Topic 9: Fluids and Swimming Low Reynold’s Number

(Chapter 12 in book – and Random Walks in Biology by H. Berg)
Overview

- What about non-diffusive transport in biology?
- Does water at the nano/micron scale behave the same way we experience it?
- What is viscosity?
- How do bacteria swim?
What determines whether flow is turbulent or laminar?

A: it’s not only the ‘thickness/stickiness’ of the fluid, but also the particles size and speed. We’ll see that a bacteria swimming in water is like us trying to swim in honey.
Consider a particle drifting down in a fluid under the influence of gravity

Because of drag, the force eventually balance and the particle attains a constant velocity

The three forces are:

\[ F_g = mg = \text{weight}, \quad F_B = m_w g = \text{buoyant force} \]

and for laminar fluids the drag force is,

\[ F_D = c \, v = \text{drag force, where } c \text{ is the ‘drag coefficient’} \]

@ equilibrium: \( F = 0 \), so

\[ mg - m_w g - cv = 0 \]
Drift velocity:

The particle achieves a constant drift velocity: \( v = \frac{(m - m_w)}{c} g \)

What is the nature of the drag coefficient? What does it depend on?

Clearly it depends on the particle size and on the properties of the liquid.

For a spherical particle moving in a viscous fluid, we have a famous result:

\[
C = 6\pi \eta R
\]

[\eta] = \text{Pa.s}

Stokes' formula is valid for a sphere in laminar flow.

It depends on size of object, \( R \) & viscosity, \( \eta \).
Drag on vesicles:

Vesicles experience a drag force since flow is laminar, you can use Stoke’s equation to make a reasonable estimate of the drag force.
Viscosity:

Viscosity is a measure of how sticky layers of fluid are to each other when a shear force is applied.

We find it’s hard to mix viscous fluids.

2 regimes: laminar flow and turbulent.

Laminar flow is reversible.

Turbulent flow is chaotic and not reversible.

Experiment: mixing a dye in glycerin.
Viscosity:

Consider the experiment on the left.

Q: how does the drag force of the moving plate depend on the parameters?

- Viscosity = shear drag force

\[ \frac{\mu}{d} \rightarrow \frac{\mu}{d} \rightarrow v_0 \]

- Consider the drag force experienced by sliding a plate of area \( A \) at speed \( v_0 \) over a liquid with viscosity \( \eta \).

- What does the drag force, \( f \), depend on?

\[ f \propto A \cdot f \propto v_0 \cdot \frac{\mu}{d} \rightarrow \text{less drag} \]
Viscosity: Critical force

So \[ f = \eta \frac{v_0 A}{d} \] (opposes motion)

- This allows us to define & measure viscosity.
- So 2 physical properties that can specify a fluid: viscosity, \( \eta \) & density, \( \rho \).
- Dimensional analysis: using just \( f \) & \( \eta \) we can define a quantity that has units of force:

\[ f_c = \frac{\eta^2}{\rho} \]

Proof: \[ [f] = \left[ \frac{ML}{LT^2} \right]^2 / \left[ \frac{M}{L^3} \right] = \frac{M^2 L^2}{L^2 T^2} \frac{M}{M} \frac{L}{T^2} = N \]
Viscosity: Critical force in biology

- Can create a dimensionless number to characterize "thickness": \( \frac{f}{f_c} \) << 1 \( \rightarrow \) thick laminar flow

- \( \frac{f}{f_c} \) >> 1 \( \rightarrow \) turbulent flow

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( \eta ) (Pa(\cdot)s)</th>
<th>( f_c ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>air</td>
<td>1</td>
<td>2\times10^{-5}</td>
<td>4\times10^{-10}</td>
</tr>
<tr>
<td>water</td>
<td>1000</td>
<td>0.0009</td>
<td>8\times10^{-10}</td>
</tr>
<tr>
<td>corn syrup</td>
<td>1000</td>
<td>5</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Water: Water will be viscous for forces \( \sim 8\times10^{-10} \) N = \( \frac{1}{10} \) nN

Biology: For a cell, forces \( \sim 1 \) pN \( \rightarrow \) water is viscous!

so cells experience water in the laminar regime – clearly very different than how we experience H2O
Reynold’s Number:

- Previous section showed that the character of a fluid depends on the size of the external force and changed at \( Re = \frac{V^2 L}{\nu} \).

- We can say more though. Consider an object in a fluid — will the flow be laminar or turbulent?

- The liquid experiences 2 forces:
  1. Inertial force due to colliding @ sphere
  2. Viscous drag force

  Idea: If inertial force > viscous force → turbulent

  If viscous force > inertial force → laminar

  \[
  \text{inertial force} = \frac{1}{\frac{v}{L}} \frac{\frac{V}{L}}{R}
  \]

  \[
  \text{viscous force} = \frac{\nu}{L}
  \]

  \[
  \text{Re} = \frac{v L}{\nu}
  \]

  \[
  = \text{Reynold's number.}
  \]
Some #’s:

\[ R = \frac{\nu R_p}{\eta} \]

\[ \text{e.g.: Swimming whale: } R \approx 10 \text{ m } \text{ls} \]

\[ R = \frac{(10 \text{ m/s})(10 \text{ m})(1000 \text{ kg/m}^3)}{0.001 \text{ Pa s}} = 10^8 \gg 1 \]

\[ \rightarrow \text{ turbulence} \]

Swimming bacteria: \( R = 10^{-6} \text{ m } \text{ls} \)

\[ R = \frac{(3 \times 10^{-6})(10^{-6})(1000)}{0.001} = 3 \times 10^{-5} \ll 1 \]

\[ \rightarrow \text{ laminar & dominated by friction.} \]
Can you swim in honey???

What determines whether flow is turbulent or laminar?

Reynold’s number = \( Re = \frac{\text{(liquid density)} \times \text{(size)} \times \text{(speed)}}{\text{(viscosity)}} \)

So it also depends on how big you are and how fast you are swimming through the liquid!

\( Re > 1000 \)  = Turbulent \hspace{1cm} \( Re < 100 \)  = laminar
Swimming: turbulent or laminar?

Swimming in water:
speed ~ 1 m /s ; size ~ 1 m

\[ Re \sim 1 \times 10^5 = \text{turbulent} \]

Swimming in honey:
Honey viscosity ~ 10000 x water

\[ Re \sim 10 = \text{laminar} \]

Bacteria swimming in water:
Speed ~ 30 x 10^{-6} m/s ; size ~ 1 x 10^{-6} m

\[ Re \sim 1 \times 10^{-5} = \text{laminar} \]

So bacteria swimming in water is like us in honey.
If you kick your feet in honey (as you would in water) will you move???

Symmetric, back-and-forth motion
Swimming strategies I: symmetric motion fails at low Re

- How do bacteria swim in such a liquid when the flow is laminar?

- Consider pushing with paddle motions

- Frictional

- Move paddles relative to body with speed $u$

\[ f_r = c_r (u - v) \]

- Liquid pushes on paddle with force $f_r = c_r (u - v)$

- Drag on bacteria $f_u = c_u u$

- Since moving with constant velocity $\Rightarrow f_r = f_u$

\[ u = \frac{c_r v}{(c_r + c_u)} \]

- Distance moved: $\Delta x = u \cdot \Delta t$
Swimming strategies I: symmetric motion fails at low Re

Proof: Consider a backstroke with the paddle moving with $v'$ for a time $dt'$, so the backward velocity of the bacteria is $u' = c_1 \frac{v'}{(c_1 + c_2)}$, but $v' dt' = v dt$, since the paddles have to return to the same spot.

So the distance travelled is $dx' = -u' dt = -c_1 \frac{v dt}{(c_1 + c_2)} = -u dt = -dx$

hence $dx' = -dx$ and you don’t go anywhere
So performing symmetric swimming motion will get you no where in honey.

How to swim in honey?? Need to perform asymmetric motion.

Many bacteria swim by using a helical propeller that exploits asymmetry in drag forces.

Key: viscous drag force depends on shape.

See Life at Low Reynold's Number, Berg and Purcell
Drag and the shape of the paddle:

\[ f_1 = c_{1} v_{1} \quad v_{1} \quad f_{n} = c_{n} v_{n} \]

Find: \[ c_{\parallel} < c_{\perp} \]

Bacteria with cilia:

\[ f_{\perp} \sim c_{\perp} v_{\perp} \quad f_{n} \sim c_{n} v_{n} \]

Power \quad backstroke

\[ f_{\perp} > f_{n} \Rightarrow \text{net force forward!} \]
E. coli have a helical flagellum.

Q: How does a helix allow it to swim at low Reynold’s number?
Forces on rotating helix:

First consider the drag on a rod which is being pulled.

Because \( \frac{f_T}{f_n} = \frac{c_T v}{c_n v_n} \neq \frac{v_T}{v_n} \)

Key: So \( f \) is not in the same direction as \( \vec{v} \).

What implication does this have for a helical flagellum?
Forces on rotating helix:

- [Equation or description of forces acting on the helix, possibly including vector diagrams and equations related to centrifugal and centripetal forces, lift, drag, and perhaps a reference to Newton's laws of motion or fluid dynamics.]
Force on bacterium:

- This net force propels bacteria

![Diagram of bacterium with RH helix and CCW rotation]

![Diagram of bacterial movement and helical bundle]

![Diagram of tumbling and random reorientation]

![Diagram of CW rotation]
E. coli switch the frequency of CCW (run) vs CW (tumble) rotation of their flagella based on their sensing of food in the environment = a biased random walk (i.e. they step more frequently in the direction of the food)

Q: how can something the size of a micron where diffusing mixes things ~ milliseconds measure a spatial gradient?

A: they store a memory of the signal and take a derivative = spatial gradient using time
Things moving in a fluid experience a drag force

This drag force depends on whether flow is turbulent or laminar.

Viscosity describes a fluid’s resistance to shearing its layers.

The Reynold’s number dictates the type of flow and depends on viscosity, the fluid’s density and the size and motion of the particle.

Cells experience life at low Reynold’s number.

They have to use asymmetric motion in order to swim.

Looked at how a helix can generate the necessary asymmetry to generate a forward force in a low Reynold’s number environment.