A Stratified Framework for Handling Conditional Preferences

Ivana Ognjanovic[1] , Dragan Gašević[2], Ebrahim Bagheri[2]
[1]University Mediterranean, [2]Athabasca University
[1]Vaka Djurovica bb, 81000 Podgorica, Montenegro
phone/fax numbers: [1]+38220409210

ABSTRACT
Representing and reasoning over different forms of preferences is of crucial importance to many different fields, especially where numerical comparisons need to be made between critical options. Focusing on the well-known Analytical Hierarchical Process (AHP) method, we propose a two-layered framework for addressing different kinds of conditional preferences which include partial information over preferences and preferences of a lexicographic kind. The proposed formal two-layered framework, called CS-AHP, provides the means for representing and reasoning over conditional preferences. The framework can also effectively order decision outcomes based on conditional preferences in a way that is consistent with well-formed preferences. Finally, the framework provides an estimation of the potential number of violations and inconsistencies within the preferences. We provide and report extensive performance analysis for the proposed framework from three different perspectives, namely time-complexity, simulated decision making scenarios, and handling cyclic and partially defined preferences.
Keywords: conditional preferences, comparative preferences, AHP method, S-AHP method, lexicographic order, well-formed preferences

1. INTRODUCTION

Different researchers have been interested in developing tools and techniques for eliciting, formalizing and interpreting stakeholders' priorities over the existing options such as the pair-wise comparison method, priority groups, networks for decision-making and cumulative ratings (Brafman & Domshlak, 2002; Berander & Andrews, 2006). The representation of preferences and its processing has been studied in many fields such as economics, especially in project and risk management, decision theory, social choice theory, with further developments and applications in areas such as operational research, databases, security analysis, and artificial intelligence. The modeling of user preferences is a great challenge, as it is difficult to express human opinion in a way that can be easily processed by computers (Yu & al., 2010).

As introduced later in this paper, there is a variety of formalisms and methods for addressing different preference structures with scales of input and output information with different semantics. Most of the current approaches collect independent preferences, under the mutual preference independence (MPI) hypothesis (Keeny & Raiffa, 1993), which means that a user's preference for an option is independent of the other options (Yu & al., 2010). However, the MPI hypothesis is not always true in practice (Stirling & al, 2007). People often express *conditional preferences* – they can state their preference for a particular option only when the state of another option is determined (Yu & al., 2010). In fact, conditional preferences appear to be more natural to the human way of thinking (Yu & al., 2010).

The best-known framework for addressing conditional preferences is introduced by CP-nets and TCP-nets (Boutilier & al., 2004; Brafman & Domshlak, 2002). They are powerful methods for representing and reasoning over different sorts of qualitatively defined conditional preferences. According to Behr et al "qualitative reasoning is helpful (not completely necessary) but certainly not sufficient for successful performance on quantitative proportional reasoning problems" (Behr, Harel G, Post, & Lesh, 1992). Thus, research problem which raised was related to providing similarly clear representations or semantics for quantitative comparisons (McGeachie & Doyle, 2011), but this has not been fully explored yet (Boutilier, Bacchus, & Brafman, 2001; McGeachie & Doyle, 2011; Mukhtar, Belaïd, & Bernard, 2009).

On the other hand, traditional elicitation methods are typically developed based on pairwise comparisons and they provide quantitative measurements over unconditional preferences. Analytical Hierarchical Process (AHP) proposed by Satty (Satty, 1980) is a widely adopted multi-criteria decision making method to make complex decisions. AHP enables decision making parties to deal with both tangible and intangible options and monitor the degree of consistency in judgments (Roper-Low, 1990). As a well-accepted method, AHP has extensively been used in many important decision making domains such as forecasting, total quality management, business process re-engineering, quality function deployment, and the balanced scorecard just to name a few (Büyüközkan, Çifçi , & Güleryüz , 2011; Chen & Wang, 2010; Amiri, 2010; Forman & Gass, 2001). It imposes the other direction for resolving the problem of providing the method of quantitative prioritization: keeping the basic characteristics of AHP prioritization method (Ishizaka & Labib , 2011), analyze how it can be extended for addressing different kinds of conditional preferences. Although AHP is simple to perform, it suffers from several problems, such as the quadratic number of comparisons, and inability to compare conceptually dissimilar options. To resolve these issues, in our previous work, we proposed the Stratified AHP (S-AHP) (Bagheri & al, 2010), which tames the number of required comparisons between the available options through the employment of a stratified two-layered approach.

In this paper, we analyze and extend the S-AHP algorithm for handling different sorts of *conditional preferences* which include partially complete and incomplete definitions with the possibility of inducing cycles. Also, a particular form of strong preference which is defined using lexicographic order (Stojmenovic, 1992) is analyzed as a special case of preferences, which is naturally to be expected in different fields and real-life problems. Accordingly, our research objective is defined as: *the formulation of different forms of conditional preferences using a two-layered structural model that handles conditional preferences and partial information over preferences.*

Our framework, called Conditional Stratified AHP (CS-AHP), provides the following major benefits to the process of prioritization and decision making:

1. It presents a framework for representing different forms of preferences over two-layered hierarchical structures;
2. It proposes an extension of the well-known AHP method that enables its use for simultaneously capturing and handling both conditional and unconditional preferences, which might also include preferences about lexicographic order, over the two-layered structure;
3. It is able to recognize the available violations of *well-formedness rules* in the defined conditional and unconditional preferences and to effectively handle them through a *resolution* process.

In the rest of the paper, we first introduce our Stratified Analytic Hierarchical Process (S-AHP) in Sect. 2. Throughout the paper, we employ a widely used benchmark case study (Sect. 2) from the area of software service selection, but it is clear that our proposed work in this paper is general enough to be applied to any prioritization process and domain. In the Sect. 3, we show how S-AHP can be extended to address our research objective within the context of conditional preferences. Sect. 4 introduces dominant relative importance relations, which define lexicographic order over two-layered hierarchical structures. Finally, Sec. 5 formalizes the CS-AHP algorithm as a stratified process for handling different kinds of preferences over the structure introduced in the previous sections. Sec. 6 presents a tool support developed for the CS-AHP practical use in the configuration process of software product lines. The performance analyses from three different perspectives, namely time-complexity, weakness under the cycles and partial definitions of preferences, and simulation analyses, are described in Sect.7. The critical review of methods and frameworks from related work is presented in Sect. 8 before the paper is concluded.

2. BACKGROUND

In this section, we formally describe the Stratified Analytic Hierarchy Process (S-AHP) by introducing a two-layered structural model for representing preferences. First, we introduce the relative importance relation, denoted as $\succ$, for the formalization of stakeholders' preferences. Also, this section introduces a running example, which practically explains the steps of the S-AHP algorithm. Throughout the paper, the same example will be extended for introducing the concepts in our approach.

2.1 Stratified Analytic Hierarchy Process

S-AHP is a prioritization technique that takes the preferences, business goals and high-level objectives of a given group of stakeholders into account in order to find the relative priority and importance of the available options (Bagheri & al, 2010)[1]. In other words, S-AHP helps the stakeholders find the most suitable set of options for their target application by creating a prioritization over all of the available options based on their preferences and objectives.

Simply stated, S-AHP takes a layered approach to the prioritization of the available options. To do this, high level objectives and goals of the stakeholders are specified and are referred to as *concerns*. Concerns are important decision making criteria for the stakeholders. For instance, the concerns of the stakeholders of a software design process can include *implementation costs*, *development time, security*, and *sales*. Once concerns are identified, the options that need to be prioritized are interrelated with the concerns. For instance, in the software design context, one of the options to be evaluated is implementation by using COTS. For the stakeholders, this option entails insignificant implementation costs, quick development time and low security. However, the other option is in-house development, which entails high implementation costs, time consuming development and high security. The formation of this interrelationship between the concerns and the available options allows S-AHP to create a prioritization over the available options by valuing options that are related to more important concerns higher. In our simple example, if security is more important for the stakeholders, in-house development will receive a higher importance and priority; likewise, if lower implementation costs are more essential, COTS-based development will be more attractive.

Formally said, the input to the S-AHP process is a triple *(O, C, QT)* where *O* is the set of available options, each of which is annotated with concerns and a set of relevant qualifier tags. Qualifier tags are different possible enumerations for each concern. For instance, the qualifier tags for the security concern can be secure, open, and vulnerable or for the implementation cost, they can be cheap and expensive; *C* is the set of defined concerns; and *QT* is the set of qualifier tags for the concerns. Concerns and their qualifier tags have a hierarchical structure (which is the basic characteristic of AHP (Satty, 1980)) with two-layers.

---

[1]In our original work on S-AHP, we experimented with the use of S-AHP for configuration of feature models, which are commonly used for modeling variability in software product lines (SPLs).

We introduce the following definitions for specifying the relation of relative importance in the two-layered structure of concerns and qualifier tags:

Definition 1 (Relative importance). Relative importance between concerns (or qualifier tags) *a* and *b* is: $a \overset{\alpha}{\succ} b$ iff concern (or qualifier tag) *a* is more important than concern (or qualifier tag) *b* with coefficient *α*, α>0. Basic characteristics of this relation are:

1. *Reflexivity:* $a \overset{1}{\succ} a$
2. α^{-1}*-Symmetry :* $a \overset{\alpha}{\succ} b \Rightarrow b \overset{1/\alpha}{\succ} a$

Here, *transitivity* does not hold, which means that if $a \overset{\alpha}{\succ} b \wedge b \overset{\beta}{\succ} c$, we cannot make any conclusions about the relation between concerns (or qualifier tags) a and c. ■

Traditionally, the values 1, 3, 5, 7, and 9 are used to represent the degree of importance of different options over each other in AHP (Satty, 1980). They show equality (or indifference), slight value, strong value, very strong and extreme value, respectively. The equality $a \overset{1}{\succ} b$ would show that concern (tag) *a* is equally important as concern (tag) *b* or that the stakeholder is indifferent about the relative importance between concerns (qualifier tags) *a* and *b*.

Based on the relative importance relations, matrices for calculating relative rankings are created in two different levels: the *level of concerns* and the *level of qualifier tags*. They can be formally defined as follows.

Definition 2 (Matrix of relative importance between concerns). Let us suppose that $C = \{c_1, ..., c_n\}$ is the set of concerns. A matrix of relative importance between concerns according to relation $\overset{\alpha}{\succ}$ is defined as $R^{\succ}_{n \times n}(C) = \{R^{\succ}[i,j] = \alpha \mid 1 \le i, j \le n, c_i \overset{\alpha}{\succ} c_j\}$. ■

Definition 3 (Matrix of relative importance for concern c). Let us suppose that concern c is annotated with a set of qualifier tags $QT = \{qt_1, ..., qt_{|QT|}\}$. A matrix of relative importance for concern *c* according to relation $\overset{\alpha}{\succ}$ is defined as follows: $R^{\succ}_{|QT| \times |QT|}(c, QT) = \{R^{\succ}[i,j] = \alpha \mid 1 \le i, j \le |QT|, qt_i \overset{\alpha}{\succ} qt_j\}$. ■

As the relation $\overset{\alpha}{\succ}$ is reflexive and α^{-1}-symmetric, matrices $R^{\succ}(C)$ and $R^{\succ}(c, QT)$ satisfy the following conditions: $R^{\succ}(C)[i,i] = 1$, i.e., $R^{\succ}(c, QT)[i,i] = 1$ and $R^{\succ}(C)[i,j] = \frac{1}{R^{\succ}(C)[j,i]}$, i.e., $R^{\succ}(c, QT)[i,j] = \frac{1}{R^{\succ}(c, QT)[j,i]}$; that is, these matrices are uniquely determined with $\{R^{\succ}(C)[i,j], i < j\}$, i.e., $\{R^{\succ}(c, QT)[i,j], i < j\}$. Accordingly, for cases when we have a set of three concerns $\{c_1, c_2, c_3\}$, its matrix of relative importance is uniquely determined with $\{c_1 \overset{\alpha_{12}}{\succ} c_2, c_1 \overset{\alpha_{13}}{\succ} c_3, c_2 \overset{\alpha_{23}}{\succ} c_3\}$. The unique determination of the matrix for one concern is defined similarly.

The matrixes introduced by Definitions 2 and 3 are filled with values of relative importance between either each pair of concerns or each pair of qualifier tags of one concern. They are defined unconditionally, and, in the rest of the paper, any set of relative importance between either each pair of concerns or each pair of qualifier tags would be considered as *stakeholders' unconditional preferences*.

The following stages can be performed consecutively to produce a valid prioritization over the available options using S-AHP:

The concern ranking step. This stage compares the set of concerns (C) using standard AHP to determine their ranks. Thus, first the relative importance of each concern with respect to the others is defined by the stakeholders and the matrix $R^{\succ}(C)$ is created. Based on the standard AHP algorithm, set $\{r_1, ..., r_n\}$, $0 \le r_i \le 1, \sum_{i=1}^{n} r_i = 1$ of relative ranks for each concern is calculated. The concerns with the highest ranks are then used in the option ranking step and the others are filtered out. In order to keep one of the basic characteristics of AHP (Satty, 1980), that is, the sum of ranks at each level must be equal to one, we choose the reciprocal division, which keeps the previous relative order of concerns.

If the set of concerns $C=\{c_1,...,c_n\}$ is reduced to $m \le n$ concerns, their recalculated ranks $\{r'_{i_1},...,r'_{i_m}\}$ are computed as follows: $r'_{i_k} = r_{i_k}\left(1+\Delta r \Big/ \sum_{j=1}^{m} r_{i_j}\right), k \in \{1,...,m\}$

(1)

where $\Delta r = \sum_{i\in\{1,...,n\}\setminus\{i_1,...,i_m\}} r_i$ is the sum of ranks of less important concerns which are filtered out.

The options ranking step. In order to find the actual rank and relative importance of the available options, the local ranks of the qualifier tags of the most important concerns are computed by performing AHP. For each concern $c_{i_k}, k \in \{1,...,m\}$, the matrix $R^{\succ}(c_{i_k}, QT_{i_k})$ is used for ranking its qualifier tags. Their local ranks are hence $r_1^{i_k},...,r_{|QT_{i_k}|}^{i_k}$. The final ranking for the qualifier tags is obtained by multiplication of its local ranks by the global rank of an appropriate concern, which gives $r_{i_k}' \cdot r_1^{i_k},...,r_{i_k}' \cdot r_{|QT_{i_k}|}^{i_k}, k \in \{1,...,m\}$. Afterwards, the rank of each option is determined based on the rank of the qualifier tags assigned to the concerns attached to each of the options. If an option is not related to a concern, its rank is considered to be zero (Satty, 1980). The goal of this stage is to assign higher ranks to the options which are related to more important concerns from the stakeholders' viewpoint.

Rankings of options are calculated by applying a predefined function *f* (i.e., minimum, maximum, or mean) on the qualifier tag value ranks for each option. The predefined function is a function that is used to select a rank for an option when an option is related to more than one concern. Let us denote with qt_j^i the j^{th} qualifier tag of the concern c_i. The final rank for an option annotated with the set of qualifier tags $\{qt_{j_1}^{i_1},...,qt_{j_k}^{i_k}\}$ is defined as

$r(qt_{j_1}^{i_1},...,qt_{j_k}^{i_k}) = f(r_{i_1}' \cdot r_{j_1}^{i_1},...,r_{i_k}' \cdot r_{j_k}^{i_k})$. **(2)**

2.2 Running example

We will use the problem of service selection in the domain of Service-oriented Computing as the running example. Service-oriented Computing argues the development of services as self-describing, platform-agnostic computational elements that can be composed into distributed applications (Papazoglou, 2003). Often, several companies provide different services with the same functionality, but with different non-functional properties, such as different security levels and costs. In such contexts, making the best decision about the selection of services that accurately satisfies the users' needs is a necessary task. The stakeholders' priorities between different non-functional properties should serve as the basis for the selection of the most appropriate service from the available set.

As an example, let us consider *payment gateways* (e.g., VeriSign, PayPal, and Cyber Source), which are actually services used for providing electronic payments in online stores. Let us also consider *security, customer ease, international sale* and *cost* as non-functional properties (concerns). For our approach, each service contains its related qualifier tags that describe the services in terms of the concerns (as shown in Figure 1). For example, the *PayPal* service contains qualifier tags (C, D, G) meaning that, PayPal has *low Security*, *high Customer ease* and *high International sale*.

Concern	High	Medium	Low
Security	**A**	**B**	**C**
Customer ease	**D**	**E**	**F**
International Sale	**G**	**H**	**I**
Cost	**J**	**K**	**L**

Tag legend

VeriSign	***Cyber Source***	***PayPal***
A, I, L	B, I, K	C, D, G

Figure 1. *Electronic payments services annotated with qualifier tags.*

In the first stage of S-AHP, concerns are ranked based on relative importance between each pair of concerns. Let us suppose that a stakeholder defines relative importance between concerns as follows $Sec \overset{3}{\succ} Cost,\ Sec \overset{5}{\succ} ISale,\ Sec \overset{5}{\succ} CustEase,\ Cost \overset{3}{\succ} ISale,\ Cost \overset{3}{\succ} CustEase,\ CustEase \overset{1/3}{\succ} ISale$. Based on Satty's standard AHP, ranks for concerns *Security, Cost, International sale* and *Customer ease* are 0.54, 0.24, 0.14, and 0.08, respectively. This shows that the stakeholders have chosen *International sale* and *Customer ease* as less important concerns. Therefore, the S-AHP approach proposes that given the fact that these concerns are less important that they should be removed and the sum of their ranks should be distributed to *Security* and *Cost* according to their ranks. To follow our example, the value of the summarized ranks of *International sale* and *Customer ease* is 0.14+0.08=0.22. This value is now distributed to *Cost* and *Security* and hence the new values are 0.54+0.54/(0.54+0.24)*0.22=0.69, and 0.24+0.24/(0.54+0.24)*0.22=0.31, respectively.

In the next stage, the available options are ranked based on the ranks of the qualifier tags of each concern. Analogous to the first stage, AHP is used for calculating local ranks of qualifier tags of each concern. Let us suppose that the calculated local ranks for high, medium and low *Security* are 0.6, 0.3 and 0.1, respectively; for high, medium and low *Cost* are 0.2, 0.3, 0.5, respectively. Global ranks of qualifier tags are obtained as multiplication of their local ranks and ranks of corresponding concerns.

The *VeriSign* service is annotated with high *Security*, low *International sale* and low *Cost*, and its final rank is an average sum of the global ranks of the annotated qualifier tags. As the *International sale* concern is recognized as a less important concern, the average sum consists only of the remaining two concerns: (0.69*0.6+0.31*0.5)/2=0.285. On the other side, any qualifier tag of concern *Cost* is not assigned to *PayPal* service, and thus, the final rank of *PayPal* is: *(0.69*0+0.31*0.1)/2=0.015*.

3. CONDITIONAL PREFRENCES IN THE TWO-LAYERED STRUCTURAL MODEL

In this section, the relation of relative importance is extended with elements of Propositional Logic for representing conditionally defined preferences. Also, the S-AHP algorithm is analyzed for addressing conditionally defined preferences and the notion of *well-formedness* rules is introduced.

3.1 Conditional preferences

Before introducing the formal concept of conditionality, let us introduce a simple example for explaining the nature of conditional preferences about relative importance of concerns and their qualifier tags.

Example 1. In the running example, let us consider a stakeholder who defines relative importance between concerns in the following manner:

Unconditional relative importance: *Cost* $\overset{3}{\succ}$ *ISale* **(3)**

Conditional relative importance: in case of a high priority of *International Sale* then *Sec* $\overset{5}{\succ}$ *CustEase*, otherwise *Sec* $\overset{3}{\succ}$ *CustEase*, *Cost* $\overset{3}{\succ}$ *Sec*. **(4)**

In the second level, i.e., the level of qualifier tags, the stakeholder might conditionally define the importance between the qualifier tags of the *International sale*, concern as follows:

In case of a high priority given to *Security*, low values of *International sale* are strongly unacceptable is comparison to high and medium values (i.e., *ISale.High* $\overset{5}{\succ}$ *ISale.Low, ISale.Medium* $\overset{5}{\succ}$ *ISale.Low*); otherwise, low values are slightly less acceptable than either medium or high values (i.e.,*ISale.High* $\overset{1}{\succ}$ *ISale.Medium, ISale.High* $\overset{3}{\succ}$ *ISale.Low, ISale.Medium* $\overset{3}{\succ}$ *ISale.Low*). **(5)**

It is important to mention that the relative importance between the qualifier tags of one concern might not be defined with only one conditional statement, i.e., the stakeholders might define more conditional statements about relative importance between the qualifier tags of *International sale* as follows.

In case of a low priority assigned to *Cost*, there is a slight preference of medium *International sale* over high *International sale* and strong preference of medium over low values of *International sale* (i.e. *ISale.Medium* $\overset{3}{\succ}$ *ISale.High, ISale.Medium* $\overset{5}{\succ}$ *ISale.Low*); otherwise, high values of *International sale* have a slight and strong priority in comparison to medium and low values, respectively (i.e. *ISale.High* $\overset{3}{\succ}$ *ISale.Medium, ISale.High* $\overset{5}{\succ}$ *ISale.Low*) **(6)** ■

We can see that the relative importance between two concerns might depend on the qualifier tags of other concerns. Also, relative importance between qualifier tags of one concern might depend on the qualifier tags of other concerns. We consider that any other way of dependency is not to be naturally expected in the two-layered structure of concerns and qualifier tags. In order to formally define the presented conditional preferences, we introduce Definitions 4 and 5.

Definition 4 (Conditional preference between concerns). Let us suppose that $C=\{c_1,...,c_n\}$ is a set of concerns and $QT=\bigcup_{i=1}^{n} QT_i$ is the set of their qualifier tags. Conditional preference between concerns $\{c_{j_1},..,c_{j_k}\}$, $j_l \in \{1,..,n\}, l \in \{1,...,k\}$ according to relation $\overset{\alpha}{\succ}$ is defined as a 5-tuple $(\Psi, C, QT, R^{\succ}(C)_1, R^{\succ}(C)_2)$ where:

1. Ψ is a propositional logic formula over a set of qualifier tags $\bigcup QT_j, QT_j \in QT / \bigcup_{l=1,k} QT_{j_l}$ as propositional variables connected with $\wedge$, $\vee$, and $\neg$ representing logical operators conjunction, disjunction and negation, respectively;
2. $QT_i \in QT$ is a set of qualifier tag values for concern c_i;
3. $R^{\succ}(C)_1$ is the relative importance matrix for concerns if Ψ is true;
4. $R^{\succ}(C)_2$ is the relative importance matrix for concerns if Ψ is false.

Whether Ψ is true or not determines which matrix of relative importance would be applied. This can be represented by pseudo-logical statements in the following format: $\Psi : R^{\succ}(C)_1 \,|\, R^{\succ}(C)_2$ ■

Informally interpreted, relative important matrix $R^{\succ}(C)_1$ represents the *then* part of a preference and matrix $R^{\succ}(C)_2$ represents the *else* part of the preference.

For example, conditional preference between three given concerns $\{c_1, c_2, c_3\}$, can be specified as $\Psi : c_1 \overset{\alpha_{12}}{\succ} c_2, c_1 \overset{\alpha_{13}}{\succ} c_3, c_2 \overset{\alpha_{23}}{\succ} c_3 \mid c_1 \overset{\beta_{12}}{\succ} c_2, c_1 \overset{\beta_{13}}{\succ} c_3, c_2 \overset{\beta_{23}}{\succ} c_3$. That is, if Ψ is satisfied, the matrix of relative importance is defined by $\{c_1 \overset{\alpha_{12}}{\succ} c_2, c_1 \overset{\alpha_{13}}{\succ} c_3, c_2 \overset{\alpha_{23}}{\succ} c_3\}$, otherwise by $\{c_1 \overset{\beta_{12}}{\succ} c_2, c_1 \overset{\beta_{13}}{\succ} c_3, c_2 \overset{\beta_{23}}{\succ} c_3\}$. Restriction on the condition of Ψ for the preference between concerns $\{c_{j_1}, .., c_{j_k}\}$, which cannot contain qualifier tags of any concern from the set $\{c_{j_1}, .., c_{j_k}\}$, is a part of Definition 4. This means that concerns cannot depend on their own qualifier tags.

The formal presentation of the preference **(4)** according to Definition 4 is: *ISale.High: Sec* $\overset{5}{\succ}$ *CustEase | Sec* $\overset{3}{\succ}$ *CustEase, Cost* $\overset{3}{\succ}$ *Sec.*

Definition 5 (Conditional preference for concern c_i). Let us suppose that C is a set of concerns and $QT = \bigcup_{i=1}^{n} QT_i$ is their appropriate set of concern qualifier tags. Conditional preference for concern $c_i \in C$ according to relation $\overset{\alpha}{\succ}$ is defined as a 5-tuple $(\Psi, c_i, QT_i, R^{\succ}(c_i, QT_i)_1, R^{\succ}(c_i, QT_i)_2)$ where:

1. Ψ is a propositional logic formula over set of qualifier tags $\bigcup_{QT_j \in QT \setminus QT_i} QT_j$ as propositional variables connected with $\wedge$, $\vee$, and $\neg$ representing logical operators conjunction, disjunction and negation, respectively;
2. c_i is the concern of interest;
3. $QT_i \in QT$ is the set of qualifier tag values for concern c_i;
4. $R^{\succ}(c_i, QT_i)_1$ is the relative importance matrix for concern c_i if the condition holds;
5. $R^{\succ}(c_i, QT_i)_2$ is the relative importance matrix of concern c_i if the condition does not hold.

Whether Ψ is satisfied or not determines which matrix of relative importance will be applied. This can be defined by pseudo-logical statements in the form $\Psi : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$. ■

For example, conditional preference for a concern with only three tag values $\{qt_1, qt_2, qt_3\}$, can be specified as $\Psi : qt_1 \overset{\alpha_{12}}{\succ} qt_2, qt_1 \overset{\alpha_{13}}{\succ} qt_3, qt_2 \overset{\alpha_{23}}{\succ} qt_3 \mid qt_1 \overset{\beta_{12}}{\succ} qt_2, qt_1 \overset{\beta_{13}}{\succ} qt_3, qt_2 \overset{\beta_{23}}{\succ} qt_3$. This means that if condition Ψ is satisfied, the matrix of relative importance is determined by $\{qt_1 \overset{\alpha_{12}}{\succ} qt_2, qt_1 \overset{\alpha_{13}}{\succ} qt_3, qt_2 \overset{\alpha_{23}}{\succ} qt_3\}$, otherwise, by $\{qt_1 \overset{\beta_{12}}{\succ} qt_2, qt_1 \overset{\beta_{13}}{\succ} qt_3, qt_2 \overset{\beta_{23}}{\succ} qt_3\}$. Restrictions on condition Ψ for relative importance between the qualifier tags of concern c_i, which cannot contain qualifier tags of concern c_i, is a part of Definition 5. This means that qualifier tags cannot depend on themselves. Furthermore, situations when options are not related to concern(s) from condition Ψ are analyzed and discussed later in the Section 5.

The formal presentation of the preference **(5)** is: *Sec.High: ISale.High* $\overset{5}{\succ}$ *ISale.Low, ISale.Medium* $\overset{5}{\succ}$ *ISale.Low | ISale.High* $\overset{1}{\succ}$ *ISale.Medium, ISale.High* $\overset{3}{\succ}$ *ISale.Low, ISale.Medium* $\overset{3}{\succ}$ *ISale.Low*.

3.2 Addressing conditional and unconditional preferences

In the previous example, none of the preferences completely defines the relative importance between each pair of concerns, i.e., each preference only reflects the relative importance between concerns that depend on the specified condition, leaving the others to be defined with new preferences or to remain undefined. More precisely, preference **(3)** defines relative importance only between concerns *Cost* and *ISale* while together with preference **(4),** relative importance between concerns *ISale* and *Sec* still remains undefined. Furthermore, relative importance for concern *Security* is defined with two different conditional preferences, **(5)** and **(6).** For all other concerns, preferences are specified either conditionally or unconditionally and only joining them together, the whole information about importance for the level of concerns can be defined. Formally, *stakeholders' preferences*, consist of parts of matrices introduced by Definitions 2-5,

$$\Re = \left\{\left\{R^{\succ}(\overline{C})\right\}_{\overline{C}\subset P_C}, \left\{R^{\succ}(c_i, QT_i)\right\}_{i\in\{1,\ldots,n\}}, \left\{\left(\Psi, \overline{\overline{C}}, QT, R^{\succ}(\overline{\overline{C}})_1, R^{\succ}(\overline{\overline{C}})_2\right)\right\}_{\overline{\overline{C}}\subset P_C}, \left\{\left(\Psi, c_j, QT_j, R^{\succ}(c_j, QT_j)_1, R^{\succ}(c_j, QT\right.\right.$$

, where P_C is a set of all subsets of the set of concerns C. The goal is to fill the whole matrix for the level of concerns (Definition 2) and n matrixes for the level of qualifier tags (Definition 3: n matrices of dimension $|QT_i| \times |QT_i|$ where $|QT_i|$ is the number of qualifier tags for the i^{th} concern). The calculation of local and global ranks is enabled only with completely filled matrices for both levels.

Let us continue with Example1 and show how the matrices should be filled based on the specified preferences.

Example 1(Continued). In this example, there are four concerns for ranking. As previously stated, preference **(3)** unconditionally defines the importance between concerns *Cost* and *International sale*; preference **(4)** defines in the *then* part of the importance between *Security* and *Customer ease* and in the *else* part between two pairs of concerns: *Security* and *Customer ease*, and *Cost* and *Security*. Now, we can see two things: first, we cannot make a decision which part of the preference (then or else) to use; and second, any part that we take, the matrix for the level of concerns will not be completely filled (six positions in the matrix should be filled, but we have at most two or three values depending on the part of the preference that we choose).

The decision about the satisfied part of a conditional preference cannot be made without concrete options which should be ranked based on the ranks of the appropriate qualifier tags Let us consider two different available options in this model: *VeriSign* and *Cyber Source* from the Section 2.2. For either option, local and global ranks for the concerns and qualifier tags should be calculated separately and based on them, the more appropriate option should be chosen.

Let us consider the first option. Low *International sale*, which annotates this option, makes the *else* part of conditional preference **(4)** satisfied. By joining preference **(4)** with unconditional preference **(3)**, we have: $Cost \overset{3}{\succ} ISale, Sec \overset{3}{\succ} CustEase, Cost \overset{3}{\succ} Sec$ .Values between other pairs of concerns are not specified. By default, we consider that undefined relative importance represents the stakeholders' indifference and according to the comments that followed Definition 1, indifference is equal to one. So, for the level of concerns, we have a matrix, which is defined by:

$$Cost \overset{3}{\succ} ISale, Sec \overset{3}{\succ} CustEase, Cost \overset{3}{\succ} Sec, Cost \overset{1}{\succ} CustEase, ISale \overset{1}{\succ} CustEase, ISale \overset{1}{\succ} Sec\ .$$

Now, Satty's standard AHP gives rank values 0.25, 0.19, 0.16 and 0.40 for concerns *Security*, *Customer ease*, *International sale* and *Cost,* respectively. **(7)**

This shows that the first stage of prioritizing concerns is completed. The next stage covers prioritization of the qualifier tags of each concern. Analogous to the level of concerns, the satisfied parts of preferences **(5)** and **(6)** should be chosen, joined together and sets of ranks calculated with the standard AHP. In the case of the first available option, the *then* parts of both preference **(5)** and **(6)** are satisfied. When joined, they give the following preference: *ISale. Medium* $\overset{3}{\succ}$ *ISale.High*, *ISale.High* $\overset{5}{\succ}$ *ISale.Low*, *ISale.Medium* $\overset{5}{\succ}$ *ISale.Low*. So, local ranks for qualifier tags of the *International sale* concern are: 0.30, 0.60 and 0.10 for high, medium and low values, respectively. Finally, their global ranks are:

$ISale.High : 0.16*0.30 = 0.05, ISale.Meidum : 0.16*0.60 = 0.10, ISale.Low : 0.16*0.10 = 0.016$.

Now, let us consider the next option, Cyber Source. The same as in the case of the first option, we have the following values for concerns *Security*, *Customer ease*, *International sale*, and *Cost*: 0.25, 0.19, 0.16 and 0.40, respectively. For the level of qualifier tags, the *else* part of preference **(5)** is satisfied which gives ($ISale.High \overset{1}{\succ} ISale.Medium$, $ISale.High \overset{3}{\succ} ISale.Low$, $ISale.Medium \overset{3}{\succ} ISale.Low$), and also the *else* part of preference **(6)** is satisfied, which gives ($ISale.High \overset{3}{\succ} ISale.Medium$, $ISale.High \overset{5}{\succ} ISale.Low$). However, these two sets of preferences cannot be joined together because they do not uniquely define the relative importance between the *ISale.High* and *ISale.Low* qualifier tags. This makes the calculation of local ranks for qualifier tags of the *International sale* concern *impossible*. ■

Situations like the one above should be recognized as non-well-formed-preferences of stakeholders. In order to detect such non-well-formed preferences, we introduce the following definition; well-formedness and its violations are discussed in Section 5.

Definition 6 (Well-formed set of preferences). A set of preferences is *well-formed* if it satisfies the following conditions:

1. Each preference is defined in the form of Definitions 2-5;
2. Each value in the matrix for the concern level is undefined or is uniquely defined by unconditional preferences and under the satisfied part (then or else) of conditional preferences;
3. Each value in the matrix for each concern in the qualifier tags level is undefined or is uniquely defined by unconditional preferences and under the satisfied part (then or else) of conditional preferences.

If any of the specified conditions is not satisfied, the set of preferences is ***not well-formed.*** ■

Finally, as we discussed in the previous example, a set of available options might be ranked based on the preferences defined on an appropriate set of concerns and their qualifier tags if the set of preferences is well-formed. This statement is formally specified and defined in Section 7.

4. DOMINANT RELATIVE IMPORTANCE

In this section, we introduce the definition of dominant relative importance, which corresponds to the definition of the well-known *lexicographic order* (Stojmenovic, 1992). Also, we explain how it can be combined with conditional and unconditional preferences introduced in the previous section and how it can be addressed by the CS-AHP algorithm.

4.1 Dominant relative importance for two-layered structure

Lexicographic order is a special way of sorting, well-known in the literature (Wilson, 2011).It is a generalization of the way the alphabetical order of words is based on the alphabetical order of letters. Lexicographic orders can be viewed as being composed of a set of particular kinds of strong preference statements where the choice of values of a variable dominates the assignments to a set of other (less important) variables (Wilson, 2011).

This special form of preference has applications in many different fields and real-life problems. In our illustrative example, concerning the selection of the most appropriate service for electronic payments in online stores, let us suppose that another stakeholder has very limited budget, but she needs high or medium security. It means that available services with low price should be the highest ranked regardless of their other characteristics (8). Difference between them should be made according to medium or high *Security (9)*. Also, it is natural to allow the stakeholder to include conditionality in preferences about relative importance between *Customer ease* and *Inter-*

national sale. We consider that it is not natural to be expected any conditionality in specifying dominant importance of low *Cost* and later, medium or high *Security*.

Dominant relative importance in two-layered structure is introduced by the following definition.

Definition 7 (Dominant relative importance of concern c_i). Let us suppose that $C = \{c_1, ..., c_n\}$ is a set of concerns and $QT = \bigcup_{i=1}^{n} QT_i$ is the set of their qualifier tags. Concern c_i has dominant relative importance to the others (denoted as $\overset{D}{\succ}(c_i)$) if the ranking over the set of options is defined as follows:

An option o_1, which is annotated with the set of qualifier tags $\{qt_{j_1^1}^{i_1^1}, ..., qt_{j_k^1}^{i_k^1}\}, k \leq m$, has higher rank than an option o_2 annotated with the set of qualifier tags $\{qt_{j_1^2}^{i_1^2}, ..., qt_{j_l^2}^{i_l^2}\}, l \leq m$, iff one of the following conditions is satisfied:

c1) a qualifier tag from the set QT_i annotates only the option o_1;

c2) no qualifier tag from the set QT_i annotates any of options o_1 and o_2 and an option o_1 is higher ranked based on qualifier tags of other concerns attached to it;

c3) when both options are annotated with qualifier tags from the set QT_i, the following two cases are distinguished:

c3.a) both options are annotated with the same qualifier tag from the set QT_i and an option o_1 is higher ranked based on qualifier tags of other concerns attached to it;

c3.b) option o_1 is annotated with the qualifier tag with higher rank from the set QT_i.

In all other cases option o_2 is higher or equally ranked to the option o_1. ■

We can see that intuitive explanation *(8)* from the previous example is formalized with *(c1)* and explanation *(9)* with *(c3)*, while *(c2)* is related to the comparison of options that are not related to the concern with dominant relative importance (i.e. comparison with no dominant relative importance).

To exemplify, let us consider the following five options: *o_1={Sec.Low, CustEase.Low, ISale.High}*, *o_2={Sec.Low, CustEase.Low, ISale.High}*, *o_3={Sec.High, CustEase.High, ISale.High}*, *o_4={CustEase.Low, ISale.High}*, *o_5={CustEase.High, ISale.High}*. Also, let us keep the dominant relative importance of concern *Security* with the highest priority for low values and lower priorities for medium and high values, respectively. Furthermore, let us suppose that combination of qualifier tags *{CustEase.High, ISale.High}* is ranked higher in comparison to *{CustEase.Low, ISale.High}…(10)*. Now, let us prioritize the mentioned five options.

According to *(c1)*, the leading three options o_1, o_2, o_3 are ranked higher than last two options o_4 and o_5. Also, according to (c2) and (10), option o_5 is higher ranked then o_4. The (c3) gives ranking between o_1, o_2, o_3 as follows: (c3.a) with (10) gives higher ranking to o_3 than to o_2; and (c3.b) gives higher ranking to o_1 in comparison to o_2, o_3. Finally, the ordering between available options from the highest ranked to the lowest is given with: o_1, o_3, o_2, o_5, o_4.

Based on the definition of dominant relative importance of one concern, we can define dominant relative importance of *k*-tuple of concerns as follows.

Definition 8 (Dominant relative importance of *k*-tuple of concerns $(c_{i_1}, ..., c_{i_k})$). Let us suppose that $C = \{c_1, ..., c_n\}$ is a set of concerns and $QT = \bigcup_{i=1}^{n} QT_i$ is the set of their qualifier tags. The *k*-tuple of concerns $(c_{i_1}, ..., c_{i_k})$ has dominant relative importance (denoted as $\overset{D}{\succ}(c_{i_1}, ..., c_{i_k})$) if concern c_{i_1} has dominant relative importance to the others, and each concern c_{i_j}, $j \in \{2,...,k\}$ has dominant relative importance to concerns $C \setminus \{c_{i_1},...,c_{i_{j-1}}\}$. ■

We can see that in the example from the beginning of this section, the pair of concerns *(Cost, Security)* has dominant relative importance. Additionally, the *low Cost* is more important than *medium* and *high*, and *medium* and *high* values of *Security* are more important than the *low* value. It is necessary to notice that, by introducing dominant relative importance of cost, with the most preferable low value, it is not important and meaningful to determine whether low cost is more or much more important than medium and high values. That information would not influence the final ranking of options, because dominant relative importance ensures that any service with low cost would be higher ranked than any other service with medium or high cost.

In that sense, stakeholders should only define *qualitative order* between qualifier tags of each concern with dominant relative importance. In the rest of the paper, we assume that the set of qualifier tags of each concern with dominant relative importance is ordered increasingly. By $r_D(qt_j^i)$, we denote the index of the j^{th} qualifier tag in increasingly ordered array of qualifier tags of the concern c_i. So, in the running example of the previous paragraph, we have: $r_D(Cost.High) = 1, r_D(Cost.Medium) = 2, r_D(Cost.Low) = 3.$

(11)

By introducing the definition of relative importance of the whole set of concerns *C*, we define lexicographic order over the two-layered structure of concerns and qualifier tags. For the sake of simplicity, let us assume that an arranged set of concerns $\{c_1,...,c_n\}$ has the dominant relative importance, and qualifier tags of each concern are ordered increasingly, i.e. qt_1^i is the qualifier tag of concern c_i with the lowest rank. The final ranking over all possible options over the set of concerns *C* and qualifier tags $QT = \bigcup_{i=1}^{n} QT_i$, with dominant relative importance of the whole set *C* is illustratively represented in Figure 2. The lowest ranked option is annotated with the lowest ranked qualifier tag of the concern with the least dominant relative importance. On the other hand, the highest ranked option is annotated with the highest ranked qualifier tags of each concern.

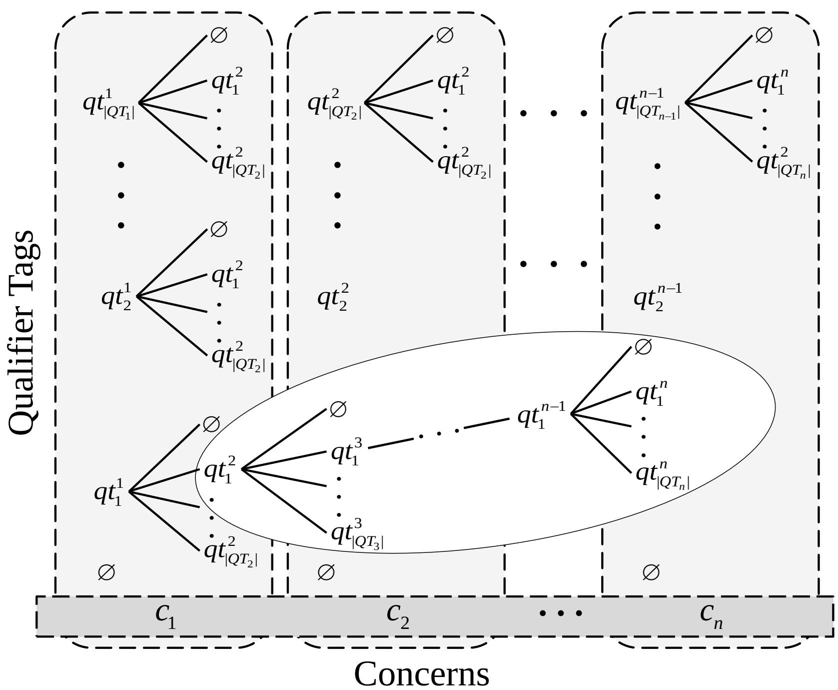


Figure 2.*Schematic representation of lexicographical order over two-layered structure of concerns and qualifier tags.*

In order to address this kind of preferences, we introduce aggregation function *F*. Based on this function, we define ranking $\bar{r}$ which addresses conditionally and unconditionally defined preferences with dominant relative importance of a subset of concerns. In the following subsection, it is also proven that in case of absence of preferences about dominant relative importance, $\bar{r}$ is reduced to the ranking introduced in Sections 2 and 3.

4.2 Addressing dominant relative importance

First, we introduce a well-known example of positional numeral systems, developed between the 1st and 5th centuries by Indian mathematicians (Porubský), which served as the inspiration for the solution of the problem of addressing dominant relative importance. Positional numeral system is a system for representation of numbers by an ordered set of numerals symbols (called digits) in which the value of a numeral symbol depends on its position. For each position, a unique symbol or a limited set of symbols is used. The value of a symbol is given by the weight of its position expressed in the bases (or radices) of the system. The resultant value of each symbol is given by the value assigned to its position (e.g., by a product of the bases) and modified (e.g., multiplied) by the value of the symbol. The total value of the represented number in a positional number is then sum of the values assigned to the symbols of all positions (Glaser, 1981).

The ordering between numbers might be interpreted as dominant importance of higher positions and the value of the number $a_n ... a_1 a_0$ in base-*b* positional system is obtained as $a_n \cdot b^n + ... + a_1 b + a_0$, where $a_i \in \{0,...,b-1\}, i \in \{0,...,n\}$.

In the two-layered structure of concerns and qualifier tags, we found an analogy to the positions and digits in concerns with dominant relative importance, and their qualifier tags, respectively. Also, by analogy to the mixed-base positional system (Porubský), we define the *k*-dimension array of bases to *k*-tuple of concerns with the dominant relative importance, as follows:

Let us suppose that $C = \{c_1,...,c_n\}$ is a set of concerns and $QT = \bigcup_{i=1}^{n} QT_i$ is the set of their qualifier tags. To the *k*-tuple of concerns $(c_{i_1},...,c_{i_k})$ which has the dominant relative importance, we assign bases defined with: $(\prod_{j=2}^{k}(|QT_{i_j}|+1), \; ..., \; (|QT_{i_{k-1}}|+1)(|QT_{i_k}|+1), \; |QT_{i_k}|+1, \; 1)$ and, finally, aggregated function for an option, which is annotated with the set of qualifier tags $\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\}$, is defined by:

$$F(qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}) = \sum_{l=1}^{k} b_l \cdot r_D(qt_{j_l}^{i_l}) + r(\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\} \setminus \{qt_{j_1}^{i_1},...,qt_{j_k}^{i_k}\}), s \le m,$$

where
$$b_l = \begin{cases} 1, & l = k \\ \prod_{j=l+1}^{k}(|QT_{i_j}|+1), & 1 \le l < k \end{cases} \qquad \textbf{(12)}$$

where *r()*and *r_D()*are previously defined, with **(2)**and **(11),** respectively.

Based on aggregated function F, we can define the final rank of the option annotated with the set of qualifier tags $\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\}$ with $\bar{r}(qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}) = F(qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}) \Big/ \prod_{j=1}^{k}\left(|QT_{i_j}|+1\right)$ **(13)**

For introducing the specified formulas in **(12)** and **(13),** we should prove that:

1) Aggregation function defined by **(12)** defines the ranking over options based on rules defined in Definitions 7 and 8 about dominant relative importance;
2) Ranking defined with **(13)** defines rank values in the interval [0, 1] ;
3) In case of the lack of concerns with dominant relative importance, formula **(13)** is reduced to formula **(2).**

Lemma. In case of dominant relative importance of *k*-tuple of concerns $(c_{i_1},...,c_{i_k})$ and arbitrary combination of qualifier tags $\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\}, s \le m$, an inequality

$$\sum_{l=u}^{k} b_u r_D(qt_{j_l}^{i_l}) + r(\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\} \setminus \{qt_{j_1}^{i_1},...,qt_{j_k}^{i_k}\}) \le b_{u-1}, \text{ where } b_0 = \prod_{j=1}^{k}\left(|QT_{i_j}|+1\right)$$

holds for each $1 \le u \le k$.

As a direct consequence of Lemma (case when *u=1*) and the fact that in case of no dominant relative importance, then sum $\sum_{l=1}^{k} b_l r_D(qt_{j_l}^{i_l})$ equals 0, we formulate the following propositions.

Proposition 1. Aggregation function defined by **(13)** defines ranking over options based on the set of conditional and unconditional preferences with relations of relative and dominant relative importance according to Definitions 7 and 8.

Detailed proofs of Lemma and Proposition 1 are given in Appendix A.

Proposition 2. For each option *o* annotated with the set of qualifier tags $\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\}, s \le m$, the following is satisfied:

1. $0 \le F(qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}) \le \prod_{j=1}^{k}\left(|QT_{i_j}|+1\right)$, independently on *s;*
2. In case of no dominant relative importance then holds: $\bar{r}(qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}) = r(qt_{j_1}^{i_1},...,qt_{j_s}^{i_s})$. ■

As a conclusion of the propositions, we have that ranking $\bar{r}$ defined in **(13)** addresses conditionally and unconditionally defined preferences with dominant relative importance of a subset of concerns in the two-layered structure of concerns and qualifier tags.

Now, let us extend our example from the beginning of this section to show how the introduced aggregated function works.

Example 2. Let us suppose that the stakeholder from Section 4.1 additionally defines the following preferences: for the level of concerns, $Sec.High : ISale \overset{5}{\succ} CustEase \mid ISale \overset{1}{\succ} CustEase$; and, for the level of qualifier tags, $CustEase.High \overset{3}{\succ} CustEase.Low, CustEase.Medium \overset{3}{\succ} CustEase.Low$

and $ISale.High \overset{3}{\succ} ISale.Low, ISale.High \overset{3}{\succ} ISale.Medium$. Based on qualitative ranks for qualifier tags of concerns with dominant relative importance, introduced in **(11)**, we have that $r_D(Cost.High) = 1, r_D(Cost.Medium) = 2, r_D(Cost.Low) = 3$. Also, the stakeholder defines that she needs high or medium security, which means that *Sec.High* and *Sec.Medium* are with the same relative importance in decision making. So, their ranks are obtained as follows: $r_D(Sec.High) = 2, r_D(Sec.Medium) = 2, r_D(Sec.Low) = 1$. Consequently, the bases assigned to the pair of concerns with dominant relative importance *(Cost, Security)* are *(3,1).*

On the other hand, based on the standard AHP method, global ranks for concerns *CustEase* and *ISale* are, in case of *Sec.High*, *0.167* and *0.833*, respectively; otherwise ranks for both concerns are equal to 0.5. Local ranks for qualifier tags of concern *CustEase* are: *0.43, 0.43*, *0.14*, respectively for high, medium and low values. Local ranks for high, medium and low *ISale* are *0.6, 0.2, 0.2,* respectively.

Finally, *VeriSign* payment service is annotated with high Security, low International sale and low Cost, and its rank is obtained, according to **(13)** with

$\bar{r}(Cost.Low, Sec.High, ISale.Low) = (9 + 2 + 0.833 \cdot 0.2)/(9 + 2 + 1) = 0.93$.

5. CS-AHP FOR ADDRESING CONDITIONAL PREFERENCES WITH DOMINANT RELATIVE IMPORTANCE

This section explains how the S-AHP algorithm might be extended for addressing the problem of ranking available options based on conditionally and unconditionally defined preferences, which might include dominant relative importance. The overall process of CS-AHP is similar to the original S-AHP previously introduced in Section 2. However, some stages have been changed and new stages have been added. So, CS-AHP consists of the following stages:

Preferences definition. At this stage, a stakeholder defines preferences on both levels – concerns and qualifier tags. The number of preferences about each pair of concerns and qualifier tags is not limited. The stakeholder is allowed to define more conditional preferences about relative importance of the same pair of concerns or qualifier tags, or even leave them to be undefined. In case of dominant relative importance, relative ranks should be defined according to **(11).**

The first condition from Definition 6 is checked at this stage, i.e., the stakeholder is not allowed to specify preferences about relative importance between concerns (qualifier tags) that depend on the qualifier tag(s) of any of those concerns.

For the next step of CS-AHP, we assume that the set of available options is specified and annotated with at most one qualifier tag per concern.

Concerns separation. Based on defined preferences, a subset of concerns with dominant relative importance is singled out.

The following steps aims at ranking the qualifier tags of the remaining concerns. It is applied in the following stages for each option separately.

Determination of the set of preferences from the level of concerns. For each option, conditions in each conditional preference are checked. If the condition is satisfied, the *then* part of the preference is chosen, otherwise its *else* part is selected. It is important to mention that not all options are annotated with qualifier tags of each concern, so, situations where a condition cannot be checked for a certain option are possible. In that case, we consider the logical value of the whole condition to be false, i.e., the option does not satisfy the condition.

Determination the ranks for the level of concerns. Based on the unconditional preferences and satisfied parts of the conditional ones, the matrix for the level of concerns is filled. In case that it cannot be uniquely filled, the set of preferences is declared not to be well-formed and the stakeholder is allowed to make a decision about multiple values for filling the matrix. Then, the ranks are calculated and less important concerns are filtered out. Their ranks should be proportionally shared with others as described in Section 2.

Determination of the set of preferences from the level of qualifier tags. Similarly to the level of concerns, each conditional preference between the qualifier tags is checked and situations when conditions cannot be checked should be addressed in the same way. If the stakeholders have filtered out a concern in the previous step, it is a sign that it is of less interest in the ranking process and conditions which include its qualifier tags should be modified. We modify the preferences, so that they cannot depend on qualifier tags that are of less interest:

If we denote by *qt* any qualifier tag of the less important concerns, each conditional preference should be transformed iteratively using the following rules:

Rule 1: Conditional preferences in the forms $\Psi \wedge qt : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$ or $\Psi \wedge \neg qt : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$ should be transformed into $\boldsymbol{\Psi : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2}$;

Rule 2: Conditional preferences in the forms $\Psi \vee qt : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$ or $\Psi \vee \neg qt : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$ need to be transformed into $\Psi : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$;

Rule 3: Conditional preferences in the forms $qt : R^{\succ}(c_i, QT_i)_1 \mid \varnothing$ or $\neg qt : R^{\succ}(c_i, QT_i)_1 \mid \varnothing$ should be considered as inappropriate ones for that option and would not be considered;

Rule 4: Conditional preferences in the forms $qt : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$ or $\neg qt : R^{\succ}(c_i, QT_i)_1 \mid R^{\succ}(c_i, QT_i)_2$ should be considered as not being well-formed; thus, the process terminates.

Rules 1 and 2 should be repeated until either of the conditions defined in Rule 3 or 4aresatisfied or a preference, which does not contain any qualifier tags of the filtered concerns, is obtained.

Determination of the ranks for the level of qualifier tags. Based on the unconditional preferences and satisfied parts of the conditional ones, the matrices for the most significant concerns are filled. In case that any of them cannot be uniquely filled, the set of preferences is declared as not being well-formed and the stakeholder is again allowed to make a decision about its resolution. Then, the local ranks for sets of qualifier tags of each concern are calculated and based on the previously calculated ranks for the set of concerns, global ranks are delivered.

Final option ranking. Final option ranking is obtained according to **(13).**

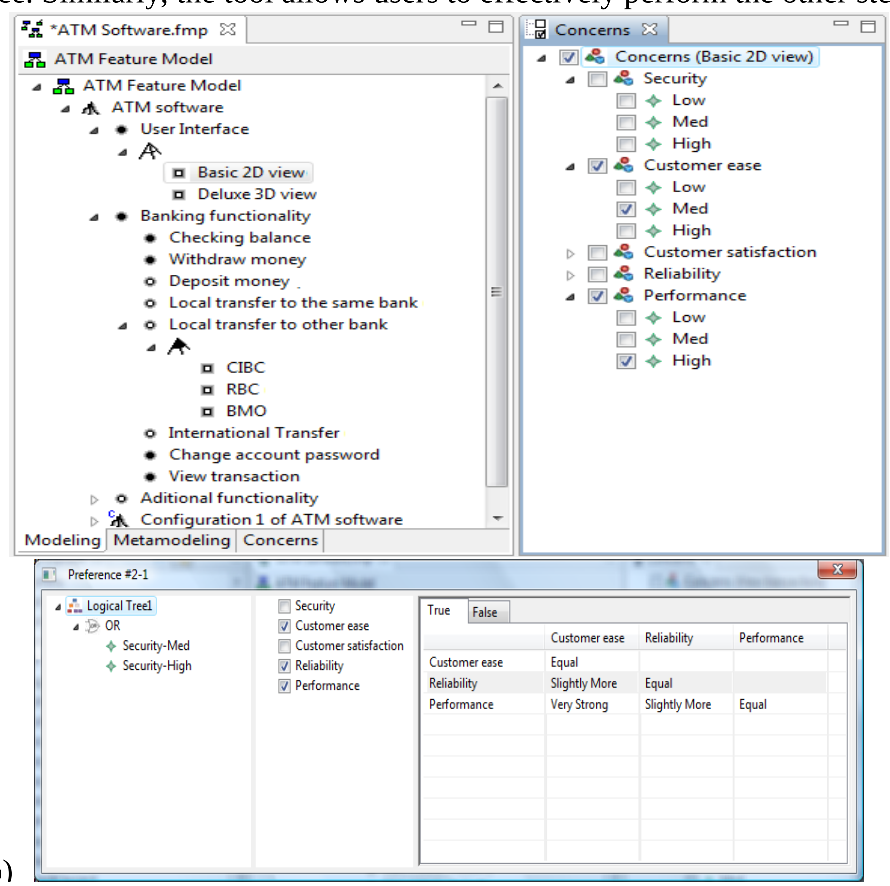
6. TOOL SUPPORT

One of the main contributions of our CS-AHP process is a wide range of its practical applications in different fields and domains. For the purpose of practical use of the CS-AHP, we experimented with the configuration process of feature models, which are commonly used for modeling variability and configuration management in software product lines (SPLs). We have created a prototype, a proof of concept, by extending the Feature Model Plug-in (*fmp*) (Czarnecki

& Kim, 2005) with the functionality required for managing the two-layered structure of concerns and qualifier tags and for supporting feature configuration enabled by CS-AHP introduced in (Bagheri & al, 2010). *fmp* is a widely used plug-in for feature modeling and configuration.

Generally, software product line engineering consists of two development lifecycles – domain engineering (development of a family of products for a particular domain) and application engineering (derivation of a concrete product by configuration of the family developed in domain engineering). The final goal of the configuration process in application engineering is the selection of the most appropriate set of features for a specific application. The target application stakeholders are allowed to define preferences of different kinds for the both the levels – concerns and qualifier tags. The CS-AHP process is used for ranking of a set of available features based on the defined preferences, in a similar manner as introduced for S-AHP in (Bagheri & al, 2010). In case violations of the well-formedness rules during the ranking process, the application stakeholders might be asked to resolve them. Finally, the selection of features is performed based on the rankings computed by CS-AHP; and features which have the least ranks are removed during the iterative steps of feature configuration process.

A screenshot of the tool’s dialog for editing concerns and their qualifier tags is given in Figure 1a, while the part for specifying conditional preferences for the level of concerns is given in Figure 1b. As it can be seen, a dialog for specifying conditional preferences consists of three major parts: the left part represents the logical tree of condition of conditional preferences; the middle part contains the concerns whose importance is specified with matrices defined on the right; and the right part represents two matrices of *then* (*True* tab) and *else* part (*False* tab) of conditional preference. Similarly, the tool allows users to effectively perform the other steps of CS-AHP.



a)

b)

Figure 3. *CS-AHP extension of the Feature Plug-in: a) Editing concerns and associating with features; b) Specifying conditional preferences on the level of concerns*

7. ANALYSES OF CS-AHP

This section reports on the results of the analyses based on three different aspects. The theoretical part of our analyses includes complexity analysis and analysis of how both cycles in dependencies and the previously recognized non-well-formedness in preferences affect the whole ranking process. In addition, the simulation results are taken as a basis for making conclusions and suggestions for the most relevant usage of the proposed framework in terms of reduction of potential violations of well-formedness in preferences.

7.1 Theoretical analysis

The presented extension of AHP, named CS-AHP, provides a framework for representing and reasoning over different forms of preferences on a two-layered hierarchical structure of concerns and qualifier tags. For the input model $(O, C, QT, \mathfrak{R})$, which has an additional $\mathfrak{R}$ dimension in addition to the S-AHP triple and the set of entered preferences, our proposed framework provides as output the ranks over elements of the set of options *O*. The stronger fact holds, as follows.

Corollary 1. In a finite number of steps, for each set of preferences and for each set of available options, the CS-AHP algorithm offers unique ranks over the set of options or concludes that the set of preferences is not well-formed.

According to the algorithm steps, presented in Section 5, we can see that the whole algorithm is divided into two levels: the level of concerns and the level of qualifier tags. For each option, appropriate matrices on both levels should uniquely be filled. Otherwise, as soon as the first preference violates the uniqueness, the algorithm stops and generates the notice about non well-formed preferences.

Even if the algorithm recognizes a violation of the well-formedness rules in the set of preferences, it may allow the stakeholder to make a decision about multiple values for filling the matrix. Alternatively, the stakeholder might not be included in the process, and instead, only the first values can be considered. That is, as all of the preferences are considered in the order in which they are entered, if any preference violates the values entered based the previous, that preference can be annotated as non-well-formed. Then, the value in the matrix will not be changed and the algorithm continues. The two mentioned solutions (with the stakeholders and by considering only the first values) allow the algorithm to always generate the ranking over the set of options. However, the algorithm will not always be unique in terms of dependences on the stakeholders' decisions about non-well-formed preferences or the order of the preferences entry.

In case of violation of the well-formedness rules, the ranking is not unique to the resolution of the violation of the well-formedness. This means that in case of a non-well formedness in preferences, its resolution has direct implications on the final ranking over the options. As each preference is checked separately for each option, it disables any *cycles* and *dependency in the processing of preferences.* It is also important to mention that preferences about dominant relative importance cannot make any violation of the well-formedness conditions, because their dominance is defined unconditionally to others (Wilson, 2011).

In preferential reasoning, different types of queries make sense in such a setting: *(i)dominance testing query* asking for relative order between two options, *(ii)ordering query,* seeking an ordering of a subset of options, and *(iii)optimization query* that look for a preferentially optimal option

(Domshlak, 2008). In preferential reasoning, all three queries are in general NP-hard (Goldsmith & al, 2008).

Corollary 2. For a given CS-AHP model *(O, C, QT, $\Re$)*, queries about dominant testing and ordering take polynomial time complexity. On the other hand, optimization queries are in the worst case NP-hard.

Let us denote with $\Re_C$ as the preferences for the level of concerns, $\Re_c^D$ as the preferences about dominant relative importance of appropriate concerns, and $\Re_{QT}$ as the preferences for the level of qualifier tags, $\Re = \Re_C \cup \Re_{QT}, |\Re_C| = n_C, |\Re_c^D| = n_c^D, |\Re_{QT}| = n_{QT}$ .As it can been seen in Section 5, for each option, each preference is considered as follows: for each option separately, the matrix on the level of concerns and $n_C - n_C^D$ matrices on the level of qualifier tags are filled and local ranks are calculated. Previously, conditions in each conditional preference are checked, which takes $O(n_R + (n_C - n_C^D)^2 + (n_C - n_C^D)n_{QT}^2)$ operations. These operations are the basic cost of pairwise methods, and additionally they are performed for each option separately, which takes $O(n_O(n_R + (n_C - n_C^D)^2 + (n_C - n_C^D)n_{QT}^2))$ operations, where n_O is the number of available options.

In case of *dominance testing* and *ordering* queries, the number of available options is finite, which gives polynomial complexity. On the other hand, an *optimization* query with the presented framework might be addressed only by backtracking the generation of all possible combinations of qualifier tags and their overall ranking. It obviously takes exponential time.

It is important to mention, that, in case of dominant relative importance of the whole set of concerns (i.e., lexicographic order), an optimization query might be addressed by simple traversal of each concern, and the set of highest ranked qualifier tags of each concern represents the optimal option. In this case, it has linear complexity. Also, in case of only unconditional preferences, an optimization query takes polynomial time for local ranking of qualifier tags of each concern and optimal option is annotated with the set of the highest ranked qualifier tags of each option.

In the previous paragraph we recognized two cases when the optimal query is addressed with linear and polynomial time complexity, which leads to conclusion that some structures of preferences optimize optimization queries - an issue left for future research.

7.2 Simulation Analysis

In this section, based on conclusions of Corollary 1, we find the characterization of each model ***(O, C, QT,*** $\Re$***)*** from the perspective of the use of the CS-AHP algorithm and get some directions about the structure of the recommended model, which would significantly reduce non-well-formedness in preferences.

7.2.1 Descriptive parameters of the model *(O, C, QT, $\Re$)*

Non-well-formed preferences might be caused independently on both levels of the two-layered process, so, the *number of non-well-formed preferences on the level of concerns* and the *number of non-well-formed preferences on the level of qualifier tags* might be considered for an analysis from the perspective of the use of the CS-AHP algorithm.

For a given CS-AHP model *(O, C, QT, $\Re$)*, we have argued that the set *O* of available options is an instrument for testing if the initial set of preferences is well-formed, and the number of non-well-formedness for both the levels are summarized up to the whole set of available options. In order to make these numbers independent of the available set of options, we use their average values. So, the usage of the CS-AHP algorithm for an appropriate model is analyzed on the basis of the *average numbers of non-well-formedness in preferences* for both the levels separately.

On the other hand, descriptive parameters of the whole model *(O, C, QT, $\Re$)* might be recognized as it follows:

- The two-layered hierarchical structure is represented with the set of concerns and their qualifier tags. So, *the number of concerns* and *the average number of qualifier tags* per each concern are considered as a pair of parameters for the characterization of the hierarchical structure. This characterization is unique due to different elements of the set of concerns and the corresponding set of qualifier tags.
- On the other hand, the set of preferences might have different structures which only depend on the way in which users define preferences related to their opportunity to specify even incomplete preferences or more preferences related to the same pair of concerns (or qualifier tags). We recognized that parameters, which can describe the set of preferences with respect to the users' freedom in defining and with potential importance for our approach, are the following:
    - *the structure of conditional preferences* which is related to the form of their specification; that is, if the preferences consist only of *then* parts or they include *else* parts, too.
    - As the number of preferences for each model is unlimited and is not related to the structure of concerns and qualifier tags, the number of preferences cannot be considered as a parameter of the model. On the level of concerns, each preference defines the relative importance between two or more concerns, so that on this level, we can define a *conditional pair of concerns* as a pair of concerns whose relative importance is defined by at least one conditional preference. Also, on the level of qualifier tags, each preference defines relative importance between pairs of qualifier tags of one concern, so that we can define a *conditional concern* as a concern whose relative importance between at least one pair of its qualifier tags is defined conditionally. Finally, *the number of conditional pairs of concerns defined on the level of concerns* and *the number of conditional concerns defined on the level of qualifier tags* are considered to be the parameters which describe the set of preferences related to the concept of conditionality.

In the following sections, we attempt to identify potential relations between the recognized descriptive parameters on the one hand, and the numbers of non-well-formedness for both the levels, on the other one. Also, we make estimations of the expected values of the numbers of non-well-formedness for both the levels separately.

7.2.2 Hypotheses

Our main goal, which is to find significant characterization of the numbers of non-well-formed preferences and their expected values, is further refined into the following more specific and concrete hypotheses:

H1. In case of different values of one descriptive parameter and fixed values for others, there are a significantly different number of non-well-formedness for both levels of each model ***(O, C, QT, $\Re$).***

H2.The average number of non-well-formed preferences for each level per model ***(O, C, QT, $\Re$)*** might be efficiently characterized at least by the number of conditional pairs of concerns (per level of concerns) and the number of conditional concerns (per level of qualifier tags).

Hypothesis H1 provides information about the impact of each recognized parameter separately on the whole model, while hypothesis H2 identifies the best subset of parameters for a significant prediction of the expected number of non-well-formedness for each model.

On the other hand, as conditionality in preferences determines different distributions of the set of preferences over the set of concerns, it would be interesting to consider it as an additional descriptive parameter. It is hard to expect to consider it as a real descriptive parameter, because of its discrete values and inability for its estimation, but its influence on the observed model is interesting for analysis. In that sense, we define an additional hypothesis.

H3. Models ***(O, C, QT,*** $\mathfrak{R}$***)*** with the same descriptive parameters have significantly different number of non-well-formedness among different distributions of conditional preferences.

These hypotheses are tested by a set of statistical techniques to understand the relationship between the number of non-well-formed preferences and recognized parameters of an appropriate model. Regression and correlation analyses are used to test if and which of the recognized parameters can (most) significantly determine the number of non-well-formedness in the model (H2). Furthermore, the ANOVA test for comparing means for multiple independent populations is used for testing if a significant difference exists in the number of non-well-formed preferences in cases of changing the value of one of the descriptive parameters (H1), and in case of different distributions of conditional preferences (H3).

As the final result, we give the estimations of expected values of violations of well-formedness rules for each group of models with the same descriptive parameters. They are obtained as a mean value of the observed average numbers of non-well-formedness on a large independent sample.

7.2.3 Experimental Setup

As previously indicated that the descriptive parameters might characterize each model under the use of the CS-AHP algorithm, random generations are used for developing different models with the same parameters. For each generated model, the number of observed non-well-formedness is determined and collected values are used for further analysis.

We have suggested that models should at most consist of 10 concerns annotated with a maximum of 7 qualifier tags, as those numbers of concerns and qualifier tags are manageable by human users. For each pair of concerns, the number of preferences is generated randomly, of which any number can be conditionally defined with random definitions of their conditions. The similar is done for the level of qualifier tags and preferences for each pair of concerns' qualifier tags.

As indicated in the comment on Corollary 1, preferences about dominant relative importance do not have any influence on the number of violations of the well-formedness conditions. Consequently, our simulations include only models with conditionally and unconditionally defined preferences and the results are later extended to the general case of additional preferences about dominant relative importance.

The number of conditional pairs of concerns on the level of concerns is in the interval *[0,n(n-1)/2]* and each pair of them might depend on a maximum of *n-2* of other concerns. Also, the total number of conditional concerns on the level of qualifier tags is in the interval *[0,n]*, and each of them might depend on a maximum of *n-1*of other concerns, where *n* is the number of concerns in the model. As explained in Sect. 5, in the stage named Determination of the ranks for the levels of concerns, less important concerns might be filtered out. During the simulations, decision about the number of concerns which are filtered out is randomly generated. Our simulation includes 1,000 random generations of models per each possible combination of descriptive parameters.

During the generation of preferences, their creation in the forms introduced by Definitions 2-5 is satisfied. In our previous study about service models from Software Product Line Tools that are publicly available and distributed by the software product line community (Bagheri & Gasevic, 2010), we observed that the number of options in those models was in range of 10 to 192 with

mean value 51.71 (SD=64.24). So, each set of preferences in our simulations is tested by obtained mean value (i.e.,52) of different options randomly configured for an appropriate model.
For testing hypotheses H1 and H2, motivated by the example introduced in Sect.3, the following three simulations are done. In the first simulation, after each randomly defined *then* part of a conditional preference, a decision is randomly made about the creation of its *else* part. In the second simulation, preferences are created only with *then* parts, i.e., if the *else* part were to be defined, it should be done with a new preference consisting of the negated condition (from the previous preference) and the *then* part. The third simulation includes the situation when each preference always contains the *else* part. During the random generation, instead of *else* parts in preferences, the empty value might be generated, too. A preference with empty the *else* part equals a preference with only the *then* part.
For testing hypothesis H3, random simulation is done with three different distributions of conditional pairs of concerns (on the level of concerns) and conditional concerns (on the level of qualifier tags), over the number of concerns that they depend on. Inverse exponential, normal and completely random distributions have well-known interpretations. In our simulations, the number of concerns, the numbers of conditional pairs of concerns (on the level of concerns) and conditional concerns (on the level of qualifier tags) are integers, so these distributions are generated over integers with integer values.
The random generation of other parameters is the same as in testing the previous two hypotheses.

7.3 Analysis

In this section, we present the employed statistical techniques, the purpose of their use, the obtained results, and their interpretation.

7.3.1 Analysis Techniques

Given the type of the collected data in the simulations, we analyzed them with standard descriptive statistics (as reported in (Blaikie, 2003) to be a common practice) including mean and standard deviation values. We can consider the parameters representing the number of concerns, the number of conditional pairs of concerns (on the level of concerns) and the number of conditional concerns (on the level of qualifier tags) as *interval data*. The structure of conditional preferences is the additional parameter, which has three possible values representing only a *then* part, an optional *else* part and a *then* with *else* part. Possible values might be annotated with 0, 1 and 2, but the annotations do not have a special numerical meaning; thus, this parameter is a *categorical type* of data. As it would be shown through the results of simulation, the descriptive parameter which represents the number of conditional pairs of concerns would be transformed into a new *categorical* parameter. Its categories represent grouped values of the number of conditional pairs of concerns (on the level of concerns).
Accordingly, the analysis of difference among groups, correlation, and regression analysis were done by using parametric tests (i.e., ANOVA, Pearson, multiple regression and multiple regression with dummy variables[2], respectively). For variables whose data were not normally distributed, we used parametric tests over log-transformed data[3]. As only one parameter is categorical with three possible values, we decided to split our data into three different groups and make a

---

[2] This approach with dummy variables is applied for representing information about group membership in quantitative terms without imposing unrealistic measurement assumptions on the categorical variables (Hardy, 1993)

[3] This approach is applied to non-parametric tests in evidence-based disciplines such as medicine (Keene, 1995). Moreover, this is consistent with the findings of previous research in psychological measurement (Rasmussen & Dunlap, 1991) and the purpose of our approach for addressing users' preferences and judgments.

regression model for each group separately and compare the results. Also, in order to find the most appropriate regression model, we use two different models and analyze the results.

7.3.2 Results

As the first step of our analysis, we hypothesize that a meaningful correlation exists between the defined descriptive parameters and the number of violations of the well-formedness conditions. Table 2 summarizes the results of correlation studies for both levels. Values for the coefficient of correlation below 0.50 generally are considered unsatisfactory (Blaikie, 2003). Accordingly, our results show that a low correlation is recognized with the average number of qualifier tags (for the level of concerns: 0.20<0.5 and for the level of qualifier tags: 0.192<0.5), so that this descriptive parameter was not considered further in our analysis.

Table 1. *The results of the correlation analysis*

Descriptive parameter	**Number of violations of well-formedness conditions (level of concerns)**	**Number of violations of well-formedness conditions (level of qualifier tags)**
	Correlation ρ-coefficient; p value	**Correlation ρ-coefficient; p value**
Number of concerns	0.515;< 0.005	0.516; < 0.005
Number of qualifier tags	-0.20; < 0.005	0.192; < 0.005
Number of conditional pairs of concerns / conditional concerns	0.672; < 0.005	0.650; < 0.005
Structure of conditional preferences	0.646; < 0.005	0.595; < 0.005

In order to analyze the influence of descriptive parameter which represents the number of concerns in case of fixed values of other descriptive parameters, we considered that our data are divided into groups according to the number of concerns. As the collected data were not normally distributed, a one way ANOVA test was used over log-transformed data to compare the means of the dependent variable (*the number of violations of the non-well-formedness conditions for both the levels*) for different groups defined by the number of concerns. The results show a significant difference in the number of violations of the well-formedness conditions between different values for the number of concerns: for the level of concerns $F(7,2460)=69.526$, $p=0.000$, and for the level of qualifier tags $F(7,772)=93.328$, $p=0.000$; the mean values per each group for both levels are represented in Table 2. Also, the Tukey post-hoc test revealed that for each group there is no significant difference compared to groups that differ in the number of concerns by 1.

The similar consideration is done for other descriptive parameters. The results show that there is a significant difference in the number of violations of the well-formedness conditions between different structures of preferences: $F(2,2465)=936.888$, $p=0.000$, on the level of concerns, and $F(2, 777)=150.227$, $p=0.000$ on the level of qualifier tags. The Tukey post-hoc test revealed that there is a significant difference between each two groups. The mean values per each group for both levels are represented in Table 3.

The results show that there is a significant difference in the number of violations of the well-formedness conditions between different numbers of conditional pairs of concerns on the level of concerns, $F(45,2422)=58.446$, $p=0.000$. In this case, the Tukey post-hoc test showed that there is no significant difference comparing each pair of groups. We decided to make supergroups, so that there is no significant difference among groups inside each of them. According to the fact that in case of n concerns, the maximum number of conditional pairs of concerns is $n(n-1)/2$ (previously discussed in the Experimental Setup Sect.), we defined supergroups as follows: the first group contains up to 3 conditional pairs of concerns, the second group contains from 4 to 6 conditionals pairs, and others, respectively, from 7 to 10, from 11 to 15, from 16 to 21, from 22 to 28, from 29 to 36 and from 37 to 45. The one-way ANOVA test showed that there is no significant difference between the mean values of the number of violations of well-formedness rules among the created supergroups. Also, it showed that there is a significant difference in the number of violations of the well-formedness conditions between different supergroups, $F(7,2460)=345.93$, $p=0.000$, while the Tukey post-hoc test showed, in comparison to the case where there is no supergroups, that there is a significant difference comparing each pair of them. The mean values of violations of well-formedness for different supergroups are presented in Table 4.

On the other hand, results showed that there is a significant difference in the number of violations of the well-formedness conditions between different number of conditional concerns on the level of qualifier tags, $F(9,770)=79.923$, $p=0.000$. The Tukey post-hoc test showed, similar to the previous analysis of the parameter which represents the number of concerns, that for each group there is no significant difference as compared to groups that differ in the number of conditional concerns by 1. The mean values of violations of well-formedness for different number of conditional concerns for the level of qualifier tags are presented in Table 5.

Consequently, the first hypothesis is proved in case of the following descriptive parameters: the number of concerns, the structure of conditional preferences, the number of conditional pairs of concerns (for the level of concerns) and the number of conditional concerns (for the level of qualifier tags); and rejected for the descriptive parameter which refers to the number of qualifier tags per each concern. For further analysis we will use the supergroups as a descriptive parameter representing conditional preferences on the level of concerns.

Discussion. As shown through testing hypothesis H1, each of the recognized descriptive parameters except the number of qualifier tags, makes significant differences in the number of violations of the well-formedness conditions for fixed values of other parameters.

Table 2. *Mean and standard deviation values of the number of violations of the well-formedness conditions for different numbers of concerns*

Number of concerns	**Violations of well-formedness conditions**		**Number of concerns**	**Violations of well-formedness conditions**	
	Level of concerns	**Level of qualifier tags**		**Level of concerns**	**Level of qualifier tags**
	Mean, Std dev			**Mean, Std dev**	
3	0.497, 0.384	0.208, 0.194	7	2.658, 2.391	2.062, 1.789
4	0.859, 0.722	0.449, 0.425	8	3.492, 3.165	3.021, 2.641
5	1.336, 1.172	0.781, 0.641	9	4.455, 4.062	4.293, 3.755
6	1.944, 1.743	1.324, 1.138	10	5.610, 5.024	5.885, 5.216

Table 3. *Mean values of the number of violations of the well-formedness conditions for different structures of conditional preferences*

Structure of conditional preference	**Violations of well-formedness conditions**		**Structure of conditional preference**	**Violations of well-formedness conditions**	
	Level of concerns	**Level of qualifier tags**		**Level of concerns**	**Level of qualifier tags**
	Mean, Std dev			**Mean, Std dev**	
then-part	0.879, 0.644	0. 861, 0.839	Then and else part	6.573, 4.793	4.889, 4.673
else-part optional	3.797, 2.675	2.925, 2.938			

Table 4. *Mean and standard deviation values of the number of violations of the well-formedness conditions for different numbers of conditional pairs of concerns*

Num of cond. pairs of concerns	Mean, Std dev	Num of cond. pairs of concerns	Mean, Std dev	Num of cond. pairs of concerns	Mean, Std dev	Num of cond. pairs of concerns	Mean, Stddev
1-3	0.565, 0.488	7-10	1.985, 1.311	16-21	4.501, 2.889	29-36	7.665, 5.068
4-6	1.159, 0.763	11-15	3.208, 2.059	22-28	5.998, 3.889	37-45	9.836, 6.184

Table 5*.Mean and standard deviation values of the number of violations of the well-formedness conditions for different numbers of conditional concerns*

Number of conditional concerns	Mean, Std dev	Number of conditional concerns	Mean, Std dev	Number of conditional concerns	Mean, Std dev
1	0.502, 0.504	4	2.262, 2.026	7	5.457, 3.876
2	1.002, 1.001	5	3.139, 2.586	8	6.881, 4.703
3	1.513, 1.597	6	4.216, 3.187	9	8.633, 5.609
				10	10.743, 6.956

Tables 6-8 summarize the results of the regression analysis for hypothesis H2. As previously explained, we divided our analysis in three separate parts, according to the structures of conditional preferences. Thus, the tables are organized as follows: each table corresponds to one of the three different structures of preferences, and, in each of the three table, results for both the levels (the level of concerns and the level of qualifier tags) are presented, separately. Also, during the testing of hypothesis H1, we transformed the descriptive parameter, which represents the number of conditional pairs of concerns, into a categorical variable with eight values. Consequently, in order to use linear regression analysis, we define seven dummy variables, as represented in Figure 4. Now, two different regression models are considered: the first one, consisting only of the number of dummy variables (on the level of concerns) and conditional concerns (on the level of qualifier tags) as the parameter with the highest correlation; and the second one, with an additional predictor being the number of concerns.

Figure 4. *Dummy coding for the number of conditional pairs of concerns on the level of concerns*

Number of pairs of conditional concerns	Dummy variable 1	Dummy variable 2	Dummy variable 3	Dummy variable 4	Dummy variable 5	Dummy variable 6	Dummy variable 7
1-3	0	0	0	0	0	0	0
4-6	1	0	0	0	0	0	0
7-10	0	1	0	0	0	0	0
11-15	0	0	1	0	0	0	0
16-21	0	0	0	1	0	0	0
22-28	0	0	0	0	1	0	0
29-36	0	0	0	0	0	1	0
36-45	0	0	0	0	0	0	1

Discussion. In all three cases of different structures of conditional preferences reported in Tables 6-8, both models accurately predict the number of expected violations of the well-formedness conditions on the both levels, of concerns and qualifier tags. We can see that, the model consisting only of the dummy variables (on the level of concerns) and the number of conditional concerns (on the level of qualifier tags) can predict above 90% of variability in the number of violations of the well-formedness conditions. It means that the descriptive parameter of the set of preferences $\Re$ might efficiently characterize the model *(O, C, QT,* $\Re$ *)* under the use of CS-AHP. Thus, the hypothesis H2 is proven.

As a value of R^2measures how much of the variability in the outcome is accounted for by the predictors, it would be used as a criterion for selecting better model for the both levels, separately. We can see that the model consisting of the number of concerns and the number of dummy variables on the level of concerns, in all three cases of different structures of conditional preferences, has greater value of R^2 than a model consisting only of the number of dummy variables (Table 6: 0.957=0.957, Table 7: 0.986>0.937, Table 8: 0.963=0.963). The same can be concluded for the level of qualifier tags, where the model consisting of the number of concerns and the number of conditional concerns is better than one consisting of only the number of conditional concerns (Table 6: 0.919>0.880, Table 7: 0.922>0.895, Table 8: 0.952>0.923). This result reasserts what we had anticipated, that the key cause for violations of the well-formedness conditions is in the number of conditionally defined pairs of concerns (i.e., concerns for the level of qualifier tags), the structure of conditional preferences, and the number of concerns in the model.

Table 6. *The results of regression analyzes for conditional preferences consisting of only then parts*

Variable		**Unstandardized coefficient**		**Standardized coefficient**	**Model Summary**
		B	**SE B**	**B**	**r^2, F(df1,df2), p**
Model 1:	Constant	0.205	0.011	0.043	0.957, F(7,812)=2606.442, 0.00<0.001
	Dummy Var.1	0.082	0.017		
	Dummy Var.2	0.276	0.016	0.152	
	Dummy Var.3	0.562	0.017	0.286	
	Dummy Var.4	0.844	0.016	0.465	
	Dummy Var.5	1,212	0.017	0.631	
	Dummy Var.6	1,645	0.019	0.760	
	Dummy Var.7	2,099	0.023	0.745	
Model 2:	Constant	0.187	0.023	0.042	0.957, F(1,811)=2280.297, 0.00<0.001
	Dummy Var.1	0.081	0.017		
	Dummy Var.2	0.273	0.017	0.151	
	Dummy Var.3	0.559	0.018	0.285	
	Dummy Var.4	,839	0.017	0.462	
	Dummy Var.5	1,206	0.018	0.628	
	Dummy Var.6	1,637	0.020	0.756	
	Dummy Var.7	2,090	0.025	0.741	
	Num of concerns	0.003	0.003	0.008	
Model 1:	Constant	-0.335	0.057	0.834	0.880, F(1,258)=214.009, 0.00<0.001
	Num of conditional concerns	0.288	0.012		
Model 2:	Constant	-1.100	0.085	0.682	0.919, F(1,257)=5.226, 0.00<0.001
	Num of conditional concerns	0.236	0.011		
	Num of concerns	0.134	0.012	0.344	

Table 7. *The results of regression analyzes for conditional preferences consisting of then parts and optional of else parts*

Variable		**Unstandardized coefficient**		**Standardized coefficient**	**Model Summary**
		B	**SE B**	**B**	**r^2, F(df1,df2), p**
Model 1:	Constant	1.081	0.091	0.042	0.937, F(7,820)=603.418,

	Dummy Var.1	0.345	0.142		0.00<0.001
	Dummy Var.2	1.185	0.137	0.151	
	Dummy Var.3	2.358	0.144	0.278	
	Dummy Var.4	3.502	0.137	0.446	
	Dummy Var.5	4.894	0.142	0.589	
	Dummy Var.6	6.282	0.155	0.671	
	Dummy Var.7	8.855	0.190	0.726	
Model 2:	Constant	0.014	0.188	0.037	0.986, F(1,819)=559.180, 0.00<0.001
	Dummy Var.1	0.304	0.139		
	Dummy Var.2	1.066	0.135	0.136	
	Dummy Var.3	2.159	0.144	0.255	
	Dummy Var.4	3.224	0.140	0.411	
	Dummy Var.5	4.537	0.149	0.546	
	Dummy Var.6	5.846	0.165	0.625	
	Dummy Var.7	8.340	0.202	0.684	
	Num of concerns	0.158	0.025	0.106	
Model 1:	Constant	-1.157	0.210	0.814	0.895, F(1,258)=198.023, 0.00<0.001
	Num of conditional concerns	0.983	0.044		
Model 2:	Constant	-3.950	0.315	0.656	0.922, F(1,257)=9.321, 0.00<0.001
	Num of conditional concerns	0.791	0.041		
	Num of concerns	0.491	0.046	0.359	

Table 8. *Results of regression analyzes for conditional preferences consisting of both then and else parts*

Variable		**Unstandardized coefficient**		**Standardized coefficient**	**Model Summary**
		B	**SE B**	**B**	**r^2, F(df1,df2), p**
Model 1:	Constant	1.516	0.077	0.043	0.963, F(7,812)=1928.460, 0.00<0.001
	Dummy Var.1	0.613	0.119		
	Dummy Var.2	2.097	0.115	0.155	
	Dummy Var.3	4.242	0.121	0.290	
	Dummy Var.4	6.357	0.115	0.469	
	Dummy Var.5	9.086	0.119	0.634	
	Dummy Var.6	12.266	0.130	0.760	
	Dummy Var.7	15.752	0.159	0.749	
Model 2:	Constant	1.118	0.162	0.042	0.963, F(1,811)=2631.574, 0.00<0.001
	Dummy Var.1	0.599	0.119		
	Dummy Var.2	2.054	0.116	0.152	
	Dummy Var.3	4.169	0.123	0.285	
	Dummy Var.4	6.254	0.120	0.461	
	Dummy Var.5	8.954	0.128	0.625	
	Dummy Var.6	12.105	0.142	0.750	
	Dummy Var.7	15.562	0.172	0.740	
	Num of concerns	0.059	0.021	0.023	
Model 1:	Constant	-1.877	0.305	0.848	0.923, F(1,258)=175.564, 0.00<0.001
	Num of conditional concerns	1.629	0.063		
Model 2:	Constant	-6.524	0.423	0.683	0.952, F(1,257)=4.112, 0.00<0.001
	Num of conditional concerns	1.311	0.055		

	Num of concerns	0.817	0.062	0.375	

From the previous hypothesis, we obtained that the structure of conditional preferences significantly influence the number of violations of the well-formedness conditions and the lowest values are in case of preferences consisting only of *then* parts. Consequently, we decided to test hypothesis H3 only in that case. The results show that there is a significant difference in the number of violations of the well-formedness conditions in different distributions of conditional preferences over concerns for the level of concerns, $F(2,2465)=44.507$, $p=0.000$; and the level of qualifier tags, $F(2,777)=40.535$, $p=0.000$. The mean values for different distributions are presented in Table 9. The Tukey post-hoc test shows that there is no significant difference between completely random simulations and normal distributions, but there is for the other remaining pairs of distributions reported in Table 9.

Table 9. *Mean and standard deviation values of the number of the violations of the well-formedness conditions for different distributions*

Distribution	Level of concerns	Level of qualifier tags
	Mean, Stddev	
Normal	1.123, 0.768	2.234, 2.005
Inverse exponential	0.876, 0.123	1.612, 1.749
Random	1.234, 0.567	2.192, 3.016

Discussion. This result confirms what we had anticipated according to the interpretation of each of the three well-known distributions. Normal distribution favors the dependency on the medium number of concerns, while inverse exponential to the low number of concerns, and consequently, it increases the number of violations of the well-formedness conditions.

7.4 Critical Recommendations

The results of the hypothesis reveal the influence of each of the recognized parameters on the number of violations of the-well-formedness conditions for both levels of an appropriate model. The analysis of hypothesis H2 shows that they can be with the highest significance characterized with the following parameters: the number of concerns and the number of conditional pairs of concerns (on the level of concern) and the number of conditional concerns (on the level of qualifier tags).

As our simulations generated 1,000 different models for each combination of parameters and their well-formedness is tested under 52 randomly generated options, the obtained results allow us to estimate expected values of violations of the well-formedness conditions with appropriate mean values. The results are presented in Tables 11 and 12 in the Appendix B of the paper.

We suggest using the values from Tables 11 and 12for the assessment of the ranks obtained with the usage of the CS-AHP algorithm on an appropriate model **(O, C, QT,** $\Re$ **)**, in the following steps:

1. The average numbers of obtained non-well formedness in the preferences $\Re$ for both the levels are compared to corresponding numbers in the tables in Appendix B. The values in Tables 11 and 12 are determined based on descriptive parameters of the observed model: the number of concerns (decreased by the number of concerns with dominant relative importance) and the number of conditional pairs of concerns (on the level of concern) and the number of conditional concerns (on the level of qualifier tags);

2. If both values are equal or less than the values in the tables, it means that the obtained values of non-well formedness is below the average and can be ignored. The obtained ranks should be considered to be appropriate for the model;
3. If any of the values is higher than the value in the table, it means that the number of violations of non-well-foremdness is above the average. The stakeholder should make a decision about the acceptance of the obtained ranks because it might be concluded that the model is below the expectation according to the usage of the CS-AHP algorithm. This means that the obtained ranks might be considered too distant from the initial set of preferences and all caused by the higher than expected value of non-well-formed preferences.

Based on the results and analyzes of the simulations, we can make recommendations about the usage of the proposed algorithm and the best ways of defining an appropriate model and its related set of preferences.

Recommendation1. The least average number of non-well-formed preferences is in cases when conditional preferences consist of only then-parts, i.e., there are a lower number of conflicts in their satisfaction;

Recommendation2.A model with less concerns and a higher number of qualifier tags is more suitable than a model with more concerns, as there is no significant increase of the number of violations of the non-well-formedness conditions by increasing the number of qualifier tags;

Recommendation3.A lower number of different concerns in the conditions is not a guarantee for a lower number of violations of the well-formedness conditions under the set of available options. It means that violations of the well-formedness in conditional preferences are caused only by situations when conditions from different conditional preferences are satisfied at the same time.

8. DISCUSSION AND RELATED WORK

8.1 Quantitative Prioritizations and Conditionally-defined Preferences

In the following, we discuss methods and formalisms from different fields that are the most related to quantitative prioritizations and conditionally defined preferences.

An example of a family of methods based on quantitative measurements (based on categorization of Larichev (Turski, 2008)) is the TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method. There are many algorithms belonging to this category, such as SAW (Simple Additive Weighting) (Zavadskas & al, Sensitivity analysis of a simple additive weight method, 2007; MacCrimmon, 1968), LINMAP (Linear Programming Techniques for Multidimensional Analysis of Preference) (Srinivasan & Shocker, 1973), CORPAS (Complex Proportional Assessment) (Zavadskas & Kaklauskas, 1996; Zavadskas & al, 2007), but their characteristics do not differ much based on the issues presented in Table10. The basic principle of the TOPSIS method (Hwang & Yoon, 1981) is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The best ideal and negative ideal solutions are found based on statically defined preferences. That is, the best price is the lowest one, the best quality is the highest one and so on. This in fact presents the limitation for its use for addressing different sorts of preferences.

An example of comparative preference methods based on pairwise comparison of alternatives is the ELECTRE family of methods developed in mid-sixties, which had a strong impact on the operational research community (Figueira, Mousseau, & Roy, 2005). The ELECTRE evaluation method allows for handling qualitative and quantitative criteria simultaneously. In contrast to the traditional approach, ELECTRE introduces the concept of an indifference threshold, and the preference relationships are redefined as follows: preferred, indifferent, and cannot be compared. The whole family of methods is developed in order to support heterogeneity of scales, interpretation of outranking relation as a fuzzy relation and situations when relative importance coefficients of criteria are not completely defined. These methods are well-known for decision optimization in the case of decisions taken in circumstances of certainty, and as in the TOPSIS methods, preferences are statically defined, which is the main reason why the ELECTRE family cannot be applied in the domain addressed by this paper.

Also related, the Bubble sort technique has been used to rank order the preference statements. Bubble sort is in essence very similar to AHP with the slight difference that preference comparisons are made to determine which preference has a higher priority, but not to what extent. It is clear that Bubble sort suffers from similar issues to those of AHP (e.g., the large number of required comparisons). There have been proposals attempting to reduce the number of required comparisons in comparison-based techniques, which are generally referred to as incomplete pairwise comparison methods (Berander & Andrews, 2006). These techniques are based on some local and/or global stopping rule, which determines when a further comparison will not reveal more useful information with regards to the prioritization of the options. Such techniques can be beneficial if used along with techniques such as AHP, S-AHP, CS-AHP and Bubble sort.

Additionally, Hierarchical Cumulative Voting (HCV) has been used to prioritize preferences where top vote-getter preferences are prioritized higher than the others (Berander & Jönsson, 2006). One of the drawbacks of this approach is that as the number of preferences (options) increases, it becomes very hard for the stakeholders (voters) to select the best voting tactic, which would reveal their preferences about the highest priority preferences. In addition, HCV assumes that it is possible to hierarchically divide the objects of interest into different levels, but does not contain any mechanism for doing so (Karlsson, Olsson, & Ryan, 1997). If we would like to extend HCV for both conditional and unconditional situations, the first solution would be to divide conditional and unconditional options in two different groups. Although in such a case it may happen that we have only one unconditional option (or even none) and a large number of conditional options which means that we have a new problem of properly dividing the options into groups. On the contrary, if all of the options are in one block, there is a problem of compensation which is well known for this method.

8.2 Conditional Preferences

Numerous studies have specifically examined conditional preferences and different sorts of preferences with conditionality. Often, they define the structure as networks and graphs for representing conditionality in preferences, and the best well-known are CP-nets/TCP-nets.

The Conditional Preference Network (CP-net) (Boutilier & al., 2004) is a formalism for compactly expressing conditional preferences in multivariate problems. It is a qualitative graphical representation of preferences that reflects conditional dependence and independence of preference statements under a *ceteris paribus* (all else being equal) interpretation. One of nice properties of the CP-net model is that determining the optimal outcome is straightforward and can be done in the linear time with respect to the number of variables by a simple topological order of a given network (Domshlak & Brafman, 2002). The situation with dominance testing is not as sharp (Domshlak & al., 2011). However, there are some studies that address certain topologies of CP-net and that optimize dominance testing query, but in general, it is NP-hard (Goldsmith & al, 2008).

Tradeoffs-enhanced CP-nets (TCP-nets) (Brafman & Domshlak, 2002) capture information about conditional independence and information about conditional relative importance. Thus, they provide a richer framework for representing user preferences, allowing stronger conclusions to be drawn, yet remain committed to the use of intuitive, qualitative information as their source. In case of a conditionally acyclic TCP-net, the formalism keeps the linear complexity time for determining an optimal outcome. Similar computational analysis for precise complexity of dominance testing in TCP-nets is still an open theoretical question, to the best of our knowledge.

Conditional Outcome Preference Network (COP-network) (Chen, Buffett, & Fleming, 2007) is one more example of users' preferences presentation with directed graphs. This formalism models preferences directly elicited from a user and then extends to indicate all preferences that can be inferred as a result. In addition to other methods, this method develops a utility function to predict all known utilities and can be used to determine quickly whether one outcome is preferred over another one. Since a COP-net contains a node for every possible outcome, run-time for building and traversing the tree is very expensive in the worst case. But, in comparison to other techniques, which only handle preferences specified over values for a particular attribute, COP-net can infer preferences over outcomes when they are specified for values across attributes as well.

CP-nets/TCP-nets are general frameworks for addressing conditionality in preferences, but they have a weak performance for dominant queries and focus on acyclicity, which is a strong restriction, limiting their potential applicability (Wilson, 2011). A few extensions have recently been proposed in different directions.

Recently an extension into a more general *cp-theory* in (Wilson, 2011) introduces a new formalism, which can be viewed as a simple logic of conditional preferences. It is shown that given semantics can represent a stronger kind of preference statements, which can be used, for example, to construct a lexicographic order on outcomes, which is not expressible within the formalisms of CP-nets and TCP-nets. Cp-theories consider weaker forms of acyclicity, which are sufficient conditions for the cp-theory to be consistent. The work on cp-theories presented in (Wilson, 2011) did not addressed the important (but very hard, as shown for CP-nets and TCP-nets in (Goldsmith & al, 2008; Brafman & Domshlak, 2002; Boutilier & al., 2004) problem of dominance testing.

There is another variant of TCP-nets, known as UCP-nets (Boutilier, Bacchus, & Brafman, 2001) that capture quantitative preferences and relative importance information using utility functions. They combine the theory of CP-nets and GAI-nets (Generalized Additive Decomposable Utility Functions) (Gonzales & Perny, 2004). By extending CP-nets with quantitative utility information, the expressive power is enhanced and dominance queries become computationally efficient. On the other hand, UCP-net shave a limitation, as they do not make any assumption on the kind of interactions between attributes that need to be prioritized.

There is a work different to the one proposed in (McGeachie & Doyle, 2011), where the TCP-nets theory is analyzed as a model for representing and reasoning over quantitative preferences (Mukhtar, Belaïd, & Bernard, 2009). Preferences and constraints are specified qualitatively and then mapped onto a quantitative utility model. The user preferences are evaluated against available options by building a preference tree containing options properties and user preference values for them. Consequently, dominance query might be addressed in polynomial time, but an optimization query additionally includes construction of a preference tree with all possible combination of options. Additionally, despite the advantages of TCP-nets, they do not allow complete quantification of relative importance over attributes.

The most related to our work is a recent extension of CP-nets with quantitative trade-off statements (McGeachie & Doyle, 2011). The extension is made by concepts from elementary geometry and usage of additive linear value function which corresponds to users' preference relations (Brafman, Domshlak, & Kogan, 2004) and representation of tradeoff statements as constraints on the partial derivatives of the value function. They have demonstrated that for each acyclic CP-net an additive value function can represent all forms of preferences (Brafman, Domshlak, & Kogan, 2004). Unfortunately, this extension also suffers of cycles induced by conditionally defined preferences as CP-nets/TCP-nets. Furthermore, priority statements specific for TCP-nets are not completely addressed and that novel representation raises many new questions for further research, especially about interaction with *ceteris paribus* preference statements (McGeachie & Doyle, 2011). This method proposes the transformation of a set of preferences into a system of linear inequalities, and in general, that system does not have a solution, which is an additional complexity to that CP-nets extension approach.

8.3 Semantics of (Conditional) Preferences

There is one more topic of interest related to our work: semantics of stakeholders' preferences with special emphasis on conditional preferences. Since conditional sentences in natural language have several interpretations (Yu & al., 2010), a statement of the conditional preferences may also be understood in different ways by different users. Addressing conditional preferences with rigorous mathematical theories has several limitations for the application in real world. First, users need to specify too many statements to draw a complete picture of preferences in a domain. Second, users should understand the statements in the way they are defined in order to be adopted (Yu & al., 2010).

In (Yu & al., 2010), the authors have proposed an ontology-based quantitative model for conditional preferences with special issues on different interpretations of the conditional preference statements such as the hierarchical relationships of the concepts in the ontology (Langley, 1995), the connotation of sufficient and necessary conditions (Brennan, 2003), and the bipolar property (Benferhat & al, 2006) of preferences in human thinking. Their experimental results show that it is possible to infer all ratings from a few rules, thus lessening users' workload. The inheritance property, which claims users' preference for an ancestor item can be inherited by its descendant items, is also proved.

8.4 Summary

The characteristics of the mentioned approaches are summarized in Table 10. For each approach, we have listed the scale of input and output data, if it supports conditionality in preferences or any other specific kind of preferences, and complexity for different reasoning queries. It is clear that a unique representational and reasoning technique which can effectively address all different kind of preferences and reasoning queries does not exist. Depending on the characteristics of a considering problem in an appropriate field, the most appropriate method should be chosen. In this paper, we recognize the problem of quantitative ordering between options based on different kinds of conditionally and unconditionally defined preferences which cannot be completely addressed by existing methods, to the best of our knowledge. We decided to use the widely accepted and adopted AHP technique and extend it for different kinds of preferences. Our use of the two-level hierarchy has several explanations, such as possibility of weak reductions of the number of comparisons (shown in (Bagheri & al, 2010)), sufficient expressiveness (according to (Yu & al., 2010)) and analogous to the concept of attributes in all developed techniques for addressing conditional preferences. We previously showed its effective usage for configuration of feature models, which are commonly used for modeling variability in software product lines (SPLs) (Ognjanovic & al, 2011).

Table 10. *Comparative analysis of prioritization techniques based on different scale of input/output data, different sort of preferences and performances for reasoning queries*

Theory/ framework	***Input data / Output data***	***Conditional preferences***	***Special kind of preferences***	***Dominant testing / ordering***	***Optimization query***
(Wilson, 2011)	ordinal scale / ordinal scale	Addressed	lexicographic order	Not completely addressed	Linear size of network in case of weaker forms of acyclicity
(Boutilier & al., 2004)	ordinal scale / ordinal scale	Not completely addressed	-	NP-hard, optimized in case of some topologies	Linear size of network in case of c acyclicity
(McGeachie & Doyle, 2011)	ordinal scale / numerical values	Not completely addressed	Continuous attributes, Multi attribute in preferences	Not completely addressed	Not completely addressed
(Brafman & Domshlak, Introducing variable importance tradeoffs into CP-nets, 2002)	ordinal scale / ordinal scale	Addressed	-	NP-hard	Linear size of network in case of conditional acyclicity
(Boutilier, Bacchus, & Brafman, 2001)	initial qualitative, transformed into numerical values / numerical values	Not completely addressed	-	Linear size of network	Linear size of network
(Mukhtar, Belaïd, & Bernard, 2009)	initial qualitative, transformed into numerical values / numerical values	Not completely addressed	-	Polynomial	NP-hard
(Chen, Buffett, & Fleming, 2007)	ordinal scale and set of utilities/ numerical values	Addressed	Multi attribute in preferences	NP-hard in the worst case	NP-hard
(Satty, 1980; Figueira, Mousseau, & Roy, 2005; Berander & Jönsson, 2006)	initial qualitative, transformed into numerical values / numerical values	-	-	polynomial	NP-hard
CS-AHP	initial qualitative, transformed into numerical values / numerical values	Addressed	lexicographic order	Polynomial	NP-hard

9. CONCLUSION AND FUTURE WORK

In this paper, we investigated how the well-known AHP technique might be extended for reasoning over different sorts of conditional preferences in a two-layered structure. The proposed CS-AHP approach satisfies the preferences of an effective prioritization technique based on the challenges and characteristics that have been introduced in (McManus, 2004):

- Stakeholders are able to define *conditional and unconditional preferences* about the available concerns and its qualifier tags. Also, they might define *preferences about dominant relative importance* as a special form of preferences. Compared with the S-AHP method that does not support conditional preferences, concerns and their qualifier tags do not have static final ranks because they depend on options which might either satisfy the conditions defined by stakeholder's preferences or not (Ognjanovic & al, 2011).
- The activities within CS-AHP are *easy to perform* and are based on a simple pair-wise comparison method. It can be inexpensively implemented in a spreadsheet program such as MS Excel with additional usage of any program for checking satisfaction of conditions in conditional defined preferences.
- During the whole process, in each step, it might be checked whether an inconsistency exists in the preferences of the stakeholders, and if so, stakeholders' intervention might be necessary (Ognjanovic & al, 2011).
- We decided to have a *local calculation* of ranks, as it disables any *cycles* and *dependency in the processing of preferences*. In comparison to all other techniques for addressing conditional preferences, *only* CS-AHP does not suffer any kind of cyclicity in preferences.
- Non-well-formedness recognized in this paper represents a logical inconsistency in defining multiple conditional preferences; consequently, such inconsistency is an obstacle for any reasoning technique. CS-AHP enables for checking non-well-formedness in preferences in each step of prioritization and in that sense represents a unique framework, which also gives the estimation of expected values of the number of non-well-formedness for an appropriate model.
- Time-complexity for dominance testing and ordering queries are polynomial which makes CS-AHP an effective technique for quantitative ordering of sets of options based on different sorts of conditional and unconditional preferences. Possibilities of defining cycles in dependencies, multiple and incomplete preferences represent additional beneficial characteristics of our proposed technique. Also, the extensive usage of AHP in many important decision making domains such as forecasting, total quality management, business process re-engineering, quality function deployment, and the balanced scorecard (Forman & Gass, 2001; Satty, 1980) can be effectively extended with a wide set of conditional preferences over the two-layered structure.

Additional two characteristics of the AHP method, not addressed in this paper, are: *(1)group decision making* (Roper-Low, 1990; Satty, 1980; Forman & Gass, 2001) –AHP considers two different approaches: aggregation of individual judgments (AIJ) and aggregation of individual priorities (AIP) (Escobar, Aguaron, & Moreno-Jimenez, 2004); *(2)analytical measure to evaluate the inconsistency* of the decision maker when eliciting the judgments, called Consistency Ratio (CR) (Satty, 2003). We believe that future research needs to be undertaken for CS-AHP in order to: *(1)*include preferences from different stakeholders with different interests and views, and *(2)*extend the existing consistency index and include the measurement of non-well-formedness introduced in this paper.

In the following we list some of the major directions for future work:

- As the main characteristic of the standard AHP algorithm is that a user needs to specify too many statements to draw a complete picture of preferences (Hsu & Wang, 2011), it is hard to expect to make any optimization in that sense. On the other hand, as two alternatives might be annotated with the same subset of qualifier tags with differences on the others, the optimization might be in reduction of checking preferences related to that intersection set. Also, another direction is in pre-recognition of inconsistencies in conditionally defined preferences. In a general case, it is a SAT problem, but the research problem is w.r.t. the recognition of situations where optimization can be done.

- The language of conditional preferences presented in this paper only allows preferences of a single variable (conditional on other variables); some natural preference statements involve preferences over more than one variable, so it would be desirable to consider more general languages (Lang, 2004; McGeachie & Doyle, 2004; Wilson, 2011).

APPENDIX 1

Proof (Lemma). According to the definition of $r_D()$ as an index of an appropriate qualifier tag in increasingly ordered array of qualifier tags of a concern with dominant relative importance, the following inequality holds:

$$\sum_{l=u}^{k} b_u r_D(qt_{j_l}^{i_l}) + r(\{qt_{j_1}^{i_1},...,qt_{j_s}^{i_s}\} \setminus \{qt_{j_1}^{i_1},...,qt_{j_k}^{i_k}\}) \le \sum_{l=u}^{k}\left(\prod_{j=l+1}^{k}\left(|QT_{i_j}|+1\right)\right)|QT_{i_l}|+1 = \prod_{j=u}^{k}\left(|QT_{i_j}|+1\right),$$

where the last equality is easily shown with mathematical induction. ■

Proof (Proposition). The proof of this proposition should be performed in two directions. In the first direction, in order to prove that, if options o_1 and $o_2$satisfy one of the conditions *(c1)-(c3)* of Definition 7, then it implies that $F(o_1)>F(o_2)$, we use mathematical induction per the number of concerns with dominant relative importance.

First, let us suppose that only concern c_i has the dominant relative importance over the others, and two arbitrary options o_1, annotated with $\{qt_{j_1^1}^{i_1^1},...,qt_{j_k^1}^{i_k^1}\}, k \le m$ and o_2, annotated with $\{qt_{j_1^2}^{i_1^2},...,qt_{j_l^2}^{i_l^2}\}, l \le m$ satisfy one of conditions *(c1)-(c3)* of Definition 7.

Let us suppose that condition *(c1)* is satisfied, e.g., an option o_1 is annotated with the qualifier tag qt_j^i of concern c_i(i.e. $r_D(qt_j^i) \ge 1$). Then, the following holds:

$$F(\{qt_{j_1^1}^{i_1^1},...,qt_{j_k^1}^{i_k^1}\}) = r_D(qt_j^i) + r(\{qt_{j_1^1}^{i_1^1},...,qt_{j_k^1}^{i_k^1}\}/\{qt_j^i\}) > 1 \ge r(qt_{j_1^2}^{i_1^2},...,qt_{j_l^2}^{i_l^2}) = F(qt_{j_1^2}^{i_1^2},...,qt_{j_l^2}^{i_l^2}).$$

In case of condition *(c2)* where options o_1 and o_2 are not annotated with qualifier tag of concern c_i and option o_1 is higher ranked then o_2 according to other concerns, it is obvious that the required inequality holds. Also, in case of condition *(c3.a)*of Definition 7 where options o_1 and o_2

are annotated with the same qualifier tag qt^i_j of concern c_i, it is obvious that required inequality holds iff $r(\{qt^{i_1^1}_{j_1^1},...,qt^{i_k^1}_{j_k^1}\}\setminus\{qt^i_j\}) > r(\{qt^{i_1^2}_{j_1^2},...,qt^{i_l^2}_{j_l^2}\}\setminus\{qt^i_j\})$, i.e., iff the option o_1 is higher ranked then $o_2$according to the other concerns.

Now, let us suppose that condition *(c3.b)* of Definition 7 is satisfied, i.e., options o_1 and o_2 are annotated respectively with the qualifier tags $qt^i_{j_i^1}$ and $qt^i_{j_i^2}$ of concern c_i, where qualifier tag $qt^i_{j_i^1}$ is higher ranked than $qt^i_{j_i^2}$. We have that the following holds:

$$F(qt^{i_1^1}_{j_1^1},...,qt^{i_k^1}_{j_k^1}) = r_D(qt^i_{j_i^1}) + r(\{qt^{i_1^1}_{j_1^1},...,qt^{i_k^1}_{j_k^1}\}\setminus\{qt^i_{j_i^1}\}) \geq r_D(qt^i_{j_i^2}) + 1 \geq r_D(qt^i_{j_i^2}) + r(\{qt^{i_1^2}_{j_1^2},...,qt^{i_l^2}_{j_l^2}\}\setminus\{qt^i_{j_i^2}\})$$
$$= F(qt^{i_1^2}_{j_1^2},...,qt^{i_l^2}_{j_l^2})$$

.

Now, if we assume that assertion holds for each number of concerns which is less then k, let us prove that it holds in case of k concerns with dominant relative importance $\overset{D}{\succ}(c_{i_1},...,c_{i_k})$. According to Definition 8, $\overset{D}{\succ}(c_{i_1},...,c_{i_k})$ holds iff $\overset{D}{\succ}(c_{i_1})$ and $\overset{D}{\succ}(c_{i_2},...,c_{i_k})$.

We have

$$F(qt^{i_1}_{j_1},...,qt^{i_k}_{j_l}) = \sum_{l=1}^{k} b_l \cdot r_D(qt^{i_l}_{j_l}) + r(\{qt^{i_1}_{j_1},...,qt^{i_k}_{j_l}\}\setminus\{qt^{i_1}_{j_1},...,qt^{i_k}_{j_k}\})$$
$$= b_1 \cdot r_D(qt^{i_1}_{j_1}) + F(\{qt^{i_1}_{j_1},...,qt^{i_k}_{j_l}\}\setminus\{qt^{i_1}_{j_1}\})$$

If any of conditions *(c1)*, *(c2)* or *(c3a)* of Definition 7 is satisfied, it is, by analogy to the previous, easy to prove the required inequality.

Now, let us consider that options o_1 and o_2 are annotated respectively with the qualifier tags $qt^{i_1}_{j_1^1}$ and $qt^{i_1}_{j_1^2}$ of concern c_{i_1}, where qualifier tag $qt^{i_1}_{j_i^1}$ is higher ranked than $qt^{i_1}_{j_i^2}$. We have that the following inequalities hold as direct implication of the proposed Lemma:

$$F(qt^{i_1}_{j_1},...,qt^{i_k}_{j_k}) = b_1 r_D(qt^{i_1}_{j_1^1}) + F(\{qt^{i_1}_{j_1},...,qt^{i_k}_{j_k}\}\setminus\{qt^{i_1}_{j_1^1}\}) \geq$$
$$b_1\left(r_D(qt^{i_1}_{j_1^2}) + 1\right) + F(\{qt^{i_1}_{j_1},...,qt^{i_k}_{j_k}\}\setminus\{qt^{i_1}_{j_1^1}\}) > b_1 r_D(qt^{i_1}_{j_1^2}) + F(\{qt^{i_1}_{j_1},...,qt^{i_l}_{j_l}\}\setminus\{qt^{i_1}_{j_1^2}\}) = F(qt^{i_1}_{j_1},...,qt^{i_l}_{j_l})$$

In the opposite direction, in order to prove that inequality $F(o_1)>F(o_2)$ implies satisfaction of one of the conditions *(c1)-(c3)*, simple reduction to the absurd and previous direction might be applied. ■

APPENDIX 2

Table 11. *Expected values (Mean value, Std dev) of the number of violations of the non-well-formedness conditions for the level of concerns*

Num of concerns / ***Num of cond. pairs of concerns***	**1-3**	**4-6**	**7-10**	**11-15**	**16-21**	**22-28**	**29-36**	**37-45**

3	**0.503**,0.386	-	-	-	-	-	-	-
4	**0.524**,0.401	**1.199**,0.819	-	-	-	-	-	-
5	**0.530**,0.419	**1.268**,0.829	**2.002**,1.374	-	-	-	-	-
6	**0.543**,0.44	**1.266**,0.818	**2.140**,1.373	**3.049**,2.116	-	-	-	-
7	**0.572**,0.508	**1.283**,0.853	**2.147**,1.377	**3.223**,2.029	**4.276**,2.965	-	-	-
8	**0.578**,0.513	**1.300**,0.870	**2.145**,1.37	**3.247**,2.038	**4.562**,2.867	**5.726**,4.008	-	-
9	**0.576**,0.538	**1.317**,0.910	**2.138**,1.376	**3.262**,2.066	**4.551**,2.852	**6.132**,3.836	**7.474**,5.211	-
10	**0.607**,0.623	**1.330**,0.933	**2.147**,1.383	**3.261**,2.094	**4.617**,2.909	**6.135**,3.844	**7.855**,4.937	**9.836**,6.184

Table 12. *Expected values (Mean value, Std dev) of the number of violations of the non-well-formedness conditions for the level of qualifier tags*

Num of concerns / ***Num of conditional concerns***	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	**0.100**, 0.060	**0.195**, 0.135	**0.346**, 0.265	-	-	-	-	-	-	-
4	**0.162**, 0.121	**0.335**, 0.214	**0.502**, 0.333	**0.824**, 0.592	-	-	-	-	-	-
5	**0.261**, 0.173	**0.526**, 0.347	**0.784**, 0.510	**1.037**, 0.685	**1.324**, 0.756	-	-	-	-	-
6	**0.371**, 0.243	**0.758**, 0.496	**1.113**, 0.732	**1.497**, 0.983	**1.887**, 1.234	**2.356**, 1.432	-	-	-	-
7	**0.523**, 0.353	**1.025**, 0.667	**1.553**, 1.023	**2.066**, 1.351	**2.584**, 1.704	**3.095**, 2.027	**3.626**, 2.327	-	-	-
8	**0.682**, 0.441	**1.357**, 0.896	**2.022**, 1.335	**2.702**, 1.783	**3.402**, 2.221	**4.072**, 2.684	**4.743**, 3.110	**5.221**, 3.453	-	-
9	**0.865**, 0.572	**1.726**, 1.135	**2.596**, 1.702	**3.456**, 2.261	**4.325**, 2.842	**5.172**, 3.406	**6.010**, 3.943	**6.892**, 4.527	**7.634**, 4.801	-
10	**1.088**, 0.716	**2.158**, 1.411	**3.214**, 2.118	**4.294**, 2.828	**5.345**, 3.503	**6.415**, 4.208	**7.462**, 4.891	**8.532**, 5.607	**9.631**, 6.327	**10.746**, 6.960

REFERENCES

Amiri, M. P. (2010). Project selection for oil-fields development by using the AHPand fuzzy TOPSIS methods. *Expert Systems with Applications, 37*(9), 6218–6224.

Ishizaka, A., & Labib , A. (2011). Review of the main developments in the analytic hierarchy process. *Expert Systems with Applications, 38*(11), 14336–14345.

Aguaron, J., & Moreno-Jimenez, J. (2003). The geometric consistency index: Approximated thresholds. *European Jouranl of operational Research, 147*, 137-145.

Bagheri, E., & al, e. (2010). Stratified analytic hierarchy process: prioritization and selection of software features. *In Proceedings of the 14th International conference on Software product lines*, (str. 300-315).

Bagheri, E., & Gasevic, D. (2010). Assessing the Maintainability of Software Product Line Feature Models using Structural Metrics. *Software Quality Journal, 19*, 579-612.

Bagheri, E; Ensan, F. (2011). Consolidating multiple requirement specifications through argumentation. *In Proceedings of the 26thACM Symposium on Applied Computing,* (str. 659-666). Tunghai University, Taichang, Taiwan.

Bagheri, E; Ghorbani, A.A. (2009). The analysis and management of non-canonical preference specifications through a belief integration game. *Knowledge and Information Systems: An International Journal, 22*(1), 27-64.

Behr, M., & al, e. (1992). Rational number ratio and proportion. U D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (str. 296-333). New York: Macmillan.

Benferhat, S., & al, e. (2006). Bipolar Possibility Theory in Preference Modeling: Representation, Fusion and Optimal Solutions. *Information Fusion, Elsevier, 7*(1), 135-150.

Berander, P., & Andrews, A. (2006). Requirements Prioritization. *Engineering and Managing Software Requirements*, 69-94.

Berander, P., & Jönsson, P. (2006). Hierarchical Cumulative Voting (HCV) – Prioritization of Requirements in Hierarchies. *International Journal of Software Engineering & Knowledge Engineering, 16*, 819-849.

Blaikie, N. (2003). *Analyzing Quantitative Data.* London: Sage.

Boutilier, C., & al., e. (2004). CP-nets: a tool for representing and reasoning with conditional ceteris paribus preference statements. *Journal of AI Research, 21*(1), 135-191.

Boutilier, C., Bacchus, F., & Brafman, R. (2001). UCP-Networks: A Directed Graphical Representation of Conditional Utilities. *In Proceedings of 17th Conference in Uncertainty in AI*, (str. 56-64). San Francisco, USA.

Brafman, R., & Domshlak, C. (2002). Introducing variable importance tradeoffs into CP-nets. *In The Proceedings of the Eighteenth Conference on Uncertainty in AI*, (str. 69-76). Canada.

Brafman, R., Domshlak, C., & Kogan, T. (2004). Compact value-function representations for qualitative preferences. *In Proceedings of the 20th Annual Conference on Uncertainty in AI*, (str. 51-58). Canada.

Brennan, A. (2003). Necessary and Sufficient Conditions. *Standard Encyclopedia of Philosophy*.

Büyüközkan, G., Çifçi , G., & Güleryüz , S. (2011). Strategic analysis of healthcare service quality using fuzzy AHP methodology. *Expert Systems with Applications, 38*(8), 9407–9424.

Chen , M. K., & Wang, S. (2010). The critical factors of success for information service industry in developing international market: Using analytic hierarchy process (AHP) approach. *Expert Systems with Applications, 37*(1), 694–704.

Chen, L.; Pu, P. (2004). Survey of Preference Elicitation Methods. *EPFL Technical Report IC/2004/67.* Switzerland: Swiss Federal Institute of Technology in Lausanne.

Chen, S., Buffett, S., & Fleming, M. (2007). Reasoning with Conditional Preferences across Attributes. *In Proceedings of the 20th Conference of the Canadian Society for Computational Studies of Intelligence on Advances in AI*, (str. 369-380).

Czarnecki, K., & Kim, C. (2005). Cardinality-based feature modeling and constraints: A progress report. *International Workshop on Software Factories, ACM.*

Domshlak, C. (2008). A Snapshot on Reasoning with Qualitative Preference Statements in AI. U *Preferences and similarities, CISM International Centre for Mechanical Sciences* (T. 504, str. 265-282).

Domshlak, C., & al., e. (2011). Preferences in AI: An overview. *Artificial Intelligence, Elsevier*, 1037-1052.

Domshlak, C., & Brafman, R. (2002). CP-nets- Reasoning and Consistency testing. *In Proceedings of the 8th International Conference on Principles of Knowledge Representation and Reasoning*, (str. 121-132). France.

Escobar, M., Aguaron, J., & Moreno-Jimenez, J. (2004). A note on AHP group consistency for the row geometric mean prioritization procedure. *European Journal of Operational Research, 153*, 318-322.

Figueira, J., Mousseau, V., & Roy, B. (2005). Electre methods, Multiple criteria decision analysis: State of the art surveys. *International Series in Operations Research & Management Science, 78*(3), 133-153.

Forman, E., & Gass, S. (2001). The analytical hierarchy process—an exposition. *Operations Research, 49*(4), 469-487.

Glaser, A. (1981). *History of binary and other non-decimal numeration.* Tomash Publishers.

Goldsmith, J., & al, e. (2008). The computational complexity of dominance and consistency in CP-nets. *Journal of AI Research*, 403-432.

Gonzales, C., & Perny, P. (2004). GAI networks for utility elicitation. *In Proceedings of the 9th International Conference on Principles of Knowledge Representation and Reasoning*, (str. 224-234). Whistler, Canada.

Hardy, M. (1993). Regression with Dummy Variables (Quantitative Applications in the Social Sciences). Sage Publications Inc.

Hsu, S., & Wang, T. (2011). Solving multi-criteria decision making with incomplete linguistic preference relations. *Expert Systems with Applications, 38*(9), 10882–10888.

Hwang, C., & Yoon, K. (1981). Multiple attribute decision making methods and applications: a state-of-the-art survey. U M. Beckmann, & H. Kunzi, *(eds.) Lecture Notes in Economics and Mathematical Systems.* Berlin: Springer-Verlag.

Karlsson, J., Olsson, S., & Ryan, K. (1997). Improving Practical Support for Large-scale Requirements Prioritising. *Requirement Engineering, 2*(1), 51-60.

Keene, O. (1995). The Log-Transformation is Special. *Statistics in Medicine, 14*(8), 811-819.

Keeny, R., & Raiffa, H. (1993). *Decisions with Multiple Objectives: Preferences and Value Tradeoffs.* Cambridge: University Press.

Lang, J. (2004). Logical preference representation and combinatorial vote. *Annals of Mathematics and Artificial Intelligence, 42*(1), 37-71.

Langley, P. (1995). *Elements of Machine Learning.* San Francisco: Morgan Kaufmann Publishers Inc.

MacCrimmon, K. (1968). Decision-making among multiple-attribute alternatives: A survey and consolidated approach. *Memorandum RM-4823-ARPA, The Rand Corporation.* Santa Monica, California.

Makki, M.; Bagheri, E.; Ghorbani, A. (2008). Automating Architecture Trade-Off Decision Making through a Complex Multi-attribute Decision Process. *In Proceedings of European Conference on Software Architecture*, (str. 264-272). Paphos, Cyprus.

McGeachie, M., & Doyle, J. (2004). Utility functions for ceteris paribus preferences. *Computational Intelligence, Willey-Blackwell, 20*(2), 158-217.

McGeachie, M., & Doyle, J. (2011). The local geometry of multiattribute tradeoff preferences. *Artificial Intelligence, 175*(7-8), 1122-1152.

McManus, J. (2004). *Managing stakeholders in software development projects.* Butterworth-Heinemann, Elsevier.

Mouhoub, M.; Sukpan, A. (2006). Conditional and Composite Temporal Constraints with Preferences'. *In Proceedings of the 13th International Symposium on Temporal Representation and Reasoning*, (str. 95-102). Budapest, Hungary.

Mukhtar, H., Belaïd, D., & Bernard, G. (2009). A quantitative model for user preferences based on qualitative specifications. *In Proceedings of International Conference on Pervasive services*, (str. 179-188). New York.

Ognjanovic, I., & al, e. (2011). Conditional Preferences in Software Stakeholders' Judgments. *In Proceedings of the 26th Annual ACM Symposium on Applied Computing*, (str. 683-690). Taichang, Taiwan.

Papazoglou, M. (2003). Service-Oriented Computing: Concepts, Characteristics and Directions. *In Proceedings of the 4th International Conference on Web Information Systems Engineering.* Washington, USA.

Porubský, S. (n.d.). *Positional Numeral Systems, Interactive Information Portal for Algorithmic Ma-thematics.* Prague: Institute of Computer Science of the Czech Academy of Sciences.

Rasmussen, J., & Dunlap, W. (1991). Dealing with non-normal data: Parametric analysis of transformed data vs nonparametric analysis. *Educational & Psychological Measurement, 51*(4), 809-820.

Roper-Low, G. (1990). The Analytic Hierarchy Process and its application to an information technology. *The Journal of Operational Research, 41*(1), 49-59.

Satty, T. (1980). The Analytic Hierarchy Process. *McGraw-Hill, New York.*

Satty, T. (2003). Decision-making with the AHP: Why is the principal eigenvector necessary. *European Journal of Operational Research, 145*(1), 85-91.

Sommerville, I.; Sawyer, P. (1997). Viewpoints: principles, problems and a practical approach to preferences engineering. *Annals of Software Engineering*(3), 101-130.

Srinivasan, V., & Shocker, A. (1973). Linear programming techniques for multidimensional analysis of privileged. *Psychometrika, 38,* 337-369.

Stirling, W., & al, e. (2007). Multicriterion Decision Making with Dependent Preferences. *IEEE Symposium on Computational Intelligence in Multicriteria Decision Making*, (str. 227-234).

Stojmenovic, I. (1992). A simple systolic algorithm for generating combinations in lexicographic order. *Computers Math.Application, 24*(4), 61-64.

Strategic analysis of healthcare service quality using fuzzy AHP methodology. (n.d.).

Turski, Z. (2008). Multi-attribute contractors ranking method by applying ordering of feasible alternatives of solutions in terms of preferability technique. *Technological and economic development, Baltic Journal on Sustainability, 14*(2), 224-239.

Wilson, N. (2011). Computational techniques for a simple theory of conditional preferences. *Artificial Intelligence, Elsevier, 175*(7-8), 1053-1091.

Yu, H.Q.; Reiff-Marganiec, S. (2008). Non-functional property based service selection: A survey and classification of approaches. *In Proceedings of the International Workshop on Non Functional Properties and Service Level Agreements in Service Oriented Computing at the 6th IEEE European Conference on Web Services.* Ireland.

Yu, Z., & al., e. (2010). Toward an Understanding of User-Defined Conditional Preferences. *In Proceedings of the 8th IEEE International Conference on Dependable, Autonomic and Secure Computing*, (str. 203-208). Washinghton, USA.

Zavadskas, E., & al, e. (2007). Multi-attribute assessment of road design solutions by using the COPRAS Method. *The Baltic Journal of Road and Bridge Engineering, 2*(4), 195-203.

Zavadskas, E., & al, e. (2007). Sensitivity analysis of a simple additive weight method. *International Journal of Management and Decision Making (IJMDM), 8*(5-6), 555-574.

Zavadskas, E., & Kaklauskas, A. (1996). Determination of an efficient contractor by using the new method of multicriteria assessment. *International Symposium for "The Organisation and Management of Construction", 2,* str. 94-104.

LIST OF FIGURES

Figure 1. Electronic payments services annotated with qualifier tags. 6

Figure 2.Schematic representation of lexicographical order over two-layered structure of concerns and qualifier tags. 13

Figure 3. CS-AHP extension of the Feature Plug-in: a) Editing concerns and associating with features; b) Specifying conditional preferences on the level of concerns 18

Figure 4. Dummy coding for the number of conditional pairs of concerns on the level of concerns 25

LIST OF TABLES

Table 1. The results of the correlation analysis 23
Table 2. Mean and standard deviation values of the number of violations of the well-formedness conditions for different numbers of concerns 24
Table 3. Mean values of the number of violations of the well-formedness conditions for different structures of conditional preferences 24
Table 4. Mean and standard deviation values of the number of violations of the well-formedness conditions for different numbers of conditional pairs of concerns 25
Table 5.Mean and standard deviation values of the number of violations of the well-formedness conditions for different numbers of conditional concerns 25
Table 6. The results of regression analyzes for conditional preferences consisting of only then parts 26
Table 7. The results of regression analyzes for conditional preferences consisting of then parts and optional of else parts 26
Table 8. Results of regression analyzes for conditional preferences consisting of both then and else parts 27
Table 9. Mean and standard deviation values of the number of the violations of the well-formedness conditions for different distributions 28
Table 10. Comparative analysis of prioritization techniques based on different scale of input/output data, different sort of preferences and performances for reasoning queries 33
Table A.1. Expected values (Mean value, Std dev) of the number of violations of the non-well-formedness conditions for the level of concerns 36
Table A.2. Expected values (Mean value, Std dev) of the number of violations of the non-well-formedness conditions for the level of qualifier tags 37

LIST OF SYMBOLS

$|A|$- cardinality of the set A
$\{\ldots\}$- set of elements
$\in, \notin$-relations of belonging and not belonging to
$\varnothing$-empty preference (i.e. empty set of relative importance)
O- set of options
c- concern
qt – qualifier tag
C- set of concerns
QT- set of qualifier tags
$\Re$ -set of preferences
$\Re_C, \Re_{QT}, \Re_C^D$- set of preferences defined respectively, on the level of concerns, on the level of qualifier tags and on the level of concerns with dominant relative importance
O()- big *O* notation
f(), F()- functions
$\sum_{i=a}^{b} e_i, \prod_{i=a}^{b} e_i$ - sum and multiplication of elements e_i where $i \in \{a,...,b\}$
$\wedge, \vee, \neg$-logical operations of conjunction, disjunction and negotiation

$\overset{\alpha}{\succ}$ - relation of relative importance with importance α

$\overset{D}{\succ}$ - relation of dominant relative importance

$r(), r_D(), \bar{r}$ - ranks obtained based on respectively, preferences defined without relation of relative importance, preferences with relation of relative importance, all sorts of preferences