

The Arithmetic of Monetary Growth and Inflation

We know that a money demand function can be described as:

$$(M / P) = y \cdot e^{-\alpha i}$$

where M is the nominal quantity of money; P is the price level, y is the level of real income, and “i” is the nominal interest rate.

For the moment we will ignore the interest rate so that the money demand equation can be written as:

$$2. (M / P) = y$$

This is a most simple form of money demand that allows us to see what happens when the rate of the money supply growth is increased.

Rewrite 2 so that the initial relationship is:

$$3. M_0 = P_0 y_0$$

Now assume that M, P and y are each displaced by a small amount, ΔM , ΔP , and Δy . This leads to a new version of 3:

$$4. (M_0 + \Delta M) = (P_0 + \Delta P) \cdot (y_0 + \Delta y) \text{ which can be multiplied out to leave:}$$

$$5. M_0 + \Delta M = P_0 y_0 + \Delta P \cdot y_0 + P_0 \cdot \Delta y + \Delta P \cdot \Delta y$$

Now, first notice that from 3, $M_0 = P_0 y_0$, so that *the first term on both sides of 5 cancel*. Second, if ΔP and Δy are both small, then multiplying two small things together leads to a product that is *very* small – so small in fact that we shall assume that it disappears! (That is, assume that $\Delta P \cdot \Delta y = 0$.)

This leaves us with 6:

$$6. \Delta M = \Delta P \cdot y_0 + P_0 \cdot \Delta y$$

Now recall that $M_0 = P_0 y_0$. Divide the term ΔM by M_0 on the left hand side (LHS), and both terms on the right hand side (RHS) by $P_0 y_0$. (This is OK because they are equal.) Now what we have is 7:

$$7. \left(\frac{\Delta M}{M_0} \right) = \left(\frac{\Delta P \cdot y_0}{P_0 y_0} \right) + \left(\frac{P_0 \cdot \Delta y}{P_0 y_0} \right).$$

But since the y_0 cancels in the first term on the RHS, and the P_0 cancels in the second term on the RHS, we are left with the expression:

$$8. \left(\frac{\Delta M}{M_0} \right) = \left(\frac{\Delta P}{P_0} \right) + \left(\frac{\Delta y}{y_0} \right).$$

Since each of these terms can be read as a percentage change, equation 8 reads as: the percentage change in the money supply is equal to the percentage change in prices plus the percentage change in real income.

If we now divide by a *change in time*, Δt , so that the change in M, ΔM is per period, Δt , then we have an equation like 9 where everything is expressed in *rates of growth* per year (or per month or whatever time period you like):

$$9. \left(\frac{1}{M_0} \frac{\Delta M}{\Delta t} \right) = \left(\frac{1}{P_0} \frac{\Delta P}{\Delta t} \right) + \left(\frac{1}{y_0} \frac{\Delta y}{\Delta t} \right).$$

Equation 9 reads as: the percentage *rate of growth* of the money supply is equal to the percentage *rate of growth* of prices plus the percentage *rate of growth* of real income.

If the rate of growth of the money supply is called, ρ (rho), and the rate of growth of the price level (otherwise known as the rate of inflation) is called, π (pi), and the rate of growth of real income is called, λ (lambda), then we can write the relationship binding the long-run growth rates of money, income and prices as:

$$10. \pi = \rho - \lambda.$$

This is a very important relationship. We have a simple expression tying the inflation rate to variables about which we know something.

We assume that the government can control the quantity of money and therefore the growth rate of the money supply, ρ . We know that the growth rate of real income changes by at most (in normal time) between +4% and -4% per year.

The rate of inflation, however, as we saw in Argentina, can be many thousands of percent per year. The culprit? Changes in the rate of growth of the money supply. It is in this sense that inflation is everywhere and always a monetary phenomenon since it reflects the difference between the rate of growth of the money supply and the rate of real income growth. As a matter of historical interest, the money supply growth rate is far more variable than the growth of real output (income) for most countries over long periods of time.

At the present time, however, the relationship is not as obvious since the rate of inflation, the rate of growth of the money supply and the rate of growth of real income are all very

low. The errors we make in measuring income, money and price growth make it hard to distinguish the underlying relationship embodied in 10. However look at other countries (with data from the back of *The Economist*, for example, and you will see 10 much more clearly.)

Why do some governments increase the rate of growth of the money supply so absurdly? Suppose a government has no revenue from income tax or taxes on raw materials or on trade. How does it raise revenue?

One way is to print money. Suppose that revenue, R , is simply generated by printing money, so that:

$$11. \quad R = \frac{\Delta M}{\Delta t}.$$

Now in real terms this leads to real revenue of:

$$12. \quad \left(\frac{R}{P}\right) = \left(\frac{1}{P}\right) \left(\frac{\Delta M}{\Delta t}\right)$$

This in turn can be rewritten by dividing and multiplying the RHS by (M/M) :

$$13. \quad \left(\frac{R}{P}\right) = \left(\frac{M}{P}\right) \left(\frac{1}{M} \frac{\Delta M}{\Delta t}\right).$$

Now if we have no real income growth – just to keep things simple – so that $\lambda=0$, then notice that the term $\left(\frac{1}{M} \frac{\Delta M}{\Delta t}\right) = \rho$, the rate of growth of the money supply is just equal to π ! (From equation 10).

In other words, real revenue, $(R/P) = \pi \cdot (M/P)$. What this means is that for governments that choose to do so, the rate of inflation created by printing money is a direct consequence and is proportional to the rate of inflation.¹

In the figure on the next page, the actual revenue from the inflation rate of π is the rectangle $\pi \cdot (M/P)_1$. You can see that compared to an initial starting point at which there was no inflation (so that $i=r$ – the nominal is equal to the real rate of interest), real balance holdings fall. Revenue is maximized when the fall in money holdings offsets the gain in revenue due to a higher rate of inflation. Geometrically, we would find the place along the demand for money schedule at which the revenue box is the largest if we wanted to find a rate of growth of the money supply that maximized revenue.

¹ It is a little more complex since the stock of real balances, (M/P) , also depends on the rate of inflation. Further analysis would show that revenue reaches a maximum at some rate of inflation, and then falls off.

Revenue From Money Creation Leading to Inflation

