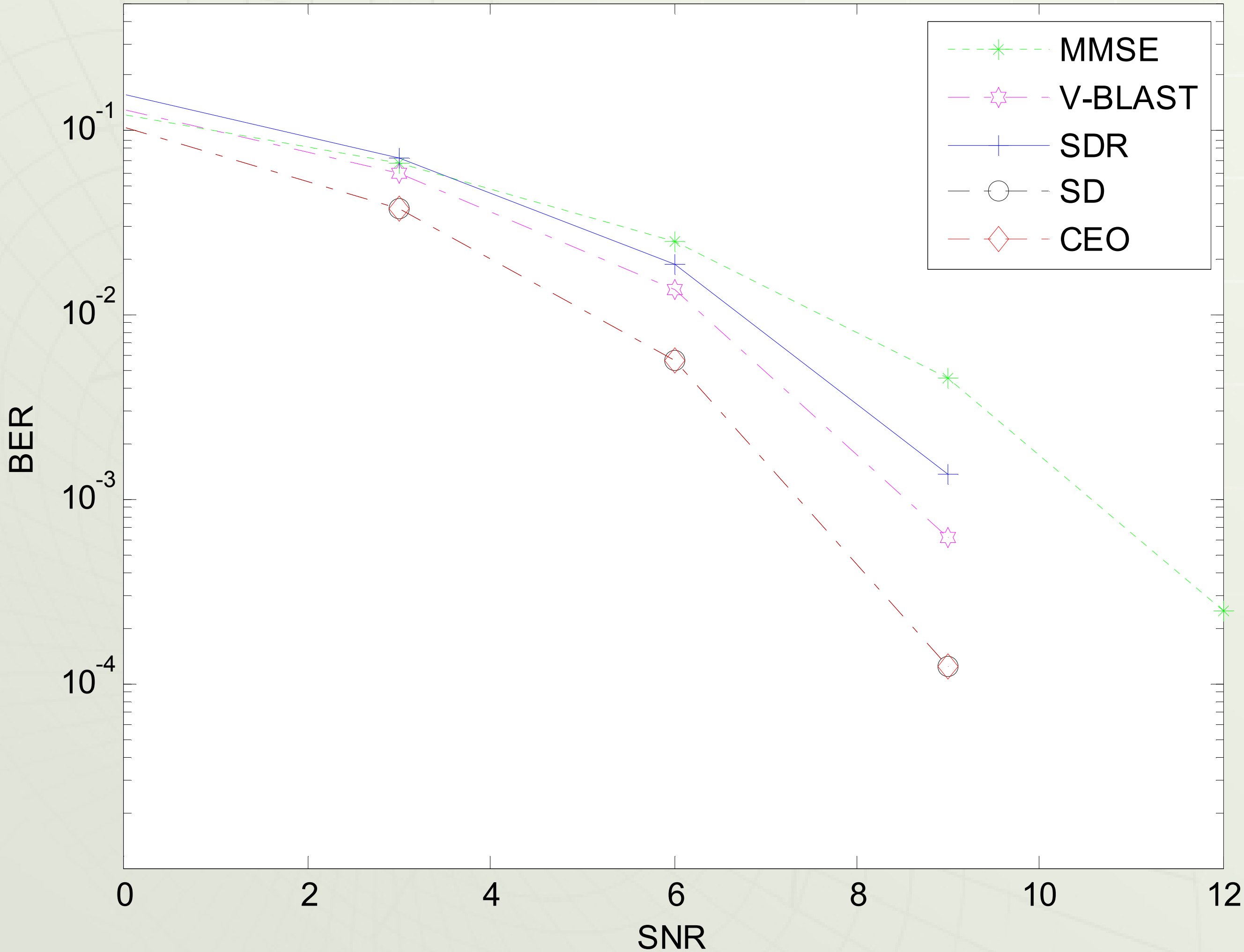
$\Gamma = 200$

$\alpha = 0.7$

$\rho = 0.3$

$m = 20$

Bit Error Rate (BER) Performance



Parameters

N_T=2

K=4

4-QAM

N_R=6

$\Gamma = 80$

$\alpha = 0.7$

$\rho = 0.3$

$m = 16$

Complexity Comparison
Problem Dimension $m \equiv N_B \log_2 M$

- ML $O\left(2^m\right)$
- MMSE $O\left(m^3\right)$
- V-BLAST $O\left(m^3\right)$
- Sphere Decoder $O\left(m^6\right)$
- CEO $O(\Gamma I)$

Conclusions

- The Optimal detector (maximum likelihood (ML) detector) for Multi-device STBC-MIMO is computationally expensive.
- The performance of CEO algorithm is close to the Sphere decoder and better than SDR.
- Complexity of CEO is less than sphere decoder and SDR.

Questions ?