Estimation of Distribution Algorithm for Sensor Selection Problems

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Motivation

The motivations to choose a subset of sensors depend on applications, but in general a small subset means less energy consumption of sensors and simpler computation than operating all sensors. Especially in sensor networks, activating only a subset of sensors can be important in prolonging the network's life time.

THE SENSOR SELECTION PROBLEM

The sensor selection problem can be viewed as a combinatorial optimization problem of selecting from potential *m* sensor measurements a subset of sensor measurements with conflicting goals of maximizing the utility of selected measurements and minimizing the cost.

Problem Formulation

- m = Total number of sensors
- k = Number of selected sensors
- n = Number of parameters to be estimated

The problem is to select *k* sensors from the set of *m* sensors in the system of estimating *n* parameters

Problem Formulation

$x_1, x_2, \ldots, x_n$, the n parameters to be estimated

$Y_1, Y_2, \ldots, Y_m$ scalar measurements from *m* sensors

- The measurements and parameters have linear relation

$$\left(Y_1, Y_2, \ldots, Y_m\right)^T = A\left(x_1, x_2, \ldots, x_n\right)^T + \left(V_1, V_2, \ldots, V_m\right)^T$$

where $A=(a_1, a_2, \ldots, a_m)^T$ is $m \times n$ real-valued matrix known to the system designer. (a_i, $i = 1,2,\ldots,m$, is an *n*-dimensional real-valued column vector.)

- $V_1, V_2, \ldots, V_m$ are i.i.d. additive white Gaussian noise with variance σ^2.

- The maximum likelihood (ML) estimator of $(x_1, x_2, \ldots, x_n)^T$ is

$$\left(A^T A\right)^{-1} A^T \left(Y_1, Y_2, \ldots, Y_m\right)^T = \left(\sum_{i=1}^{m} a_i a_i^T\right)^{-1} \sum_{i=1}^{m} a_i Y_i$$

- This is an unbiased estimator and the estimation error is

$$\left(A^T A\right)^{-1} A^T \left(Y_1, Y_2, \ldots, Y_m\right)^T - \left(x_1, x_2, \ldots, x_n\right)^T$$

$$= \left(A^T A\right)^{-1} A^T \left(V_1, V_2, \ldots, V_m\right)^T$$

- The covariance of this estimation error is

$$\Lambda \equiv \sigma^2 \left(A^T A \right)^{-1} = \sigma^2 \left(\sum_{i=1}^{m} a_i a_i^T \right)^{-1}$$

- Let us denote by $S \subset \{1, 2, ..., m\}$ the set of selected sensors. Then, the ML estimator is

$$\left(\sum_{i \in S} a_i a_i^T \right)^{-1} \sum_{i \in S} a_i Y_i$$

- The volume of the α − confidence ellipsoid of the estimator is

$$K(\alpha, n) \det(\Lambda_S)^{1/2} = K(\alpha, n) \det\left(\sigma^2 \left(\sum_{i \in S} a_i a_i^T \right)^{-1} \right)^{1/2}$$

- The mean radius of the α − confidence ellipsoid is

$$B(\alpha) \det(\Lambda_S)^{1/2n} = B(\alpha) \det\left(\sigma^2 \left(\sum_{i \in S} a_i a_i^T \right)^{-1} \right)^{1/2n}$$

A larger value of $\det\left(\sum_{i \in S} a_i a_i^T\right)$ implies smaller volume or mean radius of the α − confidence ellipsoid, thus means better utility of the estimation. Therefore, the sensor selection problem can be formulated as

$$\begin{array}{ll} \underset{S}{\text{maximize}} & \log\ \det\left(\sum_{i \in S} a_i a_i^T\right) \\ \text{subject to} & |S| = k \end{array}$$

We denote by Φ the collection of all possible sensor selections.

We denote by ϕ in Φ a selection of sensors. Each selection is represented as binary string

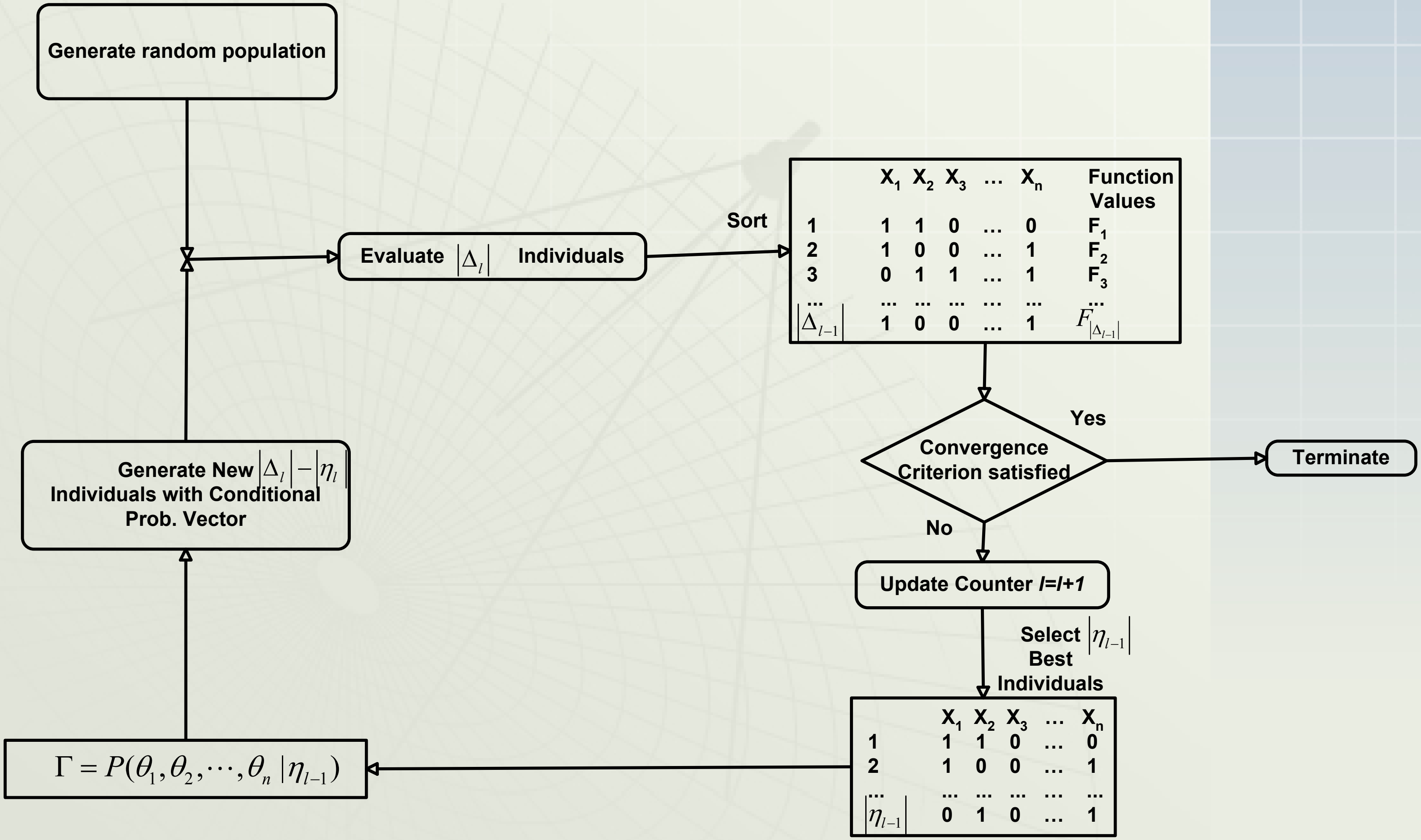
$$Z = \left[z_1 \;\; z_2 \;\; \dots \;\; z_m \right], z_i \in \{0,1\}$$

We can rewrite sensor selection problem as

$$\begin{aligned} &\text{maximize} && \log\ \det\left(\sum_{i=1}^{m} z_i a_i a_i^T \right) \\ &\text{subject to} && \Theta^T z = k \\ & && z_i \in \{0,1\},\ i = 1,\dots,m \ \ and\ \ z \in \Re^m \end{aligned}$$

The Θ is a vector with all entries equal to one.

EDA-Flow Diagram

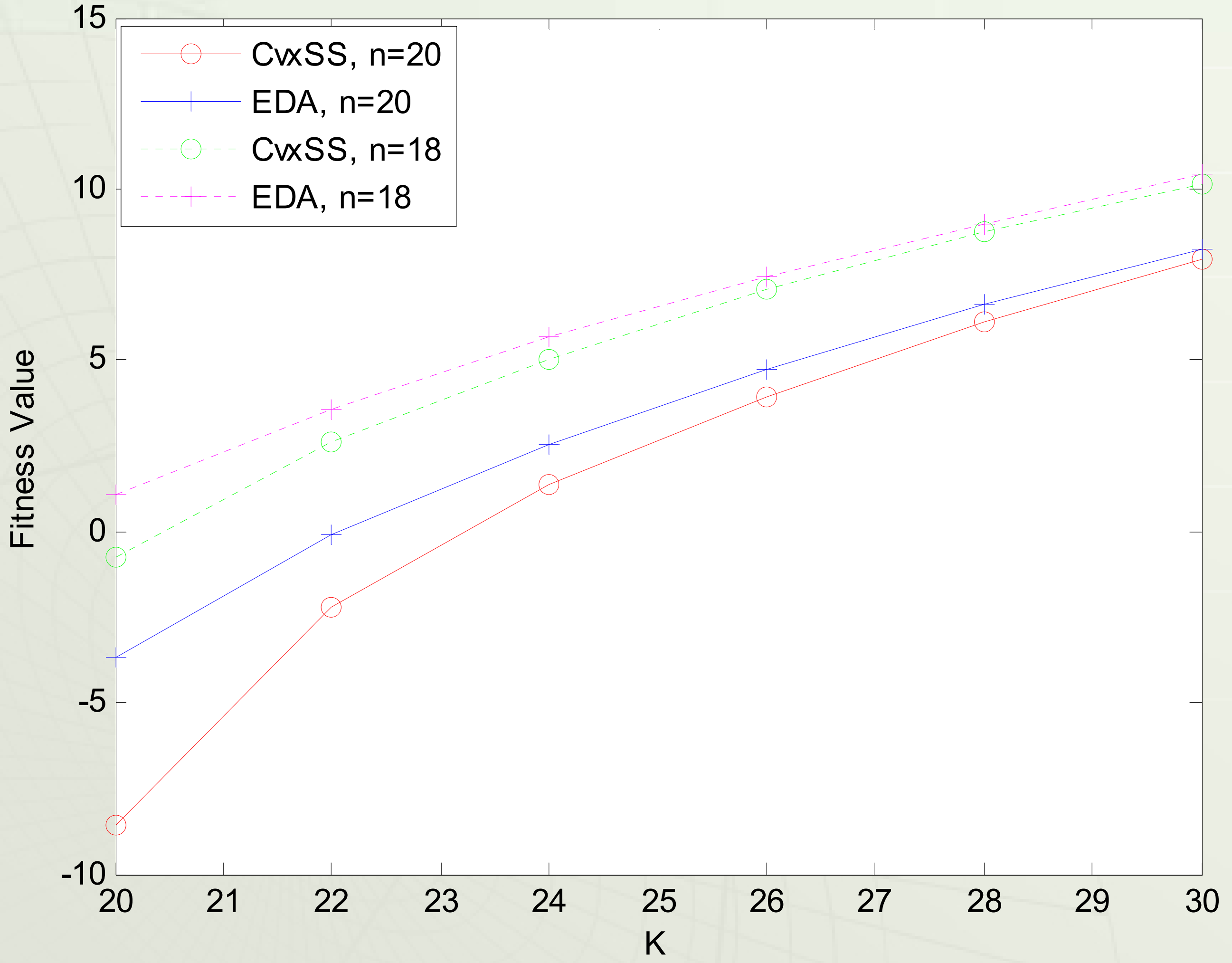


EDA

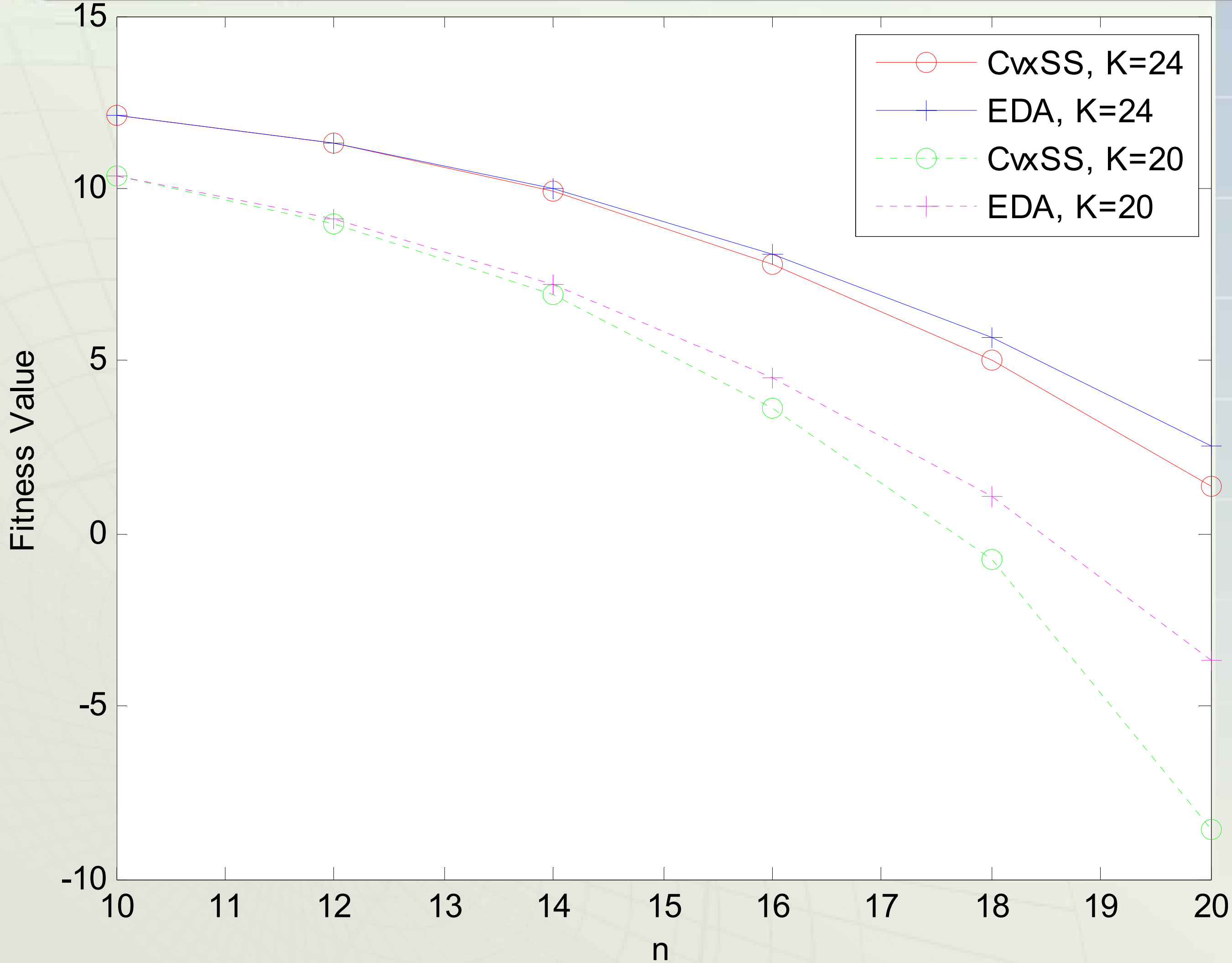
EDA can be characterized by parameters and Notations

1. I_s is the space of all potential solutions
2. F denotes a fitness function.
3. Δ_l is the set of individuals (population) at the l_{th} iteration.
4. η_l is the set of best candidate solutions selected from set Δ_l at the l_{th} iteration.
5. We denote $\beta_l \equiv \Delta_l - \eta_l \equiv \Delta_l \cap \eta_{C\,l}$ .where $\eta_{C\,l}$ is the complement of η_l.
6. p_s is the selection probability. The EDA algorithm selects $p_s|\Delta_l|$ individuals from set Δ_l to make up set η_l.
7. We denote by Γ the distribution estimated from η_l (the set of selected candidate solutions) at each iteration
8. I_{Ter} are the maximum number of iteration

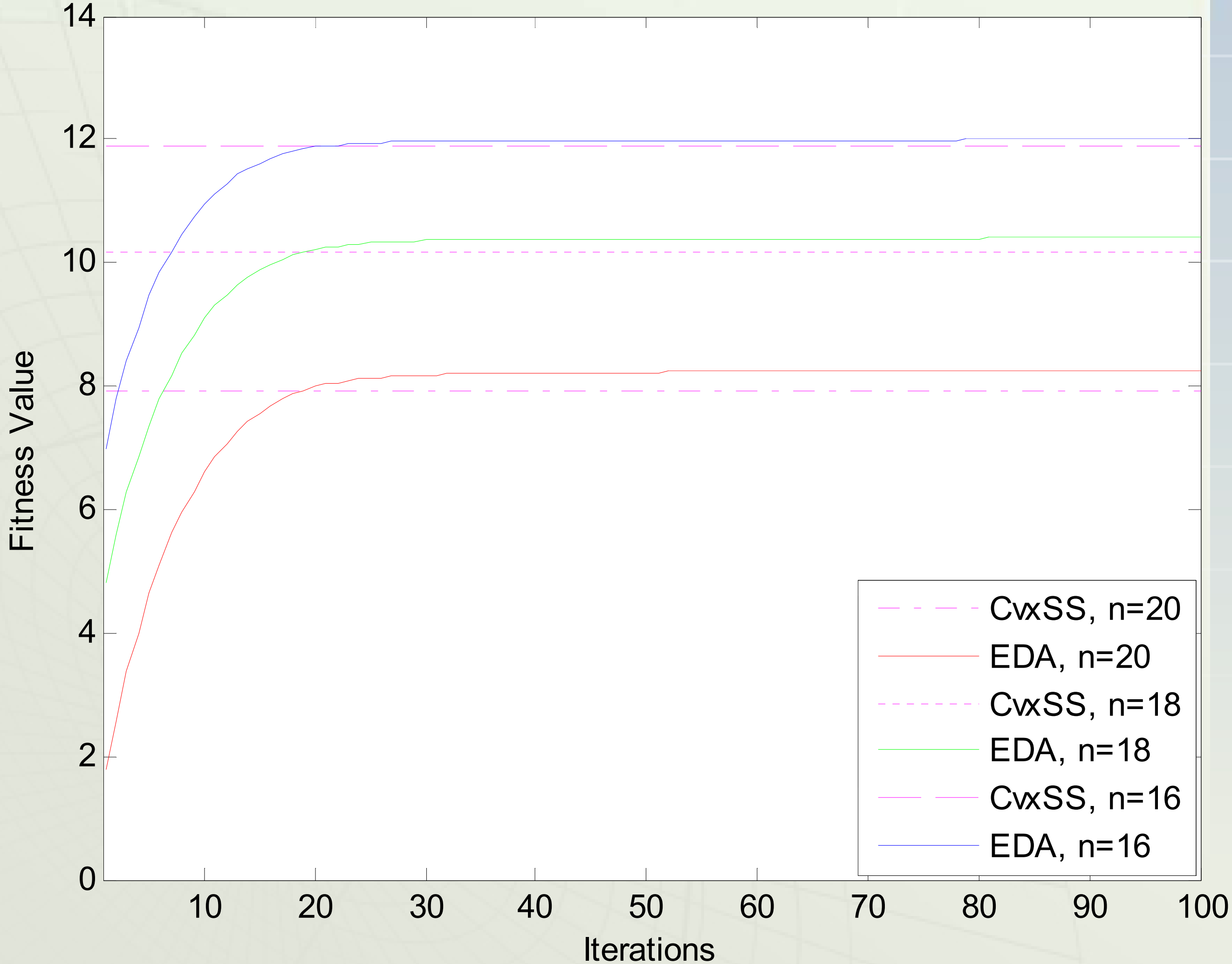
Simulation Results



Performance comparison of EDA and Convex optimization with m=100, n = {18,20}, |Δ|=100, Ψ=100 and p_s=0.3.



Performance comparison of EDA and Convex optimization with m=100, k = {20,24}, |Δ|=100, Ψ=100 and p_s=0.3.



EDA Performance per iteration with m=100, k = 30, |Δ|=100, Ψ=100 and p_s=0.3

Conclusions

- The complexity of sensor selection problem grows exponentially with the number sensors.
- The relaxation of binary constraints can only give upper bound.
- The performance of EDA algorithm is better than convex optimization.
- The EDA surpasses the convex optimization within a few iterations.

Thank You