Please let me know about typos and the like

## A First Model

Let us consider a simple model of property crime.

### Utility

We make the assumption familiar to all economists that preferences can be expressed as function of goods and that every individual has tastes that are continuous.

In this particular story it is convenient to write utility as a function of income,

$$U=U(Y) \qquad 1.$$

We would naturally assume that U'(Y)>0, since income is a good insofar as it allows you to consume more of anything, and more interestingly in this context, U''(Y)>0 *or* U''(Y)<0. In other words we do not have strong feelings about the slope of marginal utility for the moment.

Writing utility as a function of income is a useful simplification at this point. Later on in the course we will refine this so as to include such things as labour supply and time spent doing crime, but for the moment these are refinements to be thought about in the future.

### Gains to Crime

So what it is the problem to which this utility function is central? The way we think about it is to ask the following question: Are you better off committing a crime or not? What is necessary in the model to ask this question? First, you need a gain from committing the crime. Let us assume that your gain is purely financial and is G dollars. Now, what is your total income? If you are engaged in crime we will assume that you add G dollars to your initial income that we assume is $Y_0$ dollars. Notice that your total income Y is now equal to the sum of $Y_0$ and G, or symbolically:

$$Y=Y_0+G \qquad 1.$$

But this is not enough to understand the way in which property crime can be chosen. We need to introduce the element of uncertainty and the notion of losses. Let us deal with losses first.

### Losses from Crime

Obviously, no crime is without the possibility of punishment. From the perspective of the potential perpetrator, although there may be a gain in sight, G, there is also the outcome that suggests that the crime will be caught and punished. If that is the outcome, then we need to characterize the loss. In this case we might think of it as a fine, F, which is subtracted from the total income gathered by our potential criminal. In other words, if a person engages in a criminal act, and is caught, in our most basic model, income now looks like:

$$Y=Y_0-F. \qquad 2.$$

Notice that although we assume that committing the crime is successful in garnering G, the value of F could take it all away and even more!

Uncertainty and Expected Utility

We now need to link the two outcomes together. On the one hand a successful criminal gains income, $Y=Y_0+G$, and someone who is caught loses from their original level of income, $Y=Y_0-F$ and will end up with less than the endowed level of income, $Y_0$. A natural way to link these two is to look at the utility of both outcomes, $U(Y_0+G)$ and $U(Y_0-F)$ and weight them by the probability that they occur. In other words, if you are not caught, then your utility is $U(Y_0+G)$.

But there *is* some probability that you will be caught. Let us call that probability, $\pi$. Then you have the probability $(1-\pi)$ that you will not be caught, and you will be able to enjoy the gains unfettered. If we think about what decision the potential criminal makes, it now seems reasonable to assume that the decision is about a weighted average of the two outcomes for which the weights are the relevant probabilities. We can write the expected utility, EU(Y) as the weighted sum of the two possibilities: that you do not get caught and that you do get caught and fined. In other words:

$$EU(Y)= (1-\pi)\, U(Y_0+G)+\ \pi\, U(Y_0-F) \qquad 3.$$

This expression will become the workhorse of our analysis. The maximization of expected utility is what underpins much of our understanding of the economic analysis of crime. The model as displayed thus far is designed to answer the questions: Do you or do you not commit the act? There are obviously many other kinds of questions that can be asked to which refinements of this framework are required. (Students may want to ask themselves what kinds of questions are not addressable by this framework.)

Graphing Expected Utility

Our model of expected utility can be expressed in a figure. Figure 1 displays the framework. On the vertical axis is utility. On the horizontal axis is income. Utility, that is the utility function, U(Y) is the curve. Notice the shape of the utility function. First, as income (along the horizontal axis) increases, utility increases. This is a consequence of our assumption that $U'(Y)>0$: the tangent to the slope of the utility function is positive. We will also make the assumption that $U''<0$. That means that the slope of the marginal utility schedule is negative or that utility increases, but at a decreasing rate. As income increases, utility increases, but additional increments to income increase utility but increasingly less so.

So we have our utility function U(Y) which is the curve in Figure 1. We can place two points on this curve that will be of interest to us. Let us first put the point at which income is equal to the successful crime: $Y+G_0$. Let this point be on the income axis and then move vertically to point A. This is the utility, $U(Y_0+G)$, of the un-captured successful criminal. On the other hand the utility of the captured and prosecuted criminal is at point B which corresponds to $U(Y_0-F)$, where $Y_0-F$ is measured along the horizontal axis.

Now what is expected utility? Notice that from equation 3, expected utility is a *linear* function of the probability of being caught. This is critical. For a given value of $U(Y_0+G)$ and $U(Y_0-F)$, an increase in $\pi$, and a corresponding decrease in $1-\pi$, always has the same effect on EU(Y). Why is that? The values of $U(Y_0+G)$ and $U(Y_0-F)$ are both fixed (or at least are parametric to our problem thus far.) Consequently, $EU(Y)= (1-\pi)X+\pi Z$, where X and Z are simply written to emphasize that we are describing fixed values. Notice that when $\pi$ goes up, EU goes up by Z and down by X for a net effect of (Z-X). Notice further that the same net effect is true regardless of the value of Y. This is what we mean by a linear function. The increments to the expected value change only in proportion to the linear probability, since the net effect of an increase in the probability is (Z-X): a (constant) linear function of $\pi$.[1]

This is very useful. It means that the expected utility of points A and B are connected by a straight line. Why? Because the only thing that changes along the line connecting them is the probability. If the probability of being caught is 1, then we are at point B. If the probability of being caught is zero, then we are at point A. Anywhere between, there is a probability of getting caught between zero and 1. The expected value of the utility of the point A and B is the straight line between them, and exactly where depends upon the specific value of $\pi$.

## Maximizing Expected Utility

Once we have chosen expected utility as a useful characterization of the choices facing a person choosing between committing a crime or simply receiving $Y_0$, we can ask what will people do? Will they commit a crime or not? A sensible answer to this question depends on their individual decision. If the expected utility of committing the crime is greater than the utility of income without crime, then our maximizing agents do the deed. Written formally, if

$$EU(Y)>U(Y_0)$$

[1] Notice that the with the two endpoints fixed and corresponding to values of $\pi=0$ (and $(1-\pi)=1$) and $\pi=1$ (and $(1-\pi)=0$), if we choose any value for $0<\pi<1$, the value for EU (the expected value of utility) lies along a chord stretching between the two endpoints. Suppose, for example, the value of EU when $\pi=1$ is 4 and the value of EU when $\pi=0$ is 6. What is EU when $\pi=0.5$? Suppose that $Y_0=25$, F=9 and G=16. Draw the picture.

then commit the crime. If,

$$EU(Y)<U(Y_0)$$

then do not.

Our simple model does not tell us which side of the law an individual will land until we compare utility of income without crime, $U(Y_0)$, with EU(Y), the expected value of utility with crime with an uncertain outcome.

A First Result: Everyone is a potential criminal

Looking at the picture, we can see the utility of $Y_0$, $U(Y_0)$. Comparing this with E(U(Y)) – which is along the chord AB -- gives the outcome. Since we know that as the probability of detection increases, for exogenous (and appropriate[2]) values of the G and F, the expected value falls, then for a small enough $\pi$, the expected value of crime will be great enough to induce criminal behaviour. That is, as the probability of detection and punishment decreases, we move along chord AB toward point A in the figure. As constructed, eventually a small enough $\pi$ generates a value of EU(Y) greater than $U(Y_0)$, the no crime option.

What happens when things change?

We know that there are two kinds of variables: exogenous and endogenous. Exogenous variables are variables that are determined outside the system we are contemplating. For example, the individual has little impact on the fine levied on criminals. Similarly we could argue that the gains and probabilities of detection and prosecution are determined outside the scope of the model. What is endogenous is the decision to commit a crime facing the constraints and the exogenous values.

Comparative statics is a change in the endogenous variables resulting from a change an exogenous variable. In our case it is very simple because there is but a single endogenous variable, expected utility that is then evaluated vis-à-vis the decision to do the illegal act. So what we need to do is to change an exogenous variable and see how that changes the decision to engage in crime.

What are the exogenous variables? There are $\pi$, G, F, and $Y_0$. Changes in each will affect expected utility, EU(Y), which is then compared to the utility of income without crime, $U(Y_0)$. If $EU(Y)> U(Y_0)$, then crime is the "best" solution.

[2] To make an "interesting" problem, we have to assume that there is some value of G and F and $\pi$ that will cause the lawabiding individual to engage in crime. The values of G and F are the gains and fines associated with any crime. If there were no gains, obviously there would be no benefit to crime and consequently crime would never pay. If there were no fine, then getting caught would not reduce the gain and crime would always pay. Our interest, however, is in values of G and F that make it possible that crime could pay for some values of $\pi$, and yet not pay for other values of $\pi$. It is in this sense that we limit the discussion to appropriate values of G and F.

To make the issue as stark as possible, imagine that our decision-maker is on the knife edge of crime: expected utility is exactly equal to the utility of income without crime, $EU(Y)=U(Y_0)$. The configuration of probabilities, gains and rewards are such that we are at a point like C in figure 2. A change in a parameter is going to tip the balance one way or the other.

An increase in the probability of being caught

First consider what happens to expected utility when the probability of detection and punishment changes: that is, when there is an increase in $\pi$. This is really all we need to look at since we know that the utility of income without crime, $U(Y_0)$, will not change.

Consequently, a change in EU(Y) will determine what happens with respect to a decision about breaking the law. That is, imagine we are at a point like C in figure 2.[3]

Looking at the expression for expected utility from equation 3, it is clear that the

$$EU(Y)= (1-\pi)\, U(Y_0+G)+ \pi\, U(Y_0-F)$$

probability-weighted value of utility in the uncaught state will fall since a higher $\pi$ means that $(1-\pi)$ is smaller, and a higher weight is now attached to the utility of the state in which the perpetrator is caught which has lower utility. Expected utility is clearly going to fall. If we were at a point like C in figure 2, we move in the direction of the arrow. With a higher probability of detection and punishment, clearly crime will not pay relative to the alternative and the individual is tipped into the no-crime state.

Another way to see this is to differentiate expression 3 with respect to $\pi$ holding all other variables constant:

$$\frac{\partial EU}{\partial \pi} = U(Y_0 - F) - U(Y_0 + G) < 0$$

From Figure 2, notice that the second term is greater than the first term (Why?), therefore the whole expression is negative. A higher probability of being caught reduces the expected utility of the crime and leads to less crime. This is a reassuring outcome at the least!

An increase in ill-gotten gains

An increase in the gain associated with a particular crime will have the effect of moving both of the endpoint of the expected value expression. Looking at figure 3, at the lower

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[3] Why are we not at a point vertically above point C? Isn't that the utility of income that is represented by the point C? Nope. Think it through. This is in response to a question raised after class.

endpoint remains at B and at the upper endpoint moves from A to E. This has the effect of increasing the expected value of utility for given values of the other variables.

But wait, it looks as if the expected utility is lower on the dotted line. It is, but the interpretation is subtle. Suppose we look at the expected utility at E. This is when the probability of getting caught is very low or zero. Notice that this value is above any possible value for the previous level of gain, $G_0$.

Now think of a small value of getting caught (an increase $\pi$) that moves you just a little bit down the expected value line so that you just reach an expected value equal to the level of utility at A. Even though utility is the same, in the second case, $G_1$, the probability of getting caught is positive whereas at A, you have a zero probability of getting caught. In other words, for the same value of $\pi$, the expected value of the crime has increased.

More formally, we can look at the change in the expected value of utility when the gain to crime increases by taking the derivative of expression 3 holding all other variables constant:

$$\frac{\partial EU}{\partial G} = (1-\pi)U'(Y_0 + G) > 0$$

Since the marginal utility (U') is positive, the effect of an increase in the gains from crime all other things held constant is to tip the marginal actor into crime.

An increase in punishment

We can also ask what happens when we increase the punishment associated with any crime. This means that we are looking at an increase in F. In terms of figure 4, notice that only the lower endpoint at B moves down to B'. Why? The upper endpoint does not move at all. The upper endpoint is not dependent on the fine. It is the outcome associated with utility when the perpetrator is not caught and therefore not fined. However, the increase in punishment leads to a lower expected value of utility for any positive value of $\pi$. This is indicated by observing that the expected utility "line" is now below the expected utility line at the lower level of punishment. Only point A is unaffected since it is independent of the fine. (Convince yourself by using the same logic as we used for the higher expected utility associated with and increase in G.)

As before, we can look at the problem mathematically to note that expected utility unambiguously falls:

$$\frac{\partial EU}{\partial F} = -\pi U'(Y_0 - F) < 0\,.$$

The implication is simply that increasing the punishment tips the marginal actor back toward a life without crime.

Questions

1. The above is described for a utility function that is concave from below.  This is equivalent to saying that the slope of the slope of the utility function, U’’<0, so that the marginal utility of income is decreasing.  Suppose this is not the case. Suppose that U’’>0.  Draw the utility function and explain what happens to our “marginal criminal” when fine, gains and the probability of capture change.
2. Suppose $Y_0$ increases, does this make the marginal person more or less likely to engage in crime?  Does U’’ matter for your answer?

Figure 1

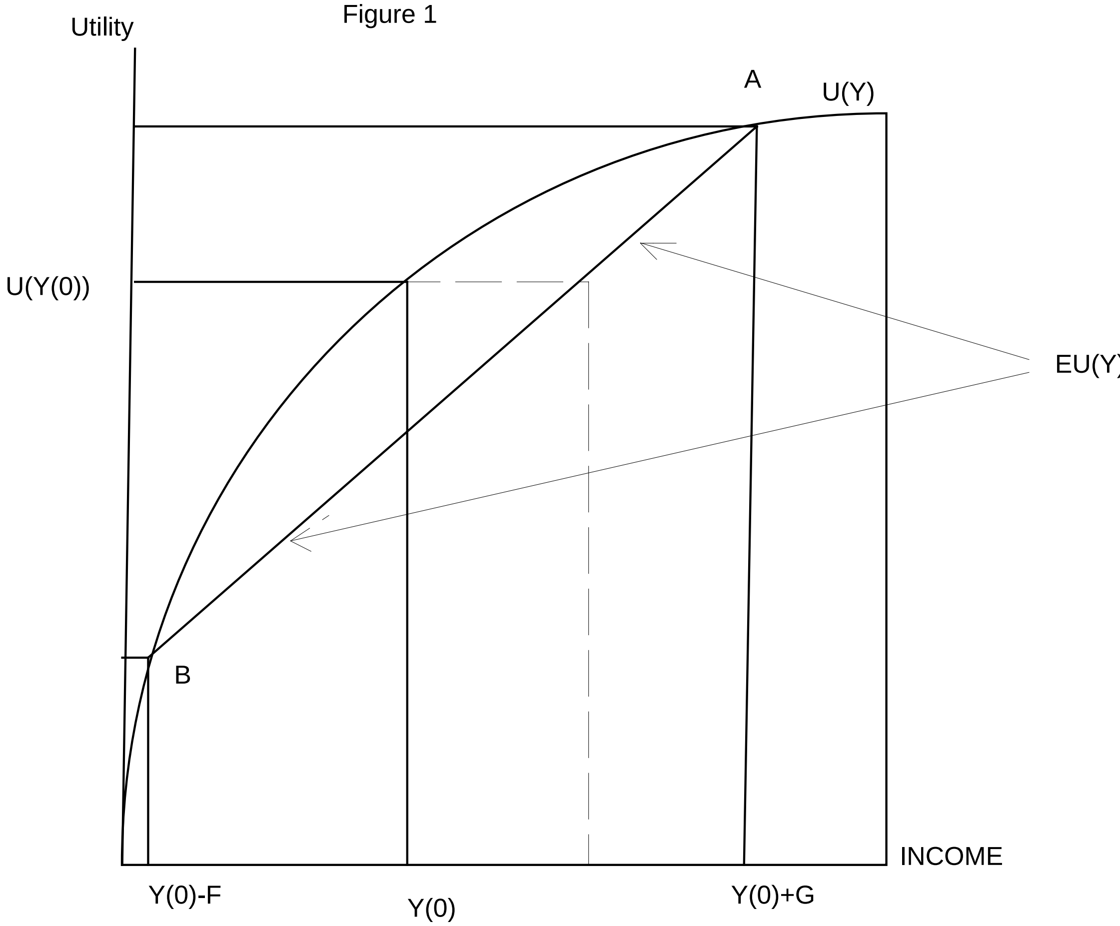

Figure 2

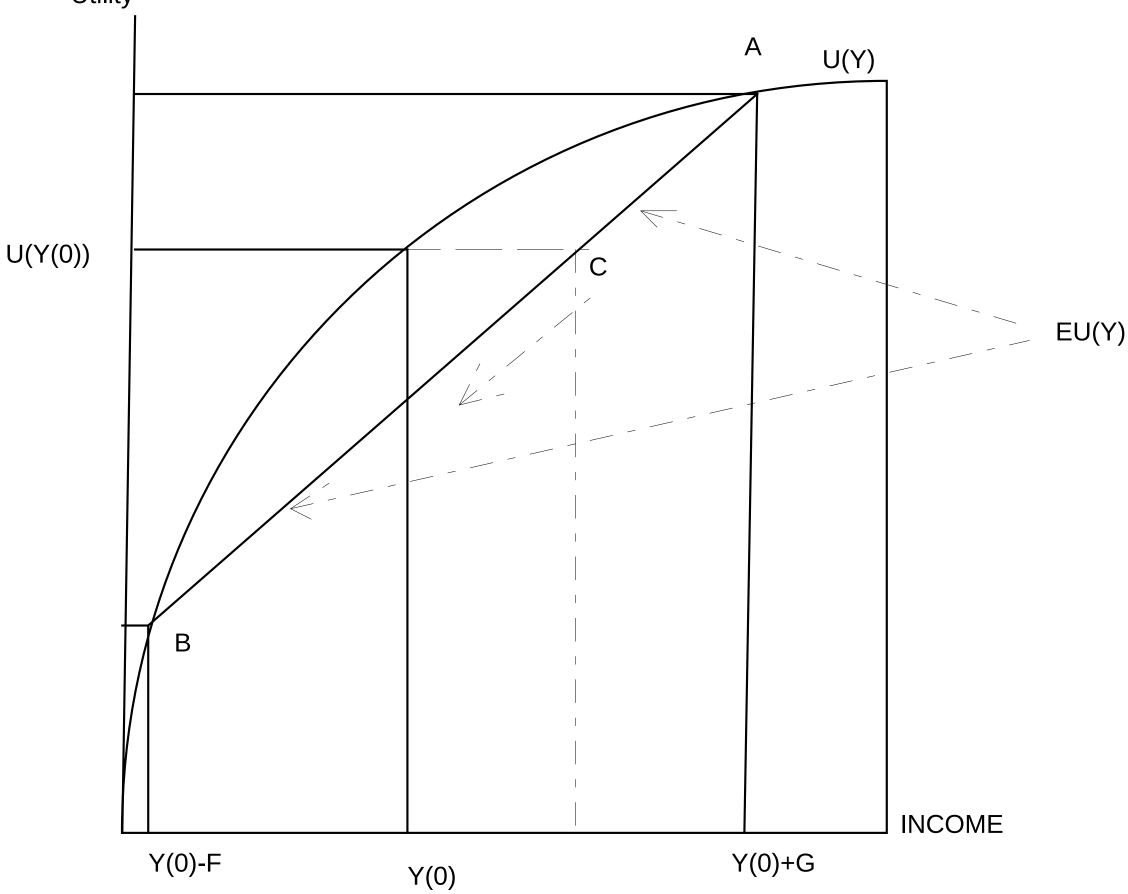

Figure 3

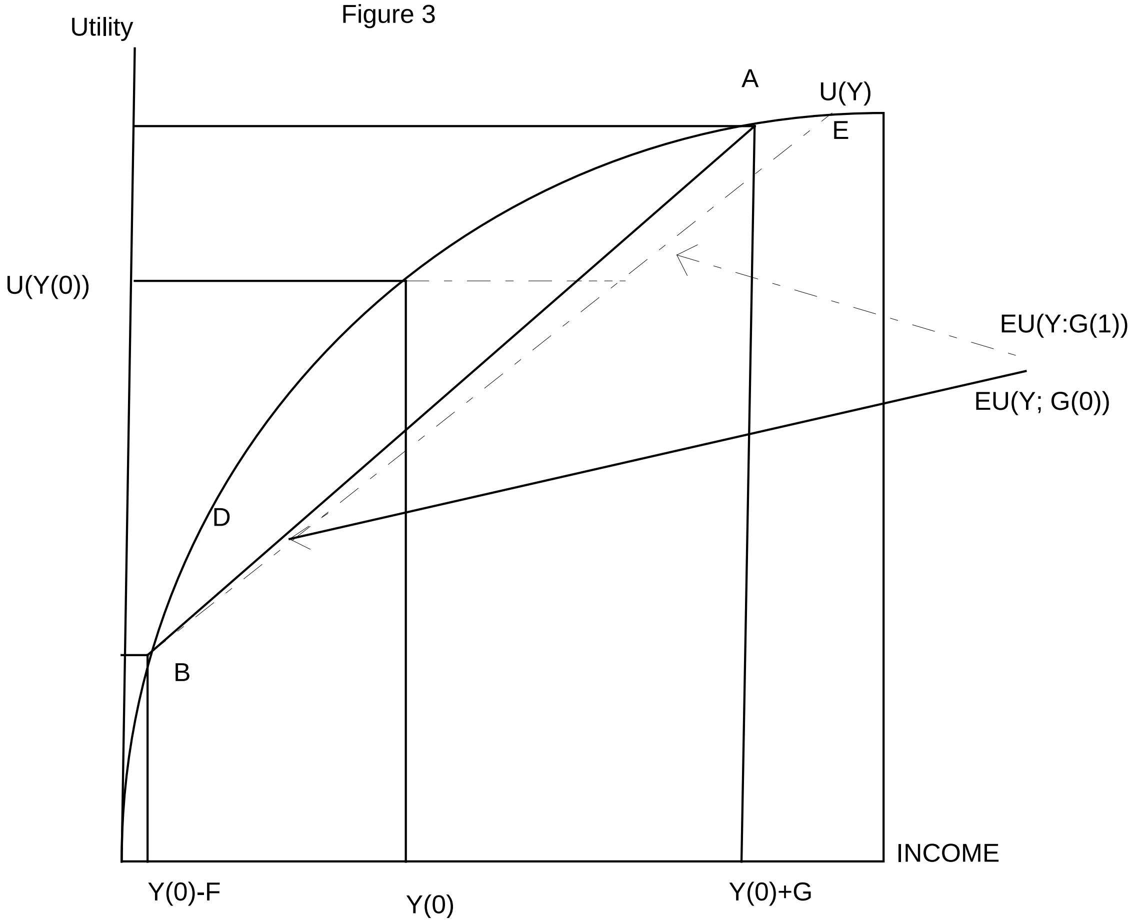

Figure 4