Please find errors and let me know.

Ehrlich's Model

Becker modeled the criminal's decision to be a criminal as an all or nothing event in the sense that the expected utility of crime guided your decision to commit a crime. If you did, you were a criminal. Otherwise you were not. This was a polar decision. The economic actor ends up either as a person engaged in crime or a person not engaged in crime.

Ehrlich () considers a more sophisticated version of the decision process that permits a more subtle variation among the alternatives. In particular, he introduces the notion of time spent at legal and illegal activities as the decision variable for the actor. Becker aggregated across individuals, each of whom had made the decision to be a criminal or not, to develop what is in effect a supply function of crime. Ehrlich develops what is known as a state preference approach to characterize the individual supply of time spent on crime. To do so, he also specifies the technology by which the time spent on crime adds to the rewards of crime that implicitly define the alternative: the time spent on legal activities

Insert Section on Graphing Indifference Curves

Begin this model by defining the alternatives available to the economic actor. State preference theory requires that all possible states be valued. In this case there are two: A, the state in which the illegal activites are detected or "caught"; and state B where the activities are not detected. Wealth in option "A" includes a initial amount, W_0, wealth derived from illegal activities, $W_i(t_i)$ that is a production function for the time that is spent on those activities, t_i, wealth that is derived from legal activities, $W_L(t\text{-}t_i)$ ($W_L(t_L)$), where t_L=t-t_i and t is total time available for activity and t_L is time spent on legal activities. A fine is leveled in the "caught" state, $F(t_i)$. This leads to writing wealth in state A as:

(1a) $$X_A = W_0 + W_i(t_i) + W_L(t - t_i) - F(t_i)$$

while the "uncaught" state is written as:

(1b) $$X_B = W_0 + W_i(t_i) + W_L(t - t_i)$$

To find the expect utility of the individual we need to weight these states of the world by the probability, π, that the illegal activity will be detected and punished, so that:

(2) $$EU(X_A, X_B) = \pi U(X_A) + (1 - \pi) U(X_B)$$

This is the expectation that is to be maximized by a choice of the amount of time spent on each of the activities. This means we need to calculate $\frac{dEU}{dt_i} = 0$ as in equation (3):

$$(3) \qquad \frac{dEU}{dt_i} = \pi U'(X_A)\frac{dX_a}{dt_i} + (1-\pi)U'(X_B)\frac{dX_B}{dt_i} = 0$$

The two terms $\frac{dX_A}{dt_i}$ and $\frac{dX_B}{dt_i}$ can be reduced to their constituent parts by directly differentiating (1a and 1b):

$$(1a') \qquad \frac{dX_A}{dt_i} = W_i^{'}(t_i) + W_L^{'}(t-t_i).\frac{d(t-t_i)}{dt_i} - F^{'}(t_i) = W_i^{'}(t_i) - W_L^{'}(t-t_i) - F^{'}(t_i)$$

$$(1b') \qquad \frac{dX_B}{dt_i} = W_i^{'}(t_i) + W_L^{'}(t-t_i).\frac{d(t-t_i)}{dt_i} = W_i^{'}(t_i) - W_L^{'}(t-t_i)$$

(Since $\frac{d(t-t_i)}{dt_i} = -1$)

The whole of (1a' and 1b') are now substituted into the expressions of $\frac{dX_A}{dt_i}$ and $\frac{dX_B}{dt_i}$ so that we have:

$$(4a) \qquad \frac{dEU}{dt_i} = \pi U'(X_A)\left[W_i^{'} - W_L^{'} - F'\right] + (1-\pi)U'(X_B)\left[W_i^{'} - W_L^{'}\right] = 0$$

This is easily rearranged to put the probabilistically weighted marginal utilities on one side of the equality and the marginal rate of transformation of wealth from time spent on legal and illegal activities on the other:

$$(4b) \qquad \frac{-\pi U'(X_A)}{(1-\pi)U'(X_B)} = \frac{\left(W_i^{'} - W_L^{'}\right)}{\left(W_i^{'} - W_L^{'} - F'\right)}$$

To see exactly what this is think about the picture developed above. Notice that the LHS of 4b is the slope of the expected utility contour. From equation 3 we have:

$$(5) \qquad \pi U'(X_A)\frac{dX_a}{dt_i} + (1-\pi)U'(X_B)\frac{dX_B}{dt_i} = 0$$

That in turn can be expressed as:

(5’) $$\left.\frac{dX_B}{dX_A}\right|_{EU} = \frac{-\pi U'(X_A)}{(1-\pi)U'(X_B)}$$

where the vertical bar means that EU is being held constant so that the expression holds along an expected utility indifference curve.