

Tales of the Specific Factor Model¹ (Let me know if you spot errors.)

The Specific Factor Model was developed by the distinguished University of Rochester economist, Ronald W. Jones in the mid-1960s. It is a cautionary tale. Although Jones had already published a much admired “Simple Structure of General Equilibrium Models” that clearly explained what was by then the standard two-factor two-good Heckscher-Ohlin model, he was unable to find a journal willing to take a piece about a model that had three factors and two goods -- what we now call the specific factor model. Eventually, in 1971 he published it in a *Festschrift* volume for Charles Kindleberger and consequently shares credit for the model with Paul Samuelson who also published an analysis in that year.² This is one of the most heavily used models in economics as it captures a number of important ingredients of economic analysis.

The Specific Factor Model

1. There are two industries which produce according to constant returns to scale technology (that is, double your inputs and you double your outputs; the marginal products of factors depend only on the relative of amounts of factors used).
2. The model is situated in an economy in which there is competition in both goods and factor markets.
3. Each industry uses two factors to produce output, however one factor is fixed in (or specific to) each industry (and is in fixed supply) and one factor is mobile between the two industries.³
4. There is a fixed overall supply of the mobile factor that can costlessly move between the industries.

The model has 5 variables that are of interest: outputs of the two industries, x_1 , x_2 and three factor rewards. To make matters concrete, we will refer to the rewards going to the three factors of production as the reward to labour, w , capital, r , and land, t . When we say they are of interest that means that we can solve the model for these variables as a function of the exogenous variables which are those that are outside the model. These are the stock of labour, L , capital, K ,

¹ These notes are to be read together with the Chapter in your text. They are complements not substitutes!

² The “Simple Structure of General Equilibrium Models” (*Journal of Political Economy*, Volume LXXIII, December 1965, Number 6): 557-572 was a clarifying exposition of the two-factor, two sector model most frequently identified in the trade literature as the Heckscher-Ohlin model. It made the model accessible to a wide professional audience. Jones, Ronald W.: “A Three-Factor Model in Theory, Trade and History,” Ch. 1 in Jagdish Bhagwati, Ronald Jones, Robert Mundell and Jaroslav Vanek, (eds.), *Trade, Balance of Payments and Growth* (North-Holland, Amsterdam) and Paul Samuelson “Ohlin Was Right,” *The Swedish Journal of Economics*, 1971 are the references to the first formal exposition of the specific factor model although Chambers, E.J. and Gordon, D. F. “Primary Products and Economic Growth: An Empirical Measurement” *Journal of Political Economy* 74, 1966: 315-22, provide a framework for Canadian economic growth that uses a three-factor, two-sector model in which capital (a specific factor) is internationally mobile. Gordon was at Rochester during this period and was aware of Jones’ modelling developments.

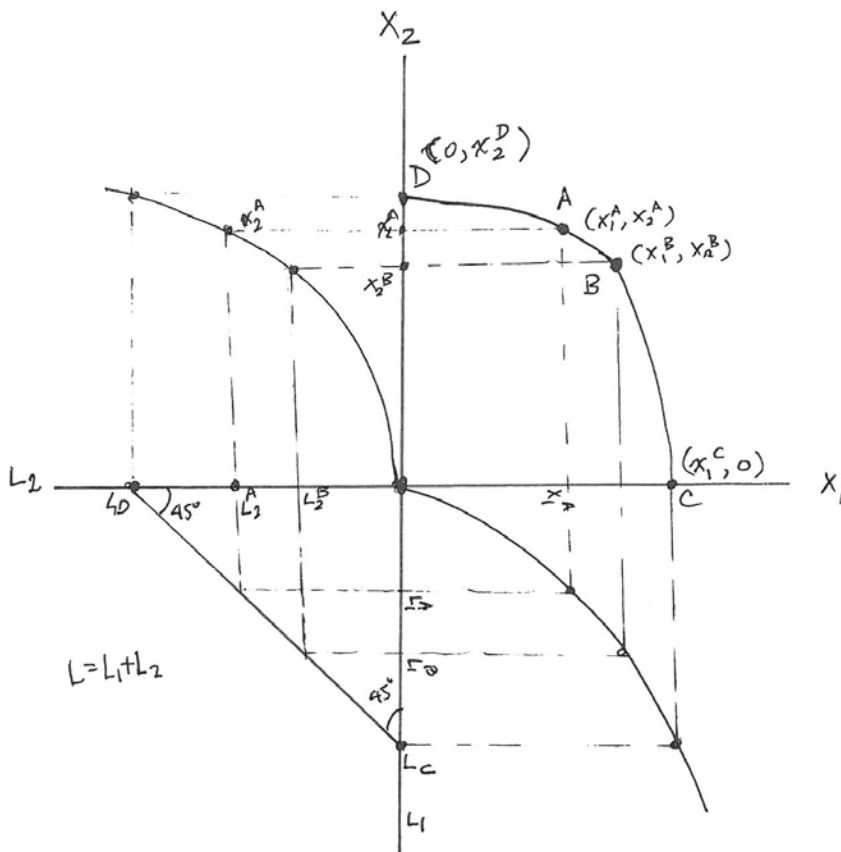
³ For a student who is a bear for punishment there is a generalization of the specific factor model in which all three factors are used in both industries in R.W. Jones and S. T. Easton, “Factor Intensities and Factor Substitution in General Equilibrium”, *Journal of International Economics* 15 (1983): 65-99.

and land, T , as well as commodity prices, p_1 , and, p_2 . If you like you can think of our country as being “small” in the sense that we take world commodity prices as given exogenously.

Deriving the Production Possibilities Frontier

To represent this model let us first derive the production possibilities frontier. In this case, it is easiest to construct the same diagram as we used in the Appendix to the Ricardian model. For concreteness, we will assume that labour is used in both industries and industry 1 uses capital (K) and industry 2 uses land (T). The difference between the diagram here and that of the Ricardo-Appendix is that the production technologies for both goods display diminishing returns to the mobile factor. The diagram is drawn for a fixed amount of the specific factor input in each industry so that the input axis is that of the mobile factor, labour, L , in each industry (although in the background we know it is really the labour to capital or labour to land ratio that determines the marginal product.)

Figure 1



With the diagram we have geometrically derived the levels of outputs for exogenous levels of the factor inputs by simply reallocating labour across the industries. Further, it should be clear that when we derive the frontier by moving labour from one industry to another, we sacrifice the

marginal product of labour in the industry that is shrinking and gain the marginal product of labour in the industry that is expanding. Thus along the frontier, the slope is: $-\frac{dx_2}{dx_1} = \frac{MPP_L^2}{MPP_L^1}$. [Can you explain why the slope is negative? Can you explain why the slope is also equal to the price ratio: $\frac{p_1}{p_2}$? Check the Appendix for help.]

Solving for Factor Prices

There are three factor prices in the approach: \mathbf{w} , \mathbf{r} and \mathbf{t} . Looking at the model structurally lets us see that there are some fundamental asymmetries since labour is used in both industries, but capital is used only in industry 1, and land is used only in industry 2. Harkening back to our description of Ricardo we can still write:

$$L = L_1 + L_2$$

But think of the equation in terms of supply and demand. On the left-hand-side is the overall all amount of labour in the economy (supply) fixed at L , while the amount of labour on the right-hand-side is the *demand for labour arising from the two industries*. In this way of seeing things, the natural step is to ask how much labour will be demanded by each industry. Since we know from our approach to Ricardo, we can write the labour constraint as:

$$L = a_{L1}x_1 + a_{L2}x_2$$

in which the a_{Lj} 's refer to the amount of labour input per unit of output and the x_j 's refer to output in each industry. We now see that the demand for labour arises from two sources: the demand from the extensive margin – the level of output, x_j , (since more output other things equal will lead to a greater demand for labour), and the intensive margin - the amount of labour demanded per unit output.

It pays to stop for a moment and ask what we want to do. Our purpose is to solve for factor prices \mathbf{w} , \mathbf{r} and \mathbf{t} . To do this, we need to have the demand for these factors arising from each industry. Industry demand depends on the marginal product of each factor.⁴ Since it is used in both industries we initially focus on the demand for labour.

Displaying our picture of labour demands in both industries leads us to a diagram in which the total labour supply is along the horizontal axis and the marginal product schedules are facing each other. This picture is displayed below⁵ and characterizes the equilibrium wage common to

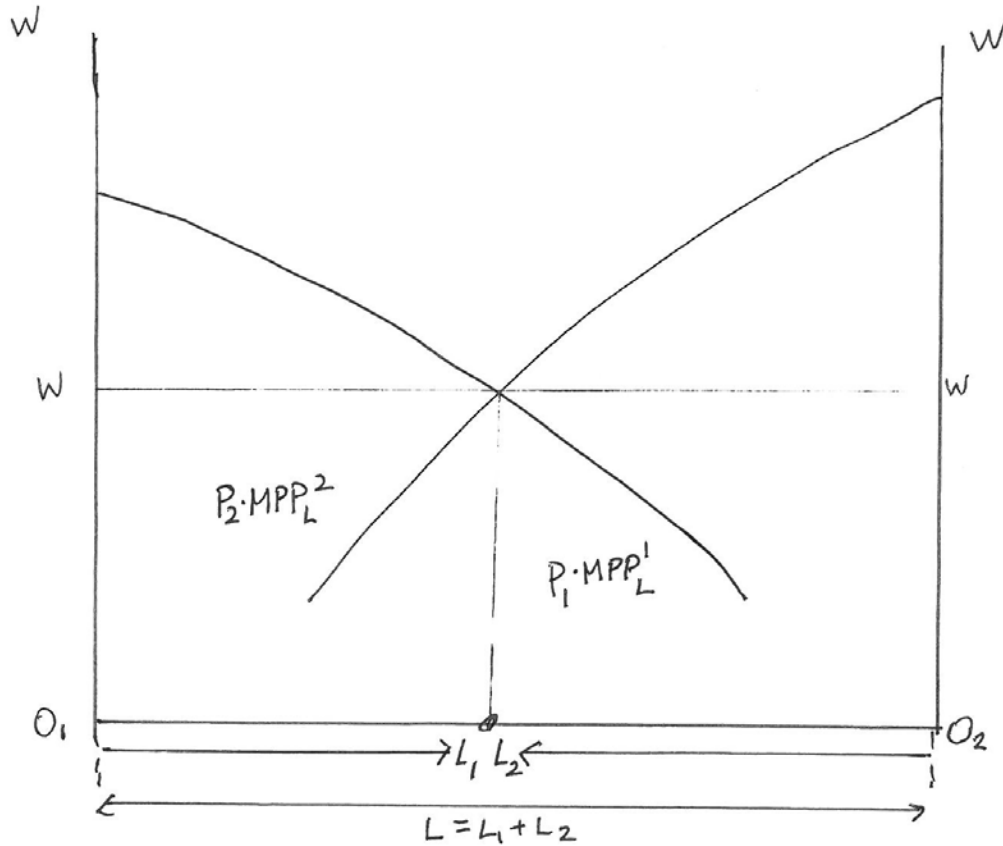
⁴ It is worth remarking that with constant returns to scale the shape of the marginal product of each factor depends only on the relative amounts of capital and labour or land and labour and not the scale of production.

⁵ The key feature of the demand for labour is its marginal product. The marginal product tells us the way in which labour is linked to the wage rate. More precisely the elasticity of the marginal product, γ_{L1} , is defined by the changes in the labor use relative to capital use when the wage (measured in terms of good 1) changes in industry 1. As before, the “^” denotes the percentage change.

$$\gamma_{L1} \equiv - \left(\frac{\hat{a}_{L1} - \hat{a}_{K1}}{\hat{w} - \hat{p}_1} \right)$$

the two industries and the allocation of labour between them. The prices are exogenous and the marginal physical products of labour are the slopes of the productions functions in quadrants II and IV in Figure 1.

Figure 2



Relative Price Changes and the Magnification Effects

The marginal (physical) product is the change in output when we change the (labour) input. Multiplying by price gives the value of this marginal product which is equal to the wage rate in our competitive economy:

$$w = p_1 \times MPP_L^1 \left(\frac{L_1}{K} \right)$$

Thus the elasticity of the marginal product schedule reflects the changes in the ratio of labour to capital usage when the wage changes in industry 1. The negative sign reminds us that the marginal product schedule has a negative slope since an increase in the wage reduces the demand for labour.

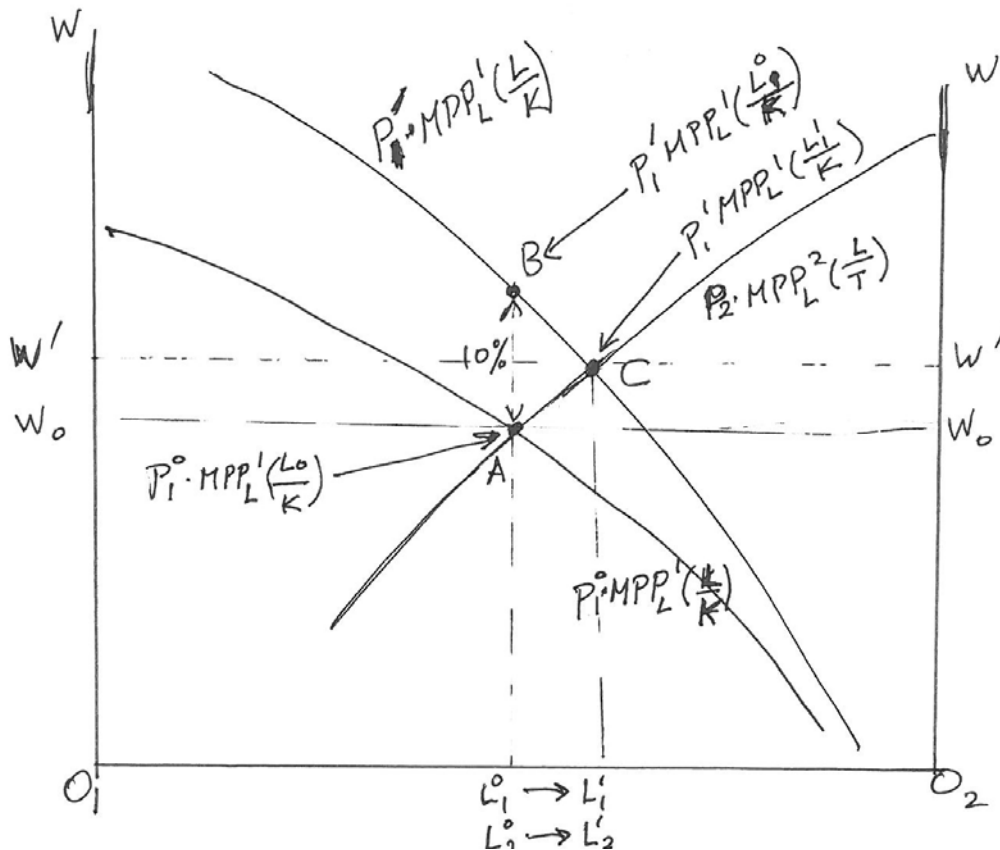
We write the same expression for industry 2:

$$\gamma_{L2} \equiv - \left(\frac{\hat{a}_{L2} - \hat{a}_{T2}}{\hat{w} - \hat{p}_2} \right)$$

The marginal physical product depends on the ratio of capital to labour (or, more conveniently, labour to capital in this case.) Thus we draw the figure in which the total labour force in each industry is along the horizontal axis with industry 1 originating on the left and industry 2 on the right. The marginal products both slope downward and are assumed to cross. That point of cross establishes the wage. (Commodity prices are exogenous.) We draw the industry 2 demand in a similar fashion. Notice that in both cases, the physical quantity of the specific factors is exogenous as is the overall supply of labour.)

Now increase the price of good 1 (holding the price of good 2 constant) by an arbitrary 10 percent: from p_1^0 to p_1^1 . In the figure, this rotates the value of the marginal product “up” so that each point on the new marginal product schedule is 10% higher than before. A new equilibrium looks like it would be at point B. But that is not to be since at point B the wage is higher in industry 1 than in industry 2. Thus labour flows out of industry 2 and into industry 1 (from L_1^0 to L_1^1 with a corresponding reduction in labour in industry 2) until the wages are equalized at point C. Thus output of industry 1 expands and output of industry 2 contracts. In fact this is simply the movement along the production possibilities frontier as relative prices change! [Think of this as a move from A to B in Figure 1.] Notice, too, that even though relative prices change, unlike Ricardo, the price change does not provoke specialization.

Figure 3

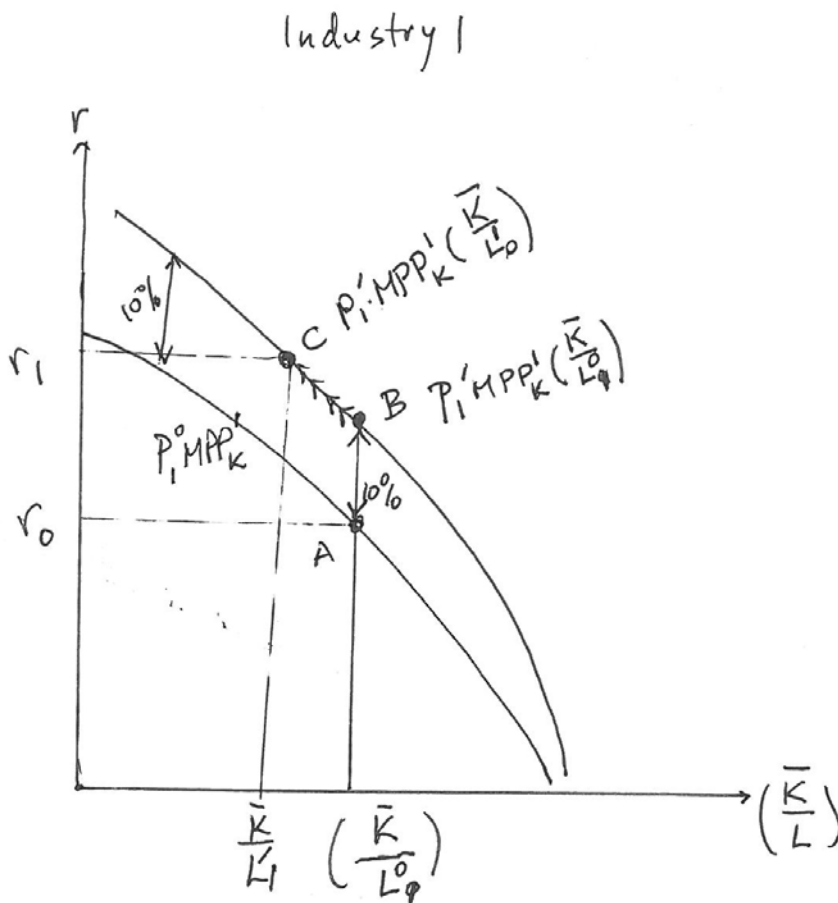


What can we say about the wage change associated with the 10 percent increase in the price of good 1: from p_1^0 to p_1' ? Notice that the wage has increased. But by how much? From the simple geometry it seems pretty clear that it has risen by less than 10 percent. Why? In this case the higher wage attracts labour from the other industry until the new equilibrium is established. That lowers the economy wide wage from the 10 percent price increase. How much lower? We don't know until we know more about the size of the two industries and the elasticities of their marginal products. But we do know that the change in the wage is trapped between the 10 percent increase in price of good 1 and the zero percent increase in the price of good 2. Thus we have established a ranking in which:

$$\hat{p}_1 > \hat{w} > \hat{p}_2 (= 0)$$

What can we say about the returns to the fixed factors arising from this 10 percent increase in the relative price of good 1 experiment? We can draw the same diagram for each of the fixed factors. A bar over a variable in the diagram indicates that it is in fixed supply.

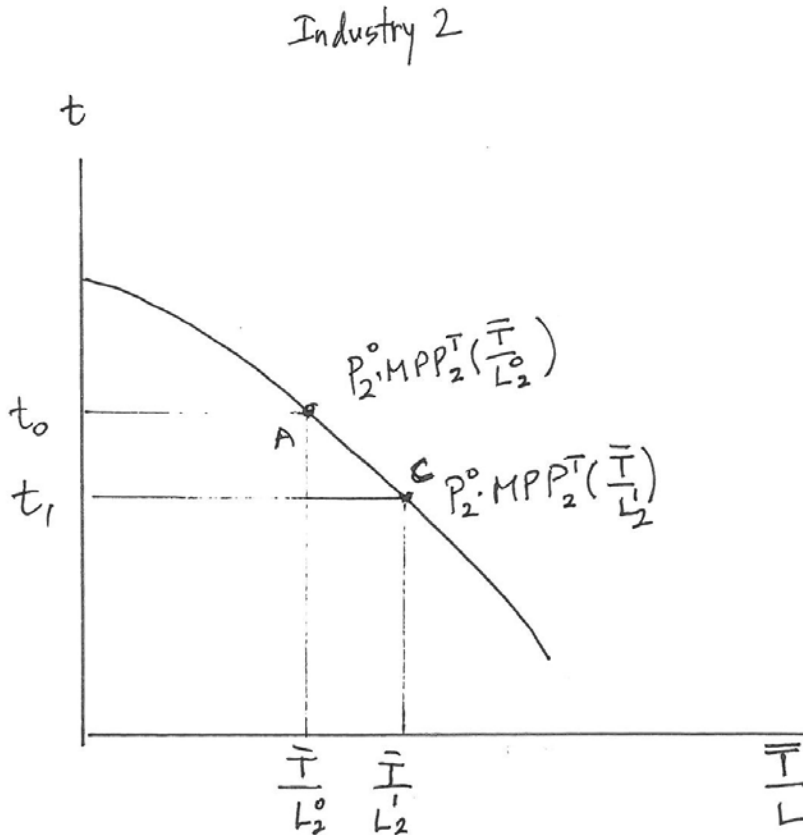
Figure 4



Notice that with the 10 percent increase in the price of good 1, the value of the marginal product schedule shifts up by 10 percent in industry 1. (Notice it does not shift the marginal product schedule in good 2 in Figure 5) In good 1, not only does the price increase, but as labour arrives

from industry 2, the capital/labour ratio decreases as well. Thus the value of the marginal physical product schedule is rotated up by 10% from the price increase, and in addition, the return to capital is driven higher as we move along the marginal physical product schedule which depends on the capital to labour ratio – which falls.

Figure 5



Dissimilarly in industry 2, there is no shift in the value of the marginal product schedule since there is no change in the price of good 2. Instead, as labour flows out from industry 2 to industry 1, the land to labour ratio will increase. This drives down the return to land along the marginal physical product schedule from A to C.

We can now complete what is termed the *magnification effects*: the change in relative prices has a magnified effect on some of the factor rewards.⁶

$$\hat{r} > \hat{p}_1 > \hat{w} > \hat{p}_2 (= 0) > \hat{t}$$

⁶ Although beyond the scope of this course, if there is no joint production – for example, two outputs produced by a single input – think of a sheep producing wool and mutton, then it will always be the case that there will be at least one factor whose price will rise more than the final product price and one factor whose price will rise less than the product price. This is simply a result of the observation that the final goods price is a weighted average of the input prices. Insofar as the weights are distributive shares and differ, the result holds.

Some algebra of the Specific Factor model⁷

1. Firms in both industries minimize their cost. Cost in each industry depends on which factors are used. In industry 1,

$$C_1 = wL_1 + rK ,$$

And, in industry 2,

$$C_2 = wL_2 + tT.$$

2. Since we have competition in the goods and factor markets, each firm is too small to affect factor prices. Consequently, individual firms take factor rewards as exogenous. In this case, firms only choose the amount of labour since the stocks of capital and land are fixed and specific to each industry. (The return to each fixed factor is a rent.) Because firms minimize their cost, $dC_1 = 0$ the first line simply identifies the sources of all changes in the firm's cost function.

$$dC_1 = L_1 dw + w dL_1 + K dr + r dK$$

But, since the factor rewards (w and r) are treated as exogenous by the firm,

$$dC_1 = 0 = w dL_1 + r dK$$

Which implies that the slope of the iso-cost function (when $dC_1=0$) depends on the wage rental ratio:

$$\frac{dK}{dL_1} = -\frac{w}{r}$$

⁷ I am indebted to Catherin Michaud for providing the edited write up of my lectures.

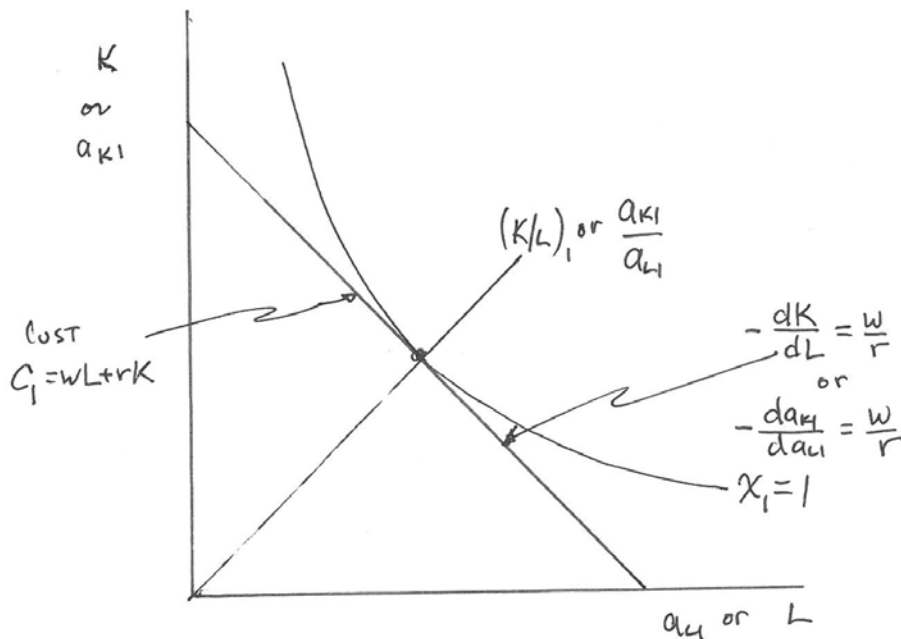
We can rewrite⁸ this in our notation so that we have:

$$\frac{da_{K1}}{da_{L1}} = -\frac{w}{r} \quad (5)$$

$$rda_{K1} = -wda_{L1}$$

$$rda_{K1} + wda_{L1} = 0 \quad (6)$$

The consequences of cost minimization identifies factor proportions as a function of factor prices. This is displayed geometrically in the figure below. The straight line is the iso-cost function for industry 1. The tangency is the minimum cost associated with (in this case) the unit isoquant. The tangency identifies the optimal capital to labor ratio in this industry.



⁸ Since we have constant returns to scale in production, we can use the unit isoquant as the representative of all isoquants. If we put a_{K1} and a_{L1} on the axes (rather than capital and labour), then with constant returns to scale, the production technology is fully specified with the “single” isoquant. Recall, too, that the ratio of factors of production alone determine the marginal products.

3. Competition ensures that the value of output is equal to the value of inputs:

$$p_1x_1 = wL_1 + rK,$$

which in turn can be expressed as the value per unit of output:

$$p_1 = \frac{wL_1}{x_1} + \frac{rK}{x_1}$$

$$p_1 = wa_{L1} + ra_{K1}$$

4. Now, we are interested in what can explain the change in price, so we take the total derivative:

$$dp_1 = a_{L1}dw + wda_{L1} + a_{K1}dr + rda_{K1} \quad (6)$$

From equation (6) which is a consequence of competition in factor markets that makes firms always (factor) price takers, we can immediately (and marvellously) simplify the determinants of the change in price at the aggregate level:

$$dp_1 = a_{L1}dw + a_{K1}dr$$

5. Again, because we are interested in the change in percentage, we multiply both sides by

$\frac{1}{p_1}$ and the RHS by $\frac{w}{w}$ and $\frac{r}{r}$:

$$\frac{dp_1}{p_1} = \frac{a_{L1}w}{p_1} * \frac{dw}{w} + \frac{a_{K1}r}{p_1} * \frac{dr}{r}$$

$$\hat{p}_1 = \theta_{L1}\hat{w} + \theta_{K1}\hat{r} \quad (7)$$

6. The same result with suitably redefined subscripts will be true for good 2 (where we

define labour's distributive share in agriculture as $\theta_{L2} = \frac{L_2w}{p_2x_2}$ so that :

$$\hat{p}_2 = \theta_{L2}\hat{w} + \theta_{T2}\hat{t} \quad (8)$$

Note that: $\theta_{L1} + \theta_{K1} = \frac{L_1w}{p_1x_1} + \frac{Kr}{p_1x_1} = \frac{L_1w+Kr}{p_1x_1} = \frac{p_1x_1}{p_1x_1} = 1$, and correspondingly,

$$\theta_{L2} + \theta_{T2} = 1.$$

Seeing the algebra of price changes makes the exercises more mechanically simple and may help guide your reasoning. For example, an increase in the supply of labour, L, ($L = L_1 + L_2$)

reduces the wage rate and increases labour in both sectors. (Presumably, this is easy to show.) How do we show what happens to the returns to capital and land? When prices are constant (and the wage decreases), arithmetically we can see that $\hat{r} > 0$ and $\hat{t} > 0$. Knowing this, you can turn to the capital and land marginal product diagrams with confidence.

Similarly, with the supply of capital (K) increased and commodity prices held constant: $\hat{r} \downarrow$ so $\hat{w} \uparrow$ and $\hat{t} \downarrow$.

Finding the slope of the production possibilities frontier (PPF)⁹

Let $x_1 = F(K, L_1)$ and $x_2 = G(T, L_2)$ be the two production functions in the specific factor model.

We are interested in the slope of the production possibilities frontier: $\frac{dx_2}{dx_1}$.

Since

$$dx_1 = MPP_K^1 dK + MPP_L^1 dL_1, \text{ and}$$

$$dx_2 = MPP_T^2 dT + MPP_L^2 dL_2,$$

Where we define the MPP as the marginal physical product of the factors in each industry. You will recall that the MPP is the change in output caused by a change in a particular factor input holding other factors of production constant.

The slope of the ppf is rise over run, or in this case, $(\Delta x_2 / \Delta x_1)$ which is:

$$\frac{dx_2}{dx_1} = \frac{MPP_T^2 dT + MPP_L^2 dL_2}{MPP_K^1 dK + MPP_L^1 dL_1}$$

⁹ It is probably worth reminding ourselves that the definition of the production possibilities frontier is the maximum amounts of the two goods for a) a given technology, b) fixed endowments of factors, c) competition in both goods and factor markets. There are other conditions, but these are sufficient for our purposes.

This is simplified if we notice first that the stocks of land and capital are fixed along the PPF so that $dK=dT=0$. Thus, we are left with:

$$(9) \quad \frac{dx_2}{dx_1} = \frac{MPP_L^2 dL_2}{MPP_L^1 dL_1}$$

However, recall that the stock of labor in the economy is in fixed supply:

$$L = L_1 + L_2$$

So that any changes in the amounts of labor between the industries is equal and opposite in sign:

$$dL = 0 \Rightarrow dL_1 = -dL_2$$

Which gives the basic result:

$$(10) \quad \frac{dx_2}{dx_1} = - \frac{MPP_L^2}{MPP_L^1}$$

But, we can do more!

Remember that the wage is equal to the value of the marginal (physical) product. Thus the wage in each industry is:

$$w = p_1 MPP_L^1$$

$$w = p_2 MPP_L^2$$

So that 10 becomes:

$$(11) \quad \frac{dx_2}{dx_1} = - \frac{p_1}{p_2}$$