

Problem 1: Sizes & Scaling:

1. The allometric scaling relationship for skeletal mass (in kg) has been found to be, $M_{skel} = (0.068) M^{1.08}$, which does not exactly scale as the body mass M . Calculate the skeletal mass for a cat that has a mass of 10 kg and an elephant that has a mass of 5000 kg. What % of total mass are the skeletons for both animals? What benefits and problems can you see with these differences?
2. Typically, large bacteria have the shape of a cylinder of fixed radius a and variable length L , where $L \gg a$.
 - (a) Show that the surface-to-volume ratio of these bacteria does not depend on the length of the bacterium. Contrast this with the surface-to-volume ratio of a sphere of radius R .
 - (b) Explain why a large cylindrical bacterium can feed itself more efficiently than a large spherical bacterium. Assume that the bacteria feed by absorbing nutrients through their surface. To do this, compare the surface areas of cylindrical and spherical bacteria that have the same volume, and thus for the same food requirements. (You should assume that the radius of the cylindrical bacteria is smaller than the radius of the spherical bacteria, i.e. $a \ll R$).

Problem 2: Allometric scaling for walking speed.

You will see on allometric scaling table that normal walking speed V is found empirically to scale with body mass as $V = 0.5M^{1/6}$ (m/s), i.e., as $V = aM^\alpha$ with $a=0.5$ and $\alpha=1/6$. I want you to derive this relation from simple physical principles, including an estimate of the amplitude a .

- (a) Estimate the amplitude a using the fact that an 80 kg adult walks naturally at about 5 km/hr. (Do not be upset if your estimate comes out a little larger than the above number)

Now, treat the leg as a simple pendulum and estimate a walking speed as the frequency of the pendulum times the step length.

- (b) Show that the walking speed scales as $V \sim M^{1/6}$. You will need to look up the frequency of a simple pendulum, and make this into a speed of stepping. The relationship will have lengths in it which you can then use our spherical human approximation to connect to mass.

Problem 3: Important Units & Scales.

1. In the cell, forces are picoNewtons (pN) and the length scales are nanometers (nm). A molecular motor name Kinesin has a mass of about 100000 Da (dalton). One $Da = 1.66 \times 10^{-27} kg$.
 - (a) Calculate the force of gravity on kinesin and express it in pN .
 - (b) Kinesin walks along microtubules and transports vesicles around the cell. The biggest force acting against it is viscous drag. kinesin can generate a maximum force of about 20 pN to overcome the drag forces that act against it. How many times its weight is this force? Think about this in comparison to how much the strongest humans can lift.
2. The most important energy scale inside a cell is k_bT , where k_b is Boltzman's constant ($k_b = 1.38 \times 10^{-23} J/K$) and T is room temperature, 295 K. Any energetic process that has an energy on the scale of k_bT can be thermally activated. Many important biological bonds are only a couple k_bT of energy.
 - (a) Calculate k_bT in terms of the familiar energy unit of Joules = 1kg.m.
 - (b) Convert your result in (i) to the unit of work/energy most useful for cell biology, $pN.nm$.

Problem 4: Estimating Numbers (Problems 2.3 & 2.1 from Physical Biology of Cell):

1. Sizing up the amount of carbon in an *E. coli*.
 - (a) Make an estimate of the number of carbon atoms needed to make an *E. coli*. Assume that 50% of the dry mass of *E. coli* is made up of carbon atoms. The dry mass of *E. coli* is 0.3 pg.
 - (b) *E. coli* are typically grown on minimal media which contains a starting concentration of glucose being 0.2 g/ 100 ml. Roughly how many *E. coli* cells can be grown on 5 ml of minimal media before the glucose runs out? The chemical formula for glucose is $C_6H_{12}O_6$. It has a molar mass of 180 g/mol.
2.
 - (a) Given that a typical bacteria (e.g. *E. coli*) is cylindrically shaped with a length $\sim 2 \mu m$ and diameter $\sim 1 \mu m$, show that it has a surface area $\sim 6 \mu m^2$ and a volume of $1 \mu m^3$. Express the volume in femtolitre. Make a corresponding estimate of the mass assuming it has the density of water.
 - (b) Roughly 2-3 kg of bacteria are harboured in your large intestine. Make an estimate for the total number of bacteria inhabiting your intestine. Estimate the total number of human cells in your body and compare the two results (assume that a human cell is spherical with a diameter $\sim 10 \mu m$).

Problem 5: Databases, databases,...

Google, “Protein Data Bank”. This will take you to the database that stores all protein structures that have been determined by x-ray crystallography or NMR.

- (a) How many structures are currently in the PDB databank?
- (b) What's the molecule of the month? what does it do?
- (c) For the molecule of the month, find its PDB identity, usually something like “1a2g”, and click on it. Try visualizing the protein using the 'QuickPDB' link. This should launch a Java program that allows you to rotate the molecule. How would you describe most of the secondary structure making up this protein?
- (d) What experimental method was used to determine its structure?
- (e) Download a molecular viewing program such as Rasmol or VMD. Then download the structure bdna.pdb from the course website. The structure is of double stranded DNA. Load into the viewer. Both viewers have documentation about how to measure bond lengths/distances that you will need to look up.

Answer the following questions about the structure:

1. the diameter of the double helix?
2. The along axis distance between base pairs
3. The distance for one full turn of the helix
4. Calculate from (2) and (3) the volume per base pair (compare with answer in Table 1.1 on p.26 of textbook).