

Notes on: A Model of Deterrence

We might, following Becker, imagine that one component of the cost of a crime is measured by an individual victim's assessment of the damage the crime has done. If we call that value, v , then multiplied by the number of crimes, vS , gives the cost of the crimes themselves. But there are other costs. In addition to the cost of the crime to the victim is the observation that communities pay to catch and incarcerate perpetrators of crimes. This adds to the cost of the crimes that are actually committed by an amount that is proportional to the amount of time that a convicted criminal spends in jail. So if there are S crimes, and there is a probability, π , that a perpetrator will be caught, then if the daily cost of jail is b , the total cost of jail time is πSbx , where x is the number of days in jail to which the criminal is sentenced.

Thus the full cost of crime in our simple, simple framework is:

$$C = vS + \pi Sbx \quad 1$$

Now consider a simple world in which the supply of offences, S , is proportional to the amount of deterrence measured in days in jail. That is imagine that criminals worry about the amount of time they will have to do and, on the margin, adjust their behaviour. Thus we assume that the supply of crimes, S , is negatively related to the amount of jail time. This means that we write that

$$S=S(x) \quad 2$$

and that $S'(x)<0$. That is since $S'=(dS/dx)$, the change in the supply of crimes when the sentence length changes, we assume that it is negative. There is some degree of deterrence.

We now can derive a simple theory of the optimum sentence. That is, since we can change the length of sentence to whatever we like (as economic "legislators") we can choose \underline{x} to minimize the cost of crime, C , in equation 1.

$$\text{By choosing, } \underline{x}, \text{ minimize } C = vS(x) + \pi S(x)bx \quad 3$$

Let us take that derivative:

$$\frac{dC}{dx} = v \frac{dS}{dx} + \pi S b \frac{dx}{dx} + \pi b x \frac{dS}{dx} = 0 \quad 4$$

Since it is a first order condition, we set it equal to zero since we are looking for a minimum. [You might imagine finding the second order condition to prove it is a minimum: $(d^2C/(dx)^2)>0$.]

From the first order condition in 4, we can solve for the value of x that (now) minimizes cost. Because economists like to use elasticities (why?), we will define the elasticity of

the change in the number of crimes in response to an increase in days in jail (ϵ), as the percentage change in S (dS/S) divided by the percentage change in x (dx/x):

$$\epsilon \equiv \frac{\frac{dS}{S}}{\frac{dx}{x}} = \frac{x}{S} \frac{dS}{dx} < 0 \quad 5$$

We assume that ϵ is negative since an increase in sentence, x , leads to a reduction in the number of crimes. We also assume that ϵ is a parameter of behaviour. That is, it is something that is determined by individual behaviour. It is not something that we, as policy makers can choose.

Now, back to the main calculation, the optimum sentence that minimizes cost:

$$x^* = \left(\frac{v}{\pi b} \right) \left(\frac{-\epsilon}{1 + \epsilon} \right) \quad 6$$

Since v, π , and b are each positive, and ϵ is negative, the entire expression is positive when $|\epsilon| < 1$ since both the numerator and denominator of the term in ϵ are then positive. [Does it make sense that $|\epsilon| < 1$? What would an “optimum” be if you knew that $|\epsilon| > 1$? Equal to 1?]

Does 6 make any sense?

Notice that the optimum sentence depends upon the value people put on personal safety, the probability of catching the crook, and the cost of incarceration. A higher valuation of the crime, $v \uparrow$, means that the optimum sentence will rise, other things equal. [Why? What is the logic behind this?] Similarly, an increase in the cost of jail, b , means that the optimum length of sentence will fall. Curiously, this is also the case if criminals are caught more frequently. Why?

There are a number of ways to read 6. First we can ask whether the time in jail in fact corresponds to our view of what is appropriate. In that case we need an estimate of both v and of ϵ . Alternatively, we could turn the formula around and see what the implicit value of a crime is by looking at the sentence handed out and knowing as we do the cost of jail, the probability of getting caught, π , and the elasticity of deterrence, ϵ .

What do some of the numbers look like?

What is the elasticity, ϵ , actually like? Well, as you might imagine this is a bone of contention to this day. Some of the estimates were done by Isaac Ehrlich, “Participation in Illegal Activities: A Theoretical and Empirical Investigation” (Journal of Political

Economy, 81 (3) May-June, 1973: 521-65.) and when all is said and done, they have looked remarkably robust...or have they. That is something you might explore for yourselves.

Here is (in a simplified way) how Ehrlich approaches it:

$$\ln \left(\frac{Q}{N} \right)_i = a + b_{1i} \ln P_i + b_{2i} \ln T_i + c_{1i} \ln W + c_{2i} \ln X + e_{1i} \ln NW + \mu_i^{38} \quad (4.1)$$

Ehrlich uses the following definitions for a regression that is estimated across the 50 US states:

Q/N = Crimes known to the police per capita

P = C/Q = number of offenders imprisoned per crimes known to the police: a measure of the probability of being caught and imprisoned

T = average time served

W = median income (in the state)

X = % of families below ½ the median income

NW = is the percentage of nonwhite families in the state

For those of you who are econometrically sophisticated, Ehrlich uses OLS, two-stage least squares and seemingly unrelated regression analysis and finds substantially the same results. He models both P, the probability of catching the crook. He is also interested in the effects of unemployment and wage rates. It is, however, a technically challenging article.

TABLE 4
 2SLS AND SUR (WEIGHTED) REGRESSION ESTIMATES OF COEFFICIENTS ASSOCIATED
 WITH SELECTED VARIABLES IN 1960: CRIMES AGAINST PROPERTY

| OFFENSE | COEFFICIENT (β) ASSOCIATED WITH SELECTED VARIABLES | | | | | |
|----------------------------------|--|-------------------------------|-------------------------|-----------------------|-----------------------|------------------------|
| | a Intercept | b_1 with $\ln \hat{P}_i$ | b_2 with $\ln T_i$ | c_1 with $\ln W$ | c_2 with $\ln X$ | e_1 with $\ln NW$ |
| A. 2SLS Estimates | | | | | | |
| Robbery: | | | | | | |
| $\hat{\beta}$ | -11.030 | -1.303 | -0.372 | 1.689 | 1.279 | 0.334 |
| $\hat{\beta}/S\hat{\beta}$ | (-1.804) | (-7.011) | (-1.395) | (1.969) | (1.660) | (4.024) |
| Burglary: | | | | | | |
| $\hat{\beta}$ | -2.121 | -0.724 | -1.127 | 1.384 | 2.000 | 0.250 |
| $\hat{\beta}/S\hat{\beta}$ | (-0.582) | (-6.003) | (-4.799) | (2.839) | (4.689) | (4.579) |
| Larceny: | | | | | | |
| $\hat{\beta}$ | -10.660 | -0.371 | -0.602 | 2.229 | 1.792 | 0.142 |
| $\hat{\beta}/S\hat{\beta}$ | (-2.195) | (-2.482) | (-1.937) | (3.465) | (2.992) | (2.019) |
| Auto theft: | | | | | | |
| $\hat{\beta}$ | -14.960 | -0.407 | -0.246 | 2.608 | 2.057 | 0.102 |
| $\hat{\beta}/S\hat{\beta}$ | (-4.162) | (-4.173) | (-1.682) | (5.194) | (4.268) | (1.842) |
| Larceny and auto: | | | | | | |
| $\hat{\beta}$ | -10.090 | -0.546 | -0.626 | 2.226 | 2.166 | 0.155 |
| $\hat{\beta}/S\hat{\beta}$ | (-2.585) | (-4.248) | (-2.851) | (4.183) | (4.165) | (2.603) |
| Property crimes: | | | | | | |
| $\hat{\beta}$ | -6.279 | -0.796 | -0.915 | 1.883 | 2.132 | 0.243 |
| $\hat{\beta}/S\hat{\beta}$ | (-1.937) | (-6.140) | (4.297) | (4.246) | (5.356) | (4.805) |
| B. SUR Estimates | | | | | | |
| Robbery: | | | | | | |
| $\hat{\beta}$ | -14.800 | -1.112 | -0.286 | 2.120 | 1.409 | 0.346 |
| $\hat{\beta}/S\hat{\beta}$ | (-2.500) | (-6.532) | (-0.750) | (2.548) | (1.853) | (4.191) |
| Burglary: | | | | | | |
| $\hat{\beta}$ | -3.961 | -0.624 | -0.996 | 1.581 | 2.032 | 0.230 |
| $\hat{\beta}/S\hat{\beta}$ | (-1.114) | (-5.576) | (-4.260) | (3.313) | (4.766) | (4.274) |
| Larceny: | | | | | | |
| $\hat{\beta}$ | -10.870 | -0.358 | -0.654 | 2.241 | 1.785 | 0.139 |
| $\hat{\beta}/S\hat{\beta}$ | (-2.52) | (-2.445) | (-1.912) | (3.502) | (2.983) | (1.980) |
| Auto theft: | | | | | | |
| $\hat{\beta}$ | -14.860 | -0.409 | -0.233 | 2.590 | 2.054 | 0.101 |
| $\hat{\beta}/S\hat{\beta}$ | (-4.212) | (-4.674) | (-1.747) | (5.253) | (4.283) | (1.832) |

NOTE.—The underlying regression equation is

$$\ln \left(\frac{Q}{N} \right) = a + b_{1i} \ln \hat{P}_i + b_{2i} \ln T_i + c_{1i} \ln W + c_{2i} \ln X + e_{1i} \ln NW + \mu_i. \quad (4.3)$$

Coefficients From Ehrlich Table 5

| | Constant | P | T | W | X | NW |
|----------------------------------|----------|--------|---------------|-------|-------|------|
| Murder | -1.198 | -0.9 | -0.018 | 0.186 | 1.152 | 0.54 |
| t-value | -0.33 | -3.06 | -1.7 | 0.36 | 2.1 | 8.65 |
| Rape | 0.93 | -0.913 | -0.44 | 0.33 | 0.425 | 0.65 |
| t-value | 0.19 | -6.6 | -2.3 | 0.5 | 0.7 | 0.8 |
| Assault | -6.6 | -0.72 | -0.78 | 1.14 | 1.15 | 0.46 |
| t-value | -1.1 | -4 | -2 | 1.7 | 1.8 | 3.8 |
| Crimes against the Person | 1.6 | -0.8 | -0.5 | 0.3 | 0.6 | 0.4 |
| t-value | 0.4 | 6 | 3 | 0.6 | 1.1 | 4.3 |
| All Crimes | -1.4 | -0.99 | -1.1 | 1.3 | 1.8 | 0.3 |
| t-value | -0.3 | -5.9 | -4.5 | 2.6 | 4.2 | 5.1 |

Which leads to a calculation like:

| Offence | Coefficient on Time Served | Implied $-E/(1+E)$ |
|---------------------------|----------------------------|--------------------|
| Robbery | 0.3 | -0.23 |
| Burglary | 0.99 | -0.50 |
| Larceny | 0.6 | -0.38 |
| Auto Theft | 0.2 | -0.17 |
| Murder | -0.018 | 0.02 |
| Rape | -0.44 | 0.79 |
| Assault | -0.78 | 3.55 |
| Crimes against the Person | -0.5 | 1.00 |
| All Crimes | -1.1 | -11.00 |

Now all we need are:

The cost of incarceration and the probability that a person doing the crime is in jail.

Can you find out what the cost of incarceration is?

How about the proportion of people doing jail time for specific crimes?

What about the value of a crime? How much do you think an average robbery is “worth preventing”? A murder? Rape? Common assault?